

Mechanics II

CHAPTER 8

Surface Tension and Viscosity

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8

Surface Tension and Viscosity

INTRODUCTION

In this chapter, we will discuss two important properties of liquid, i.e. (i) surface tension and (ii) viscosity. Surface tension is a special property of free surface of liquid. This is because of intermolecular forces of attraction by the virtue of which any free surface of a liquid behaves as a stretched membrane. This helps us to understand the walking of insects, floating of leaves etc., on water surface. Here we will also discuss the effect of surface tension in capillary action. The capillary action explains how the water rises from the inner core to the earth's surface and how the roots of the trees soak water from the soil, how the oil lamp glows by soaking oil by capillary action of the wick, how capillary action takes place in bloating paper, perspiration, water purifier/filter, etc.

Then the second property called viscosity arises from the haphazard (random) molecular motion between the liquid layers. This helps us to understand the real motion of liquid and gases in pipes, motion of water in channels and rivers, motion of rain drops in air, settlement of sediments in water, etc.

SURFACE TENSION

In gravity free space it is found that liquid always takes the shape of a spherical drop. It is because, for a given volume, a sphere has the least surface area. It means that the surface of every liquid has always a tendency to have least surface area and in this respect, it behaves like a stretched membrane having a tension in all directions parallel to the surface. This tension in the surface of a liquid is called surface tension.

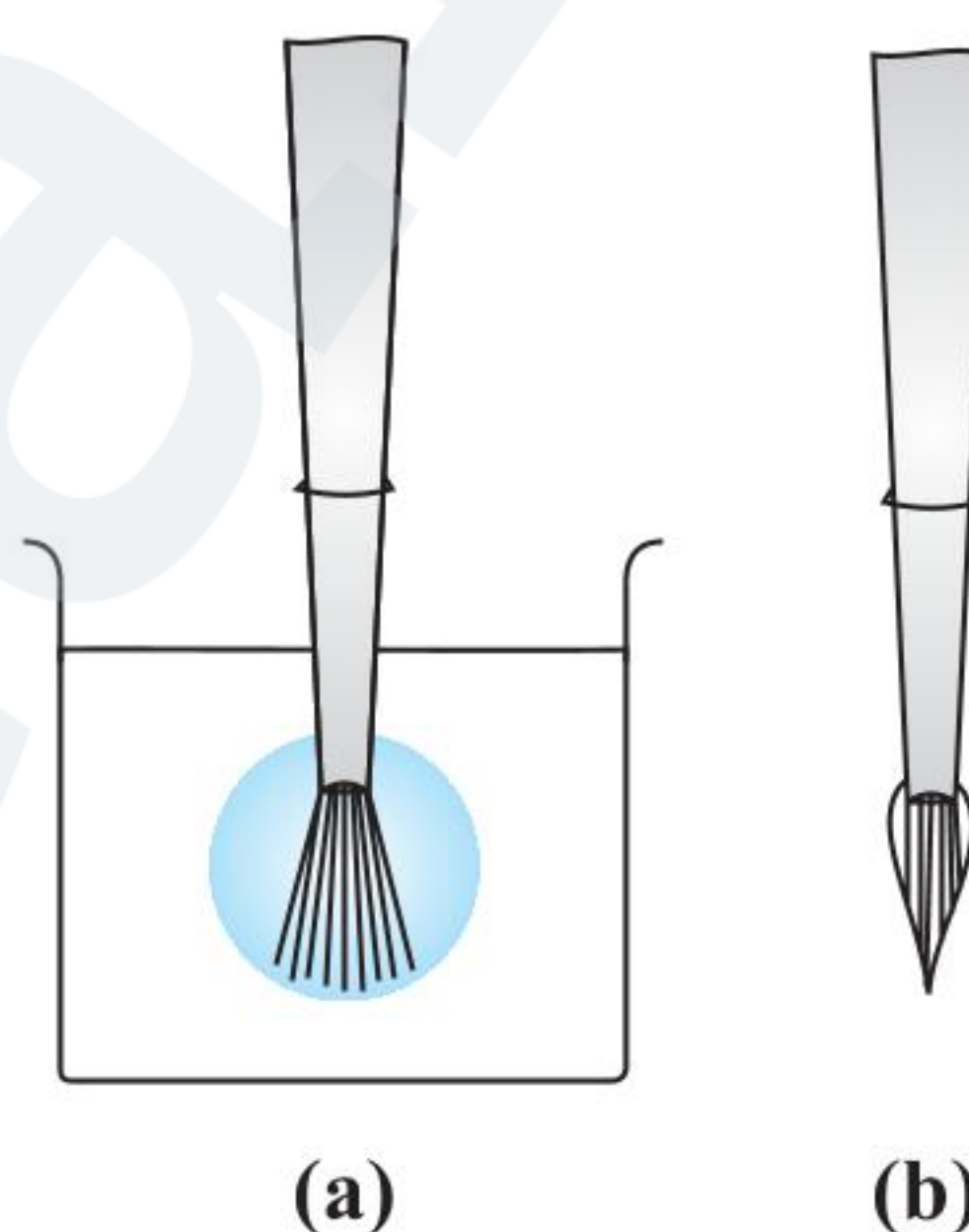
Thus, the property of a liquid at rest by virtue of which the free surface of a liquid at rest behaves like a stretched elastic membrane and hence has a natural tendency to occupy as small an area as possible is called surface tension.

SOME OBSERVED PHENOMENA BASED ON SURFACE TENSION

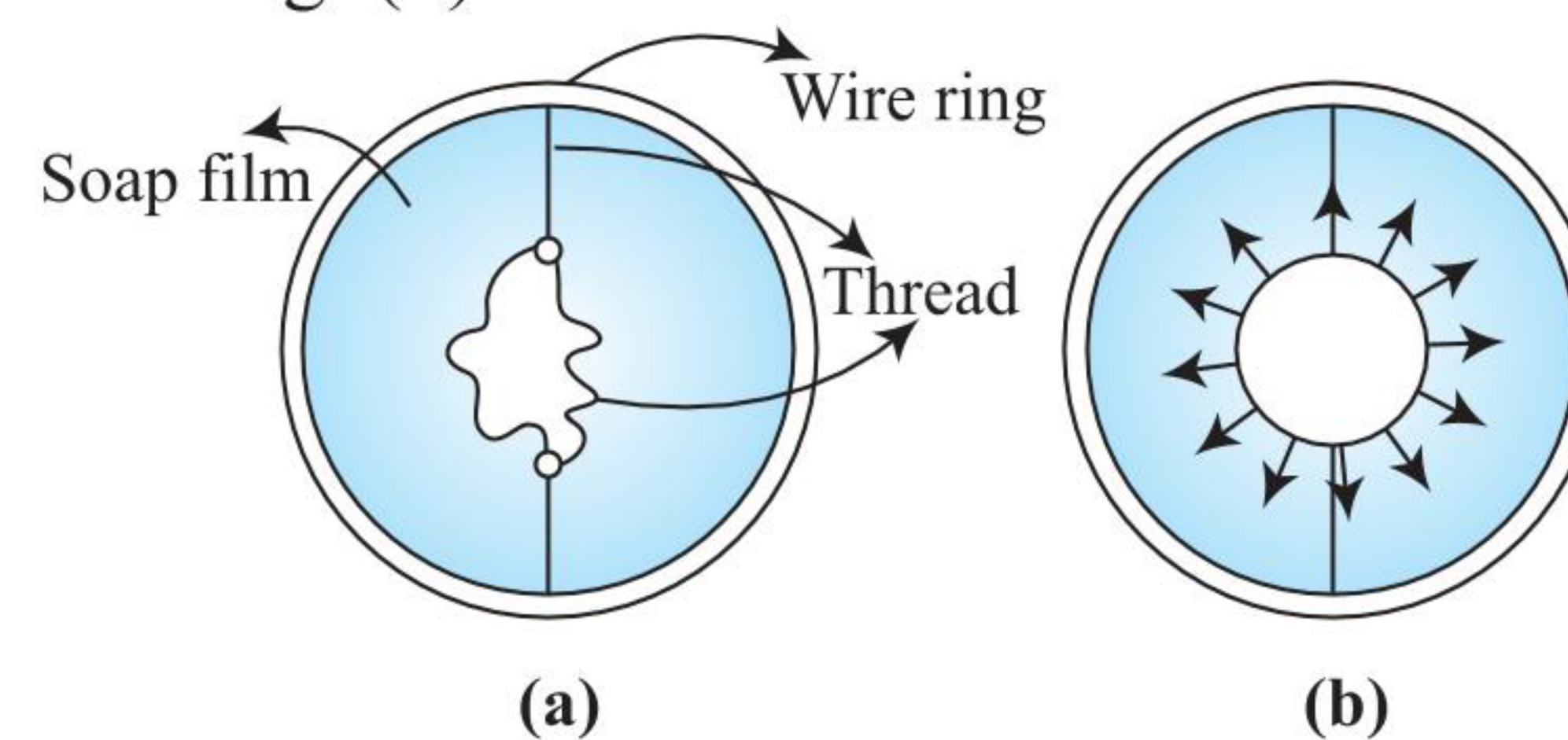
The following experiments demonstrate the property of surface tension.

1. Insects, like mosquitoes, move about freely on the surface of ponds without sinking. This is on account of the support they get from the stretched surface film of water.
2. If we gently place a greased needle on the free surface of water at rest. We shall find that the needle does not sink. A careful observation shows that the surface of water below the needle gets slightly depressed. This shows that the free surface of the liquid layer supports the needle in the same way as a stretched rubber membrane does.

3. Hair of a brush, when dipped in water spread out [Fig. (a)]; but as soon as it is taken out, its hair cling together as shown in Fig. (b).



4. When a wet loop of cotton thread is placed on a film of soap solution. Let us trap a thin soap film on a circular wire by dipping it into a soap solution and taking it out carefully. On placing a wet loop of cotton thread on the film, it is found to lie as it is and has no tendency to change its form as shown in Fig. (a). Let the film inside the loop be now pricked with a hot needle end. The loop at once takes up a circular form and the film inside it disappears as shown in Fig. (b).



The analogy of an elastic membrane to the free liquid surface is not wholly correct. In case of an elastic membrane, the tension increases with stretching and is directly proportional to it within elastic limits. But a liquid surface experiences a constant tension.

TYPES OF MOLECULAR FORCES

If we dip a clean glass rod in mercury bath, it is found that mercury does not stick to the glass rod. On the other hand, if we dip a glass rod into water, some of the water sticks to the rod. For explaining this we need to learn about intermolecular forces. The intermolecular forces of attraction may be classified as follows:

1. **Cohesion:** The force of attraction between molecules of the same substance is called the force of cohesion. The force of cohesion is maximum in solids, lesser in liquids and least in gases. It is for this reason that the solids have definite shape and resist all deforming forces.

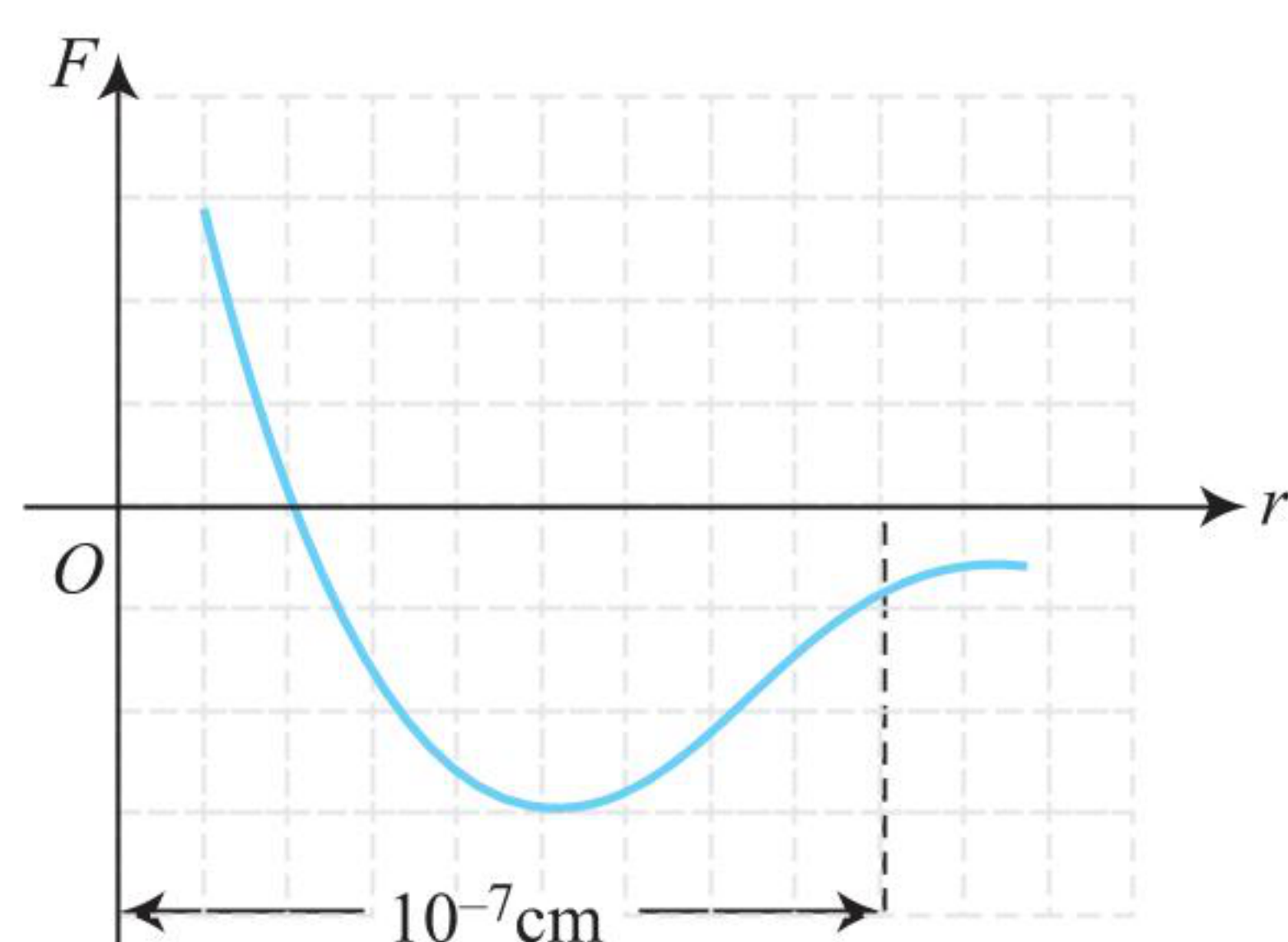
2. **Adhesion:** The force of attraction between the molecules of the different substances is called the force of adhesion. The examples of adhesion are the sticking of glue to the wood, paint to the wall, starch to the fingers, chalk particles to the black board, water to its container, etc.

Now, we can explain why mercury does not stick to the glass rod. The reason is that the force between two molecules of mercury (cohesive force) is greater than the force between a mercury molecule and the molecule of the glass (adhesive force). Also, the water wets the glass rod is explained by the fact that the force between two water molecules (cohesive force) is quite small as compared to the force between a water molecule and a glass molecule (adhesive force).

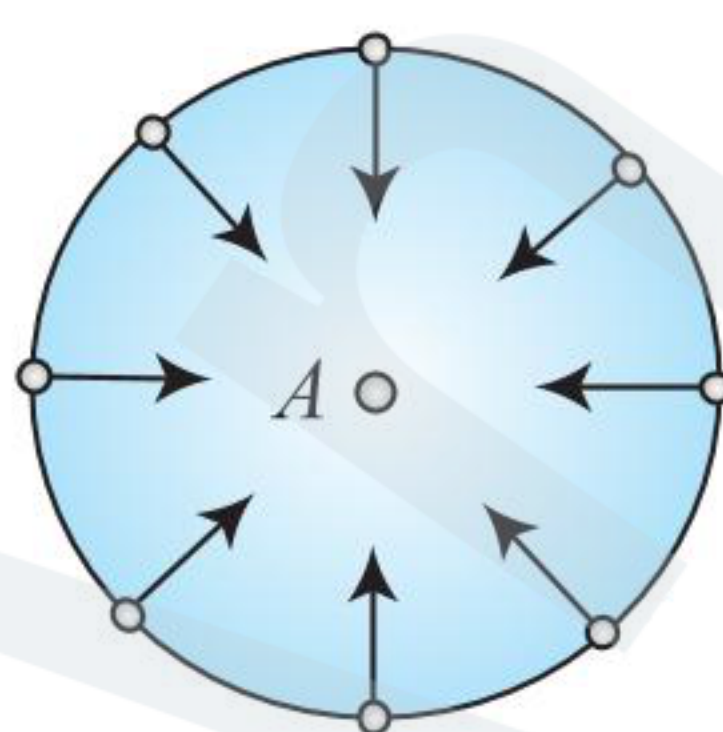
MOLECULAR THEORY OF SURFACE TENSION

For understanding the phenomenon of surface tension, first we need to have some idea about the intermolecular forces, which operate to keep the liquid molecules together.

Sphere of influence: To understand the cause of surface tension we need to understand the molecular interaction of molecules at different places inside a liquid. Referring to the F - r diagram we find that the force of attraction between any two molecules decreases to a negligible value at $r = 10^{-7}$ cm.

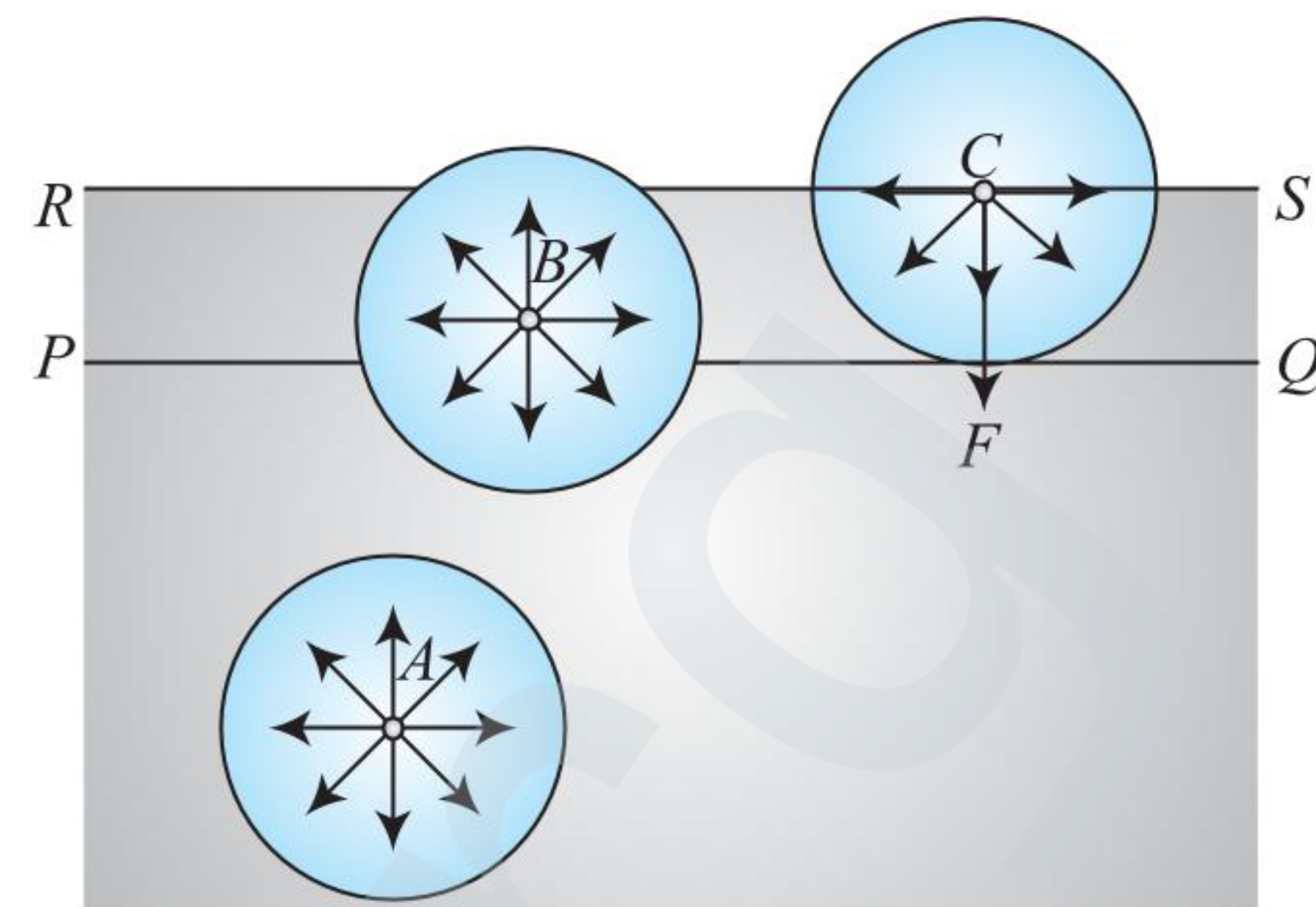


The cohesive force is effective up to a certain distance called **molecular range** (of the order of 10^{-7}). With the molecule as centre and molecular range as radius, if a sphere is drawn, it is called the **sphere of influence**. This is due to the reason that all molecules lying in this sphere are attracted by the molecules at the centre and vice versa. The uppermost layer of the liquid with thickness equal to the molecular range is called **surface film**.



Force on different molecules: Let $PQRS$ be a surface film of a liquid contained in a glass trough as shown in the figure. Now, consider three molecules A , B , and C ; the sphere of influence of the molecule A lies completely inside the liquid. Hence, all surrounding molecules within the sphere of influence attract the molecule equally in all directions. Consequently, the net force acting on the molecule A is zero. It means that the molecule A is free to move inside the liquid with its thermal speed. Now come to the molecule B which is situated little below the liquid surface

such that its sphere of influence is mostly lying inside the liquid. There are few molecules more below the molecule B than above that molecule.



Hence, the net downward force is little bit more than the net upward force of the molecules lying inside the sphere of influence on the molecule B . Then, the molecule B experiences a little downward pull.

The molecule C lies on the top of the surface film. The upper half of its sphere of influence lies completely outside the liquid and contains no liquid molecules in it except a few gases or vapour molecules which exert a negligible upward force on C . The molecule C , therefore, experiences maximum downward force.

Hence, we conclude that no resultant force acts on a molecule so long as it lies below the surface film. But for all the molecules lying in the surface film, there is a resultant downward force of attraction. This force goes on increasing as we move from the bottom (PQ) of the surface film to its top (RS). This means that to take a molecule into the surface film from anywhere below it, some work is to be done. This work is stored in the film as the potential energy of the film. Therefore, molecules on the free surface of a liquid have potential energy. Further, we know that any system, when free, tends to have minimum potential energy. Therefore, for minimum potential energy of the surface film, the number of molecules in it should be minimum. For this, the surface film should occupy minimum volume which would be so if the film has the least surface area as its thickness is fixed.

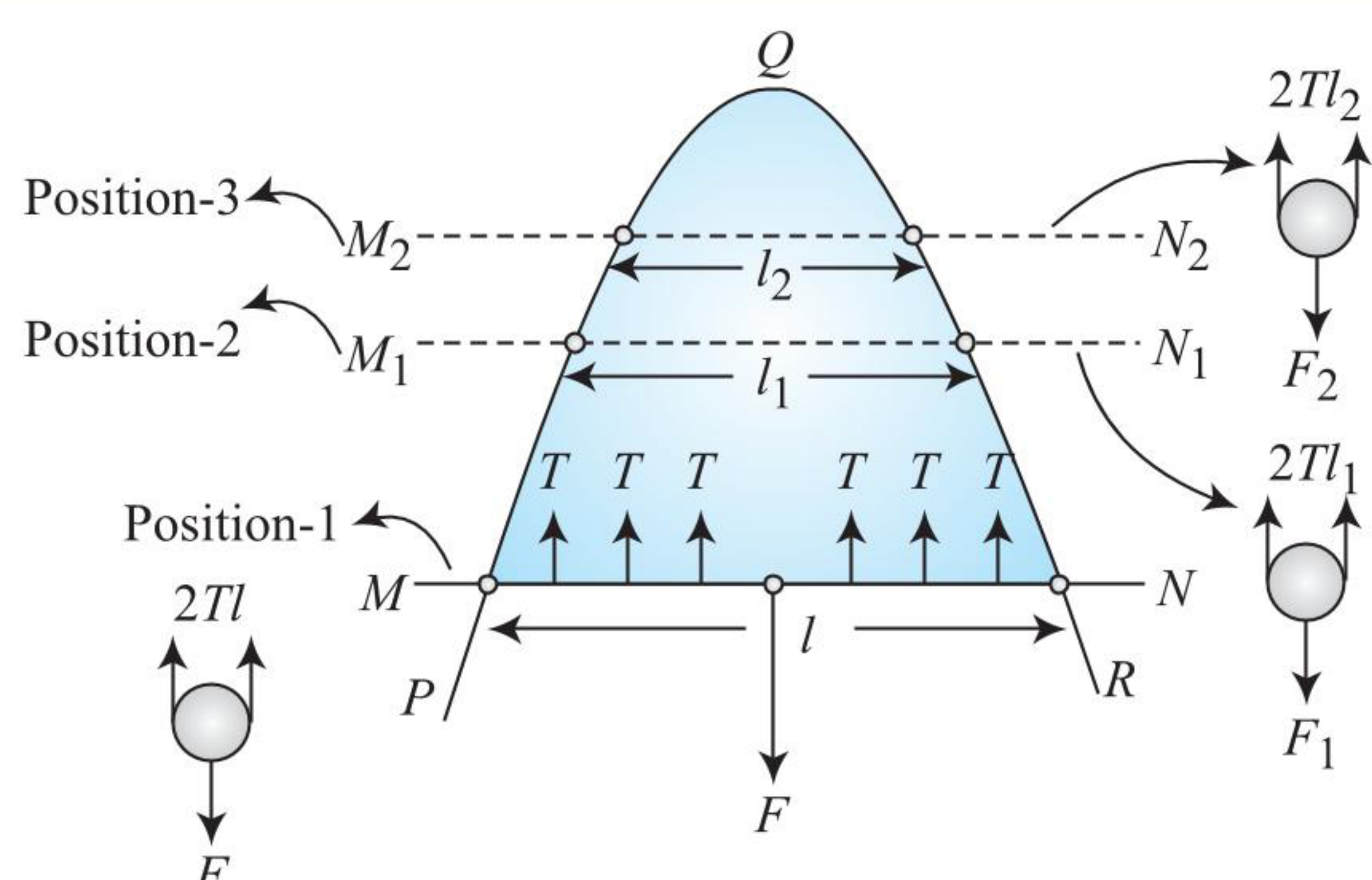
Thus, we observe that for a liquid at rest, the area of its surface film should be as small as possible. In trying to acquire least area, the surface film tends to contract and hence behaves as a stretched membrane.

Under the action of surface tension forces, the free surface of a liquid tends to have the least area for a given volume of liquid in order to minimize its potential energy. Since a sphere has minimum area for a given volume, therefore, rain drops and small droplets of mercury are approximately spherical in shape.

HOW TO DEFINE SURFACE TENSION

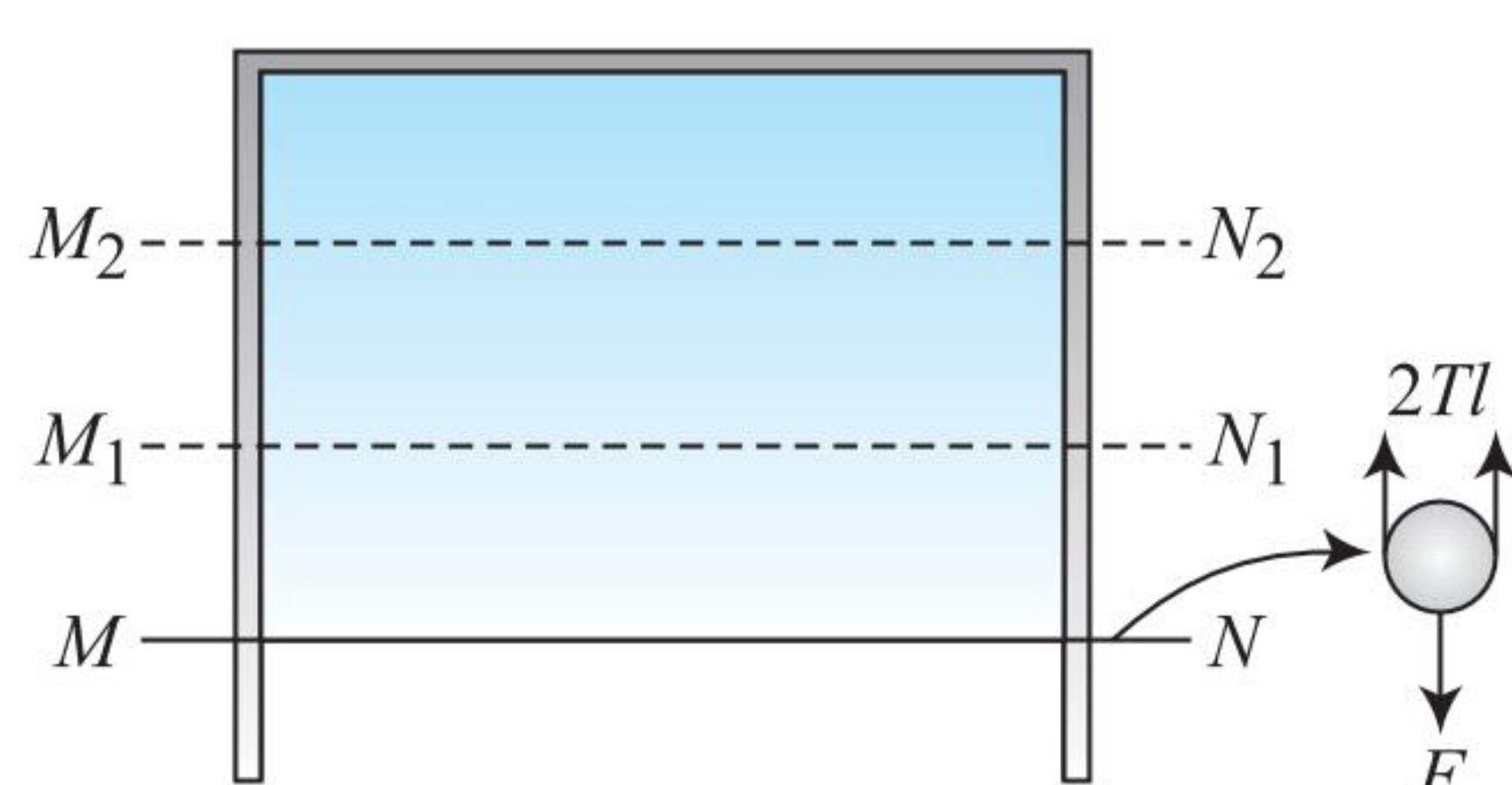
Let us consider a liquid film of the shape shown in the figure formed between a bent wire PQR and a metal wire MN .

The film has a tendency to contract and as such it exerts a pull on the wire MN tending to move it upwards. Obviously, to keep the wire MN in equilibrium, an equal force F has to be applied as shown in the figure. This clearly shows that there acts a force of tension at the boundary of the liquid film and it acts inwards along its surface.



If we repeat this experiment at constant temperature by changing the length of the wire MN in contact with the film. This can be done by changing the position of MN to M_1N_1 , M_2N_2 ... etc. We shall find that the force (F_1) required to keep the wire in equilibrium while at position M_1N_1 is greater than the force (F_2) required to keep it in equilibrium while at M_2N_2 . Further, both F_1 and F_2 are less than F . Thus, it is obvious that F increases with the increase in length of the wire MN in contact with the film.

If we repeat the same experiment with a rectangular shape, we find that F is independent of the area of the soap film.



In the figure regardless of the position of the wire MN , F does not change. Thus, when the wire MN is moved to M_1N_1 , though the area of the film increases, F remains the same. Thus, it should be borne in mind that F changes because of a change in length (l) and not on account of the change in the area of the film. In other words, F is directly proportional to l . Thus,

$$F \propto 2l \text{ or } F = T \times 2l \quad \dots(i)$$

Factor 2 has been introduced because the film has two free surfaces.

Here, T is a constant of proportionality and is called the coefficient of surface tension or simply the surface tension of the liquid. Further, force of surface tension does not act only on the boundary line but on both sides of any other imaginary line, XY on the surface film, in a direction perpendicular to the line and tangential to the liquid surface.

$$\text{As } F = T \times 2l, \quad T = \frac{F}{2l} = \text{force/length}$$

Hence, the surface tension of a liquid can be measured as the force per unit length on an imaginary line drawn on the liquid surface, which acts perpendicular to the line on its either side at every point and tangentially to the liquid surface.

$$T = \frac{\text{Total force on either side of the imaginary line (F)}}{\text{Length of the line (l)}}$$

Since the direction of surface tension is perpendicular to any imaginary line, surface tension has no preferred direction. Hence, we can treat the surface tension T as a scalar, whereas force is

a vector. SI unit of surface tension is N/m and its dimensional formula is $[MT^{-2}]$.

Important Points:

- Surface tension acts over the free surface of a liquid only and not within the interior of the liquid.
 - At the boundary, the surface tension acts only on one side, i.e., the liquid side. But at any other point of the surface, it acts on either side of the imaginary line.
 - The surface tension of a liquid is given as the linear density of surface tension force (cohesive force along the liquid surface) which is given as $T = \frac{F}{l}$ (or $\frac{dF}{dl}$).
- T is constant at a given temperature.
- The magnitude of T depends on the temperature of the liquid and on the medium on the other side of the free surface. The surface tension of a liquid decreases by increasing temperature.
 - The surface tension of a liquid decreases by adding impurities.

DETERMINATION OF COEFFICIENT OF SURFACE TENSION

One of the commonly used methods for the determination of coefficient of surface tension is shown in Fig. (a). This method involves the measurement of a force F needed to pull a platinum ring through the surface of a liquid. The ring forms one arm of a balance while weights are added to the other side. The total length of the liquid surface to be broken is twice the circumference of the ring as there are two surfaces to the film [Fig. (b)]. Thus, if r is the radius of the ring, the additional force needed for balance will be

$$F = 2(2\pi r)T = 4\pi rT \text{ or } T = \frac{F}{4\pi r} \quad \dots(i)$$

Thus, knowing F and r ; T can be calculated. It is to be noted that surface tension depends upon temperature and as such temperature must be kept constant throughout the experiment.

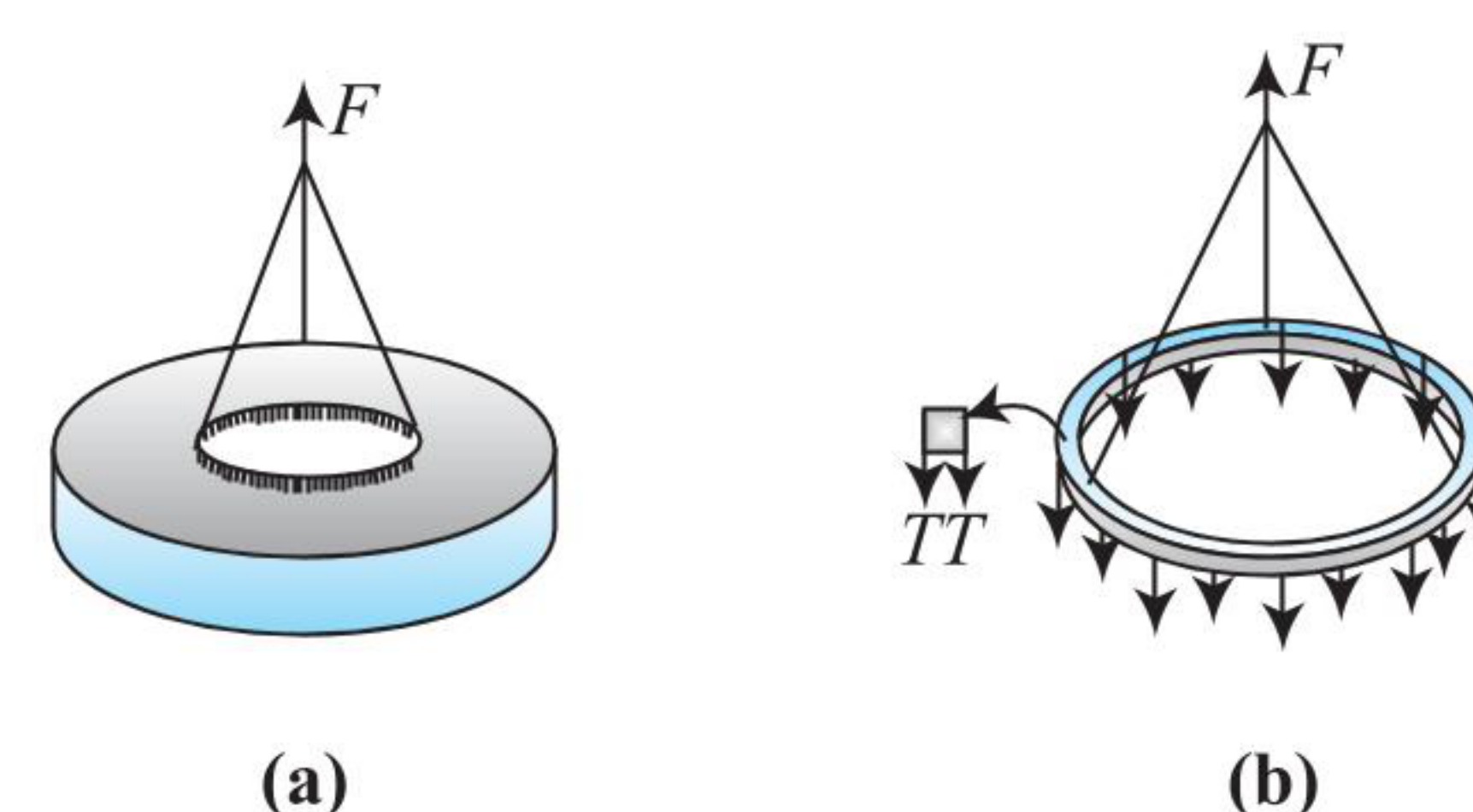


ILLUSTRATION 8.1

A wire ring of 5 cm radius is rested on the surface of a liquid and then raised. The pull required is 3.14 g more before the film breaks than it is after. Find the surface tension of the liquid.

Sol. When the wire ring is resting on the liquid surface, a thin film of liquid is in contact with the ring. Force due to surface tension acts along the circumference. Therefore, the force required to lift the ring is equal to the sum of the weight of the ring and the force due to surface tension. Hence, the extra pull

required to break the film is equal to the force on the ring due to surface tension, i.e.

Pull required to break the film = Force on the ring due to surface tension

Here, pull required to break the film = $3.14 \text{ gf} = 3.14 \times 1000 \text{ dyne}$ and radius of the ring, $r = 5 \text{ cm}$

The liquid is in contact with the ring both along its inner and outer circumferences.

Therefore, force on the ring due to surface tension

$$= 2(2\pi rT) = 2 \times 2\pi \times 5 \times T = 20\pi T$$

Hence, $20\pi T = 3.14 \times 1000$

$$\text{Or } T = \frac{3.14 \times 1000}{20\pi} = 50.0 \text{ dyne cm}^{-1}$$

ILLUSTRATION 8.2

A U-shaped wire loop having a slider (of negligible mass) is dipped in a soap solution and then removed. A thin soap film is formed between the wire and the light slider. It is found that the film can support a weight of 0.006 N , before it will break. If the length of the slider is 0.1 m , what is the surface tension of the film?

Sol. Due to surface tension, the soap film tries to make its surface area minimum. As it has two free surfaces, the total force due to surface tension on the slider will be

$$F = 2(T \times l)$$

In equilibrium, total force due to surface tension on the slider will be equal to its weight, i.e.

$$2(T \times l) = Mg$$

Here, $Mg = 0.006 \text{ N}$; $l = 0.1 \text{ m}$

$$T = \frac{Mg}{2l} = \frac{0.006}{2 \times 0.1} = 0.03 \text{ N m}^{-1}$$

ILLUSTRATION 8.3

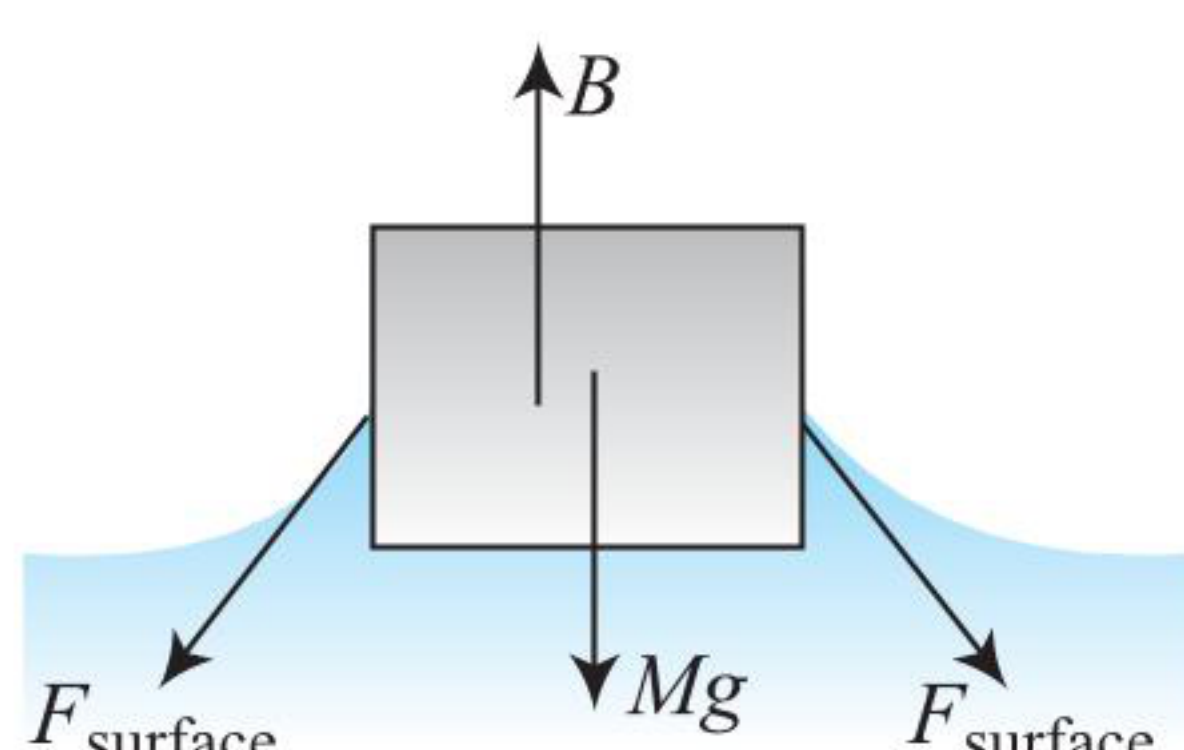
A cube of mass $m = 800 \text{ g}$ floats on the surface of water. Water wets it completely. The cube is 10 cm on each edge. By what additional distance is it buoyed up or down by surface tension? Surface tension of water = 0.07 N m^{-1} .

Sol. If surface tension is neglected the condition for floating gives

$$800 \times 10^{-3} g = (0.1^2 x \rho) g \text{ or } x = 0.08 \text{ m}$$

$$(\rho = 1000 \text{ kg m}^{-3} \text{ for water})$$

Since water wets the cube, the angle of contact is zero and force of surface tension acts vertically downwards. So, it is buoyed down by surface tension.



$$\therefore 800 \times 10^{-3} g + 4 \times 0.1 \times 0.07 = (0.1^2 x' \rho) g$$

$$\text{or } x' = 0.08 + \frac{0.028}{98} = 0.08 + 2.8 \times 10^{-4}$$

Therefore, the additional distance = $2.8 \times 10^{-4} \text{ m}$

ILLUSTRATION 8.4

Find the maximum possible mass of a greased needle floating on water surface.

Sol. Let the mass of the needle be m . As the liquid surface is distorted, the surface tension forces acting on both sides of the needle make an angle θ , say, with vertical. Since the forces acting on the needle are F , F and mg , resolving the forces vertically for its equilibrium, we have

$$\sum F_y = F \cos \theta + F \cos \theta - mg = 0$$

$$\text{This gives } m = \frac{2F \cos \theta}{g}$$

where $F = Tl$

$$\text{Then } m = \frac{2Tl \cos \theta}{g}$$

For m to be maximum, $\cos \theta = 1$

$$\text{Hence, } m_{\max} = \frac{2Tl}{g}$$

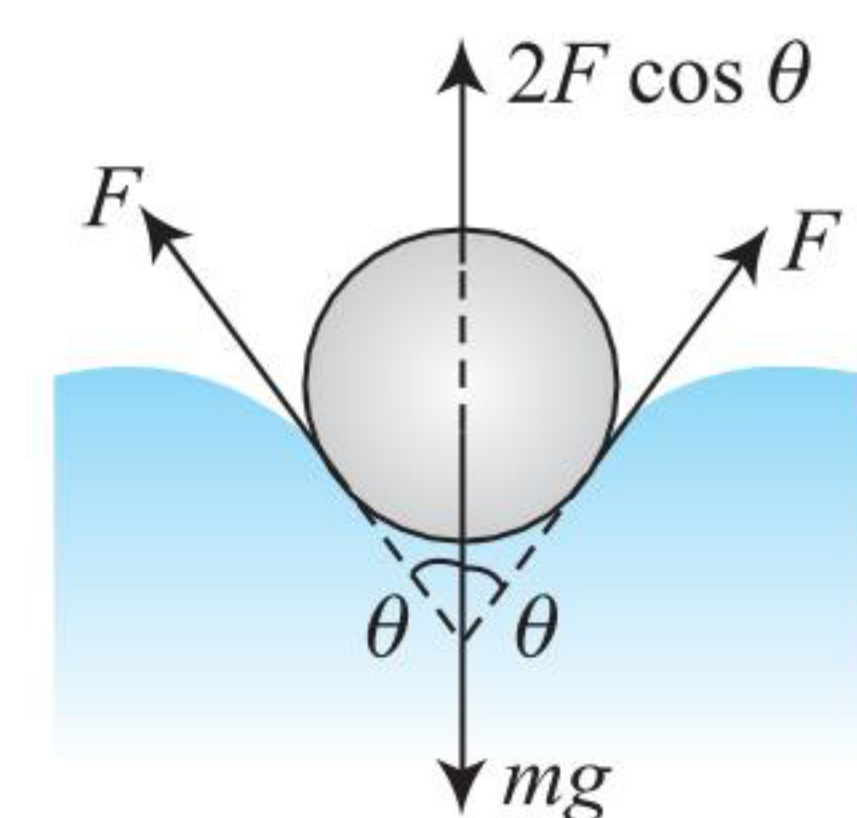
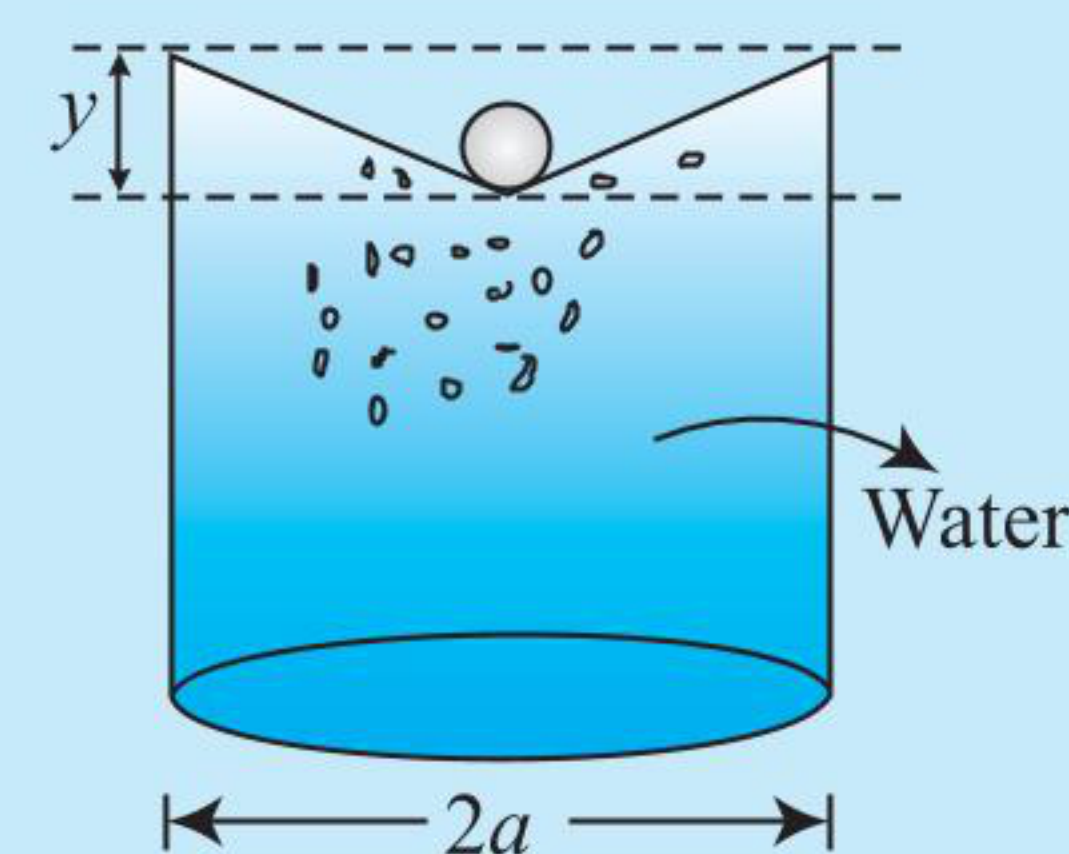
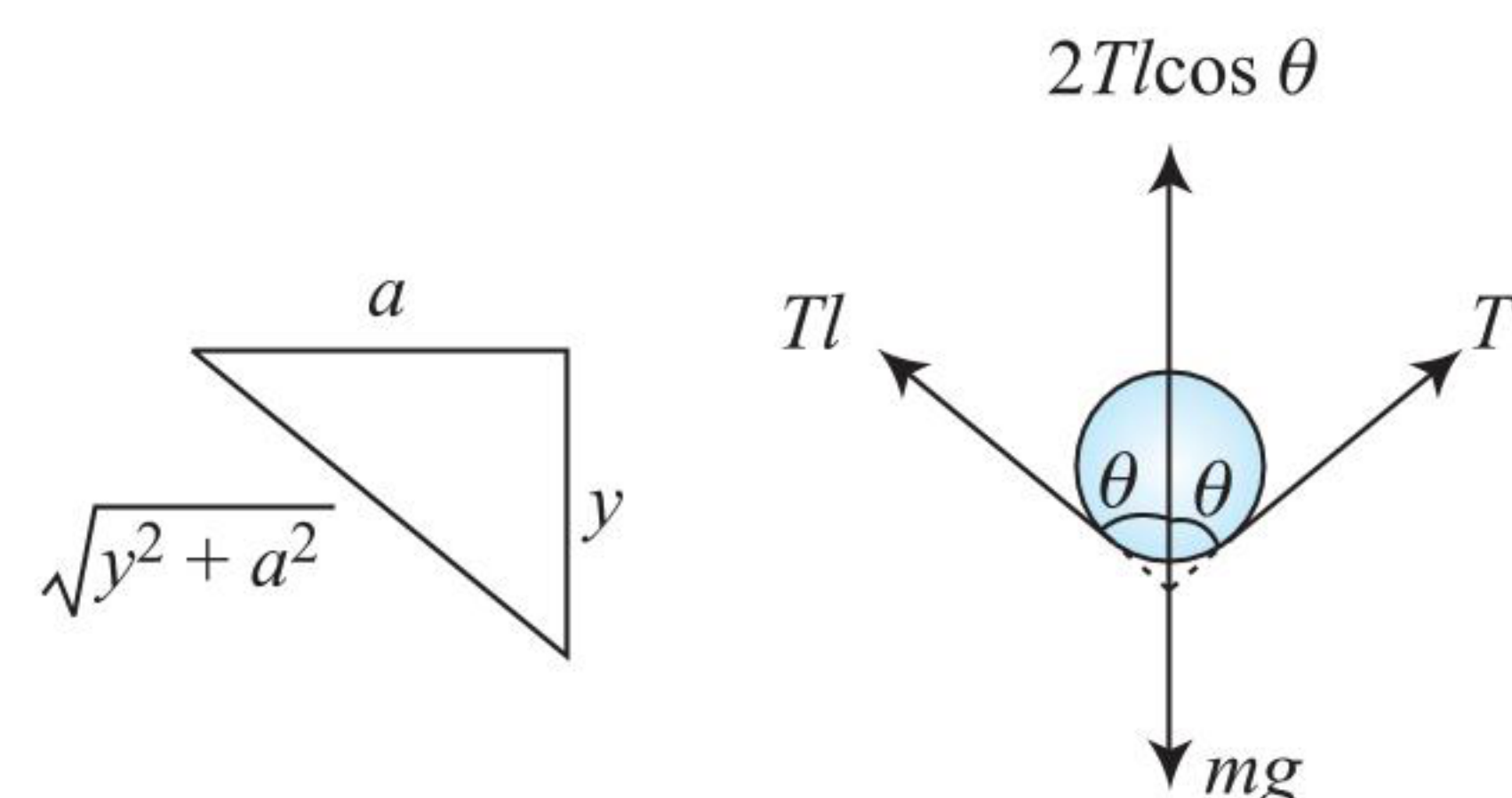


ILLUSTRATION 8.5

A container of width $2a$ is filled with a liquid. A thin wire of weight per unit length k is gently placed over the liquid surface in the middle of surface as shown. As a result the liquid surface is depressed by a distance y ($y \ll a$). Determine the surface tension of the liquid.



Sol. The free-body diagram of wire is shown in the figure. Let l be the length of wire. If λ is mass per unit length of wire, the weight of wire = $(l\lambda)g$ (acting vertically downward).



Vertical force due to surface tension = $2Tl \cos \theta$ (upward)

Therefore, for vertical equilibrium $2Tl \cos \theta = (l\lambda)g$

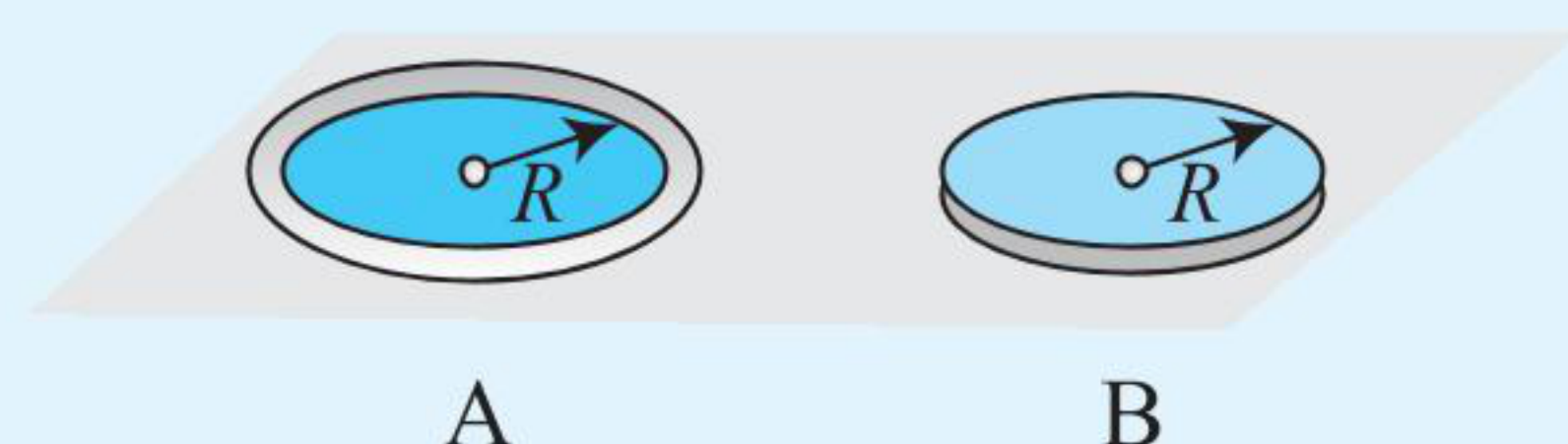
$$T = \frac{\lambda g}{2 \cos \theta}$$

From the figure, $\cos \theta = \frac{y}{\sqrt{y^2 + a^2}} = \frac{y}{a}$ (for $y \ll a$ given)

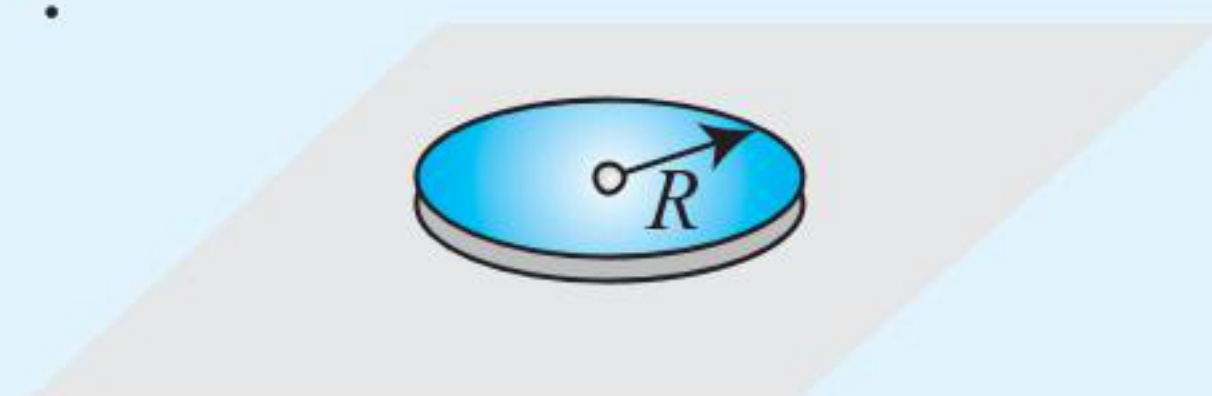
$$T = \frac{\lambda g}{2\left(\frac{y}{a}\right)} = \frac{\lambda a g}{2y}$$

CONCEPT APPLICATION EXERCISE 8.1

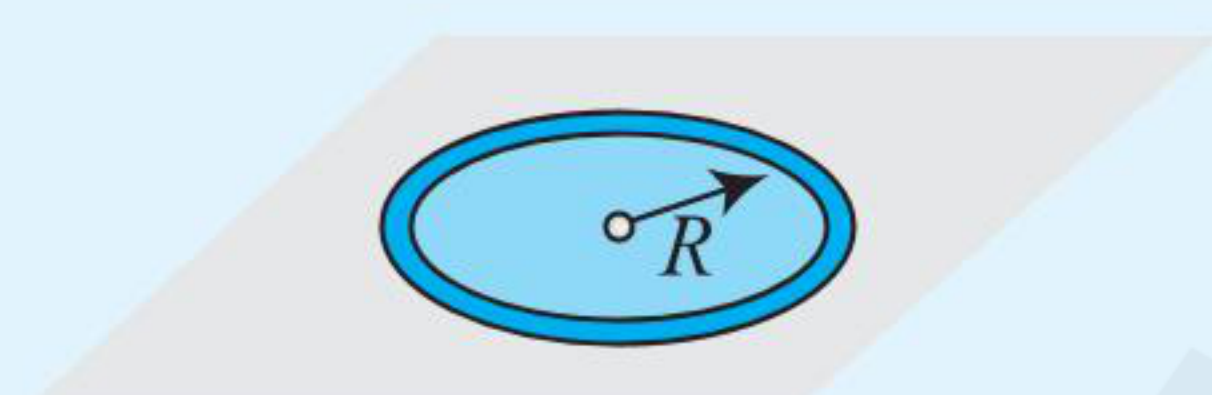
1. A rigid ring *A* and a rigid thin disk *B* both made of same material, when gently placed on water, just manage to float due to surface tension as shown in the figure. Both the ring and the disk have same radius. What can you conclude about their masses?



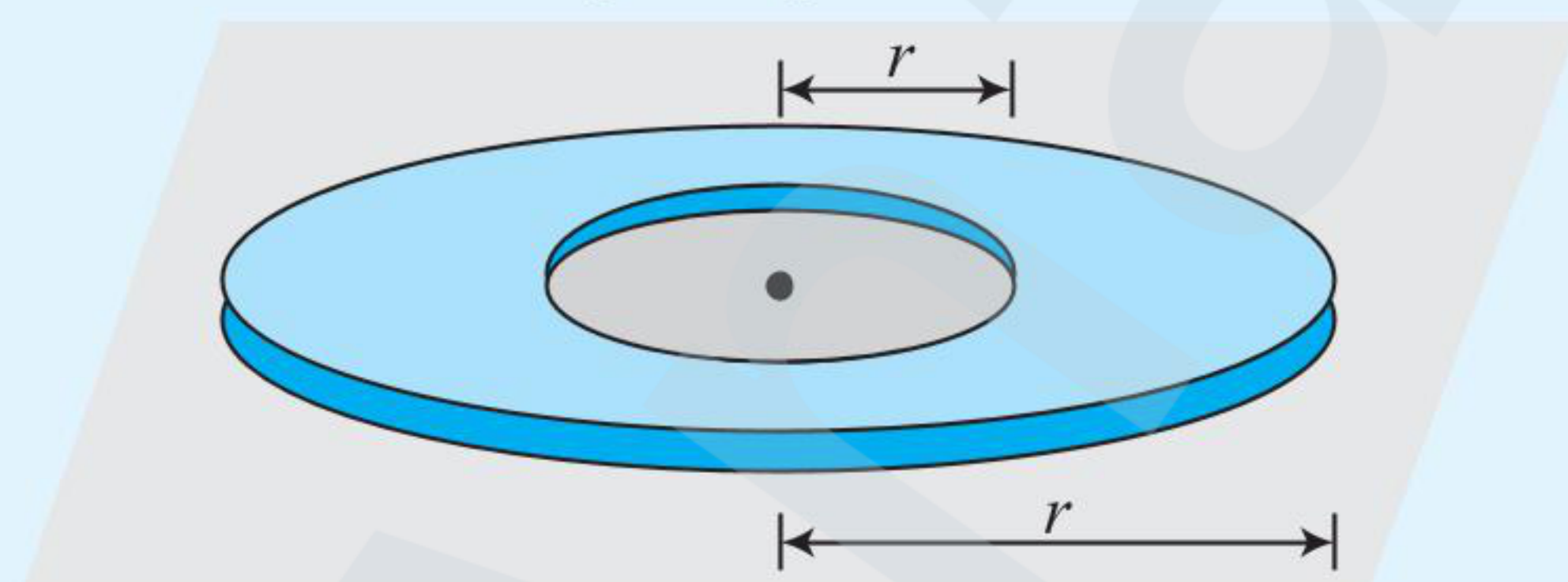
- (a) Both have the same mass.
 (b) Mass of the ring is half of that of the disk.
 (c) Mass of the ring is double to that of the disk.
 (d) More information is needed to decide.
2. A plate of radius 10 cm is placed on a water surface. Calculate the force required to take away the plate from the surface of water. Surface tension of water = 0.07 N m^{-1} .



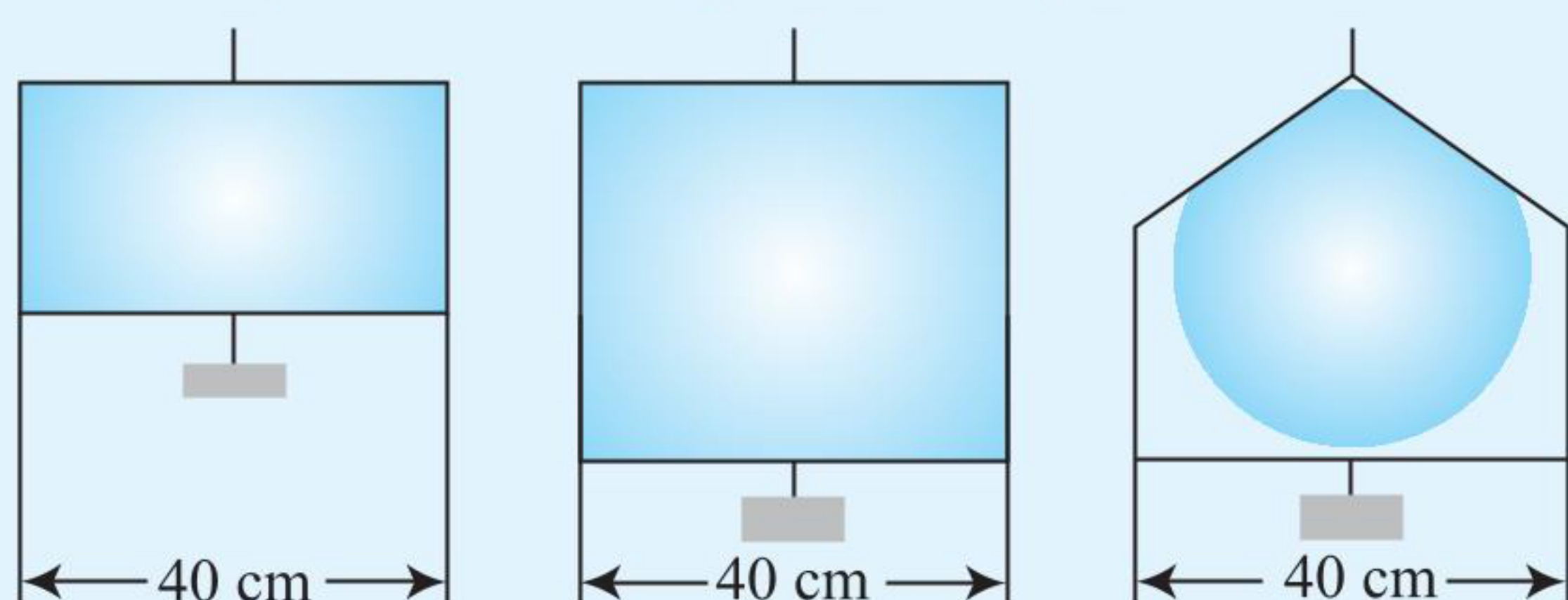
3. A ring substitutes the circular plate in the previous problem. Calculate the force required to take away the plate from the surface of water.



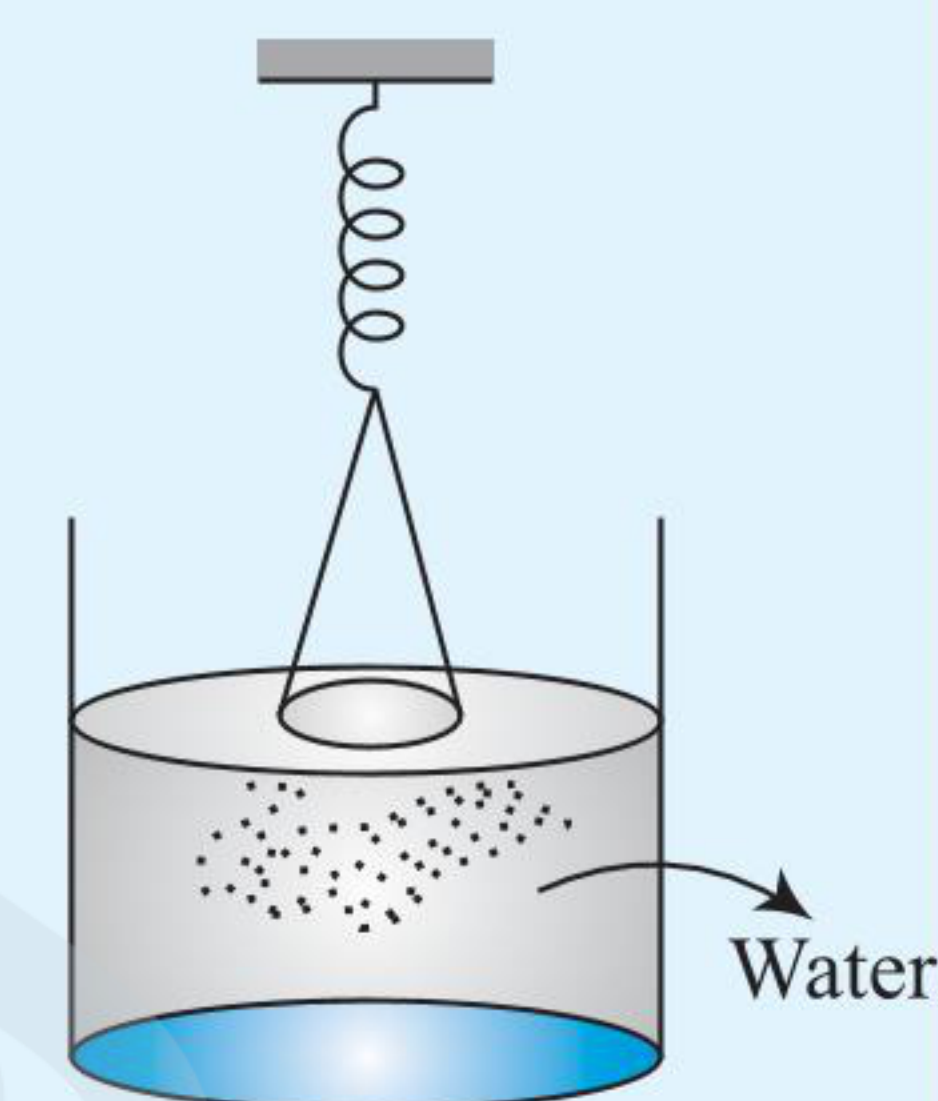
4. In the previous problem, if we take an annular disc of radius $r = 10 \text{ cm}$ and $R = 20 \text{ cm}$, Calculate the force required to take away the plate from the surface of water.



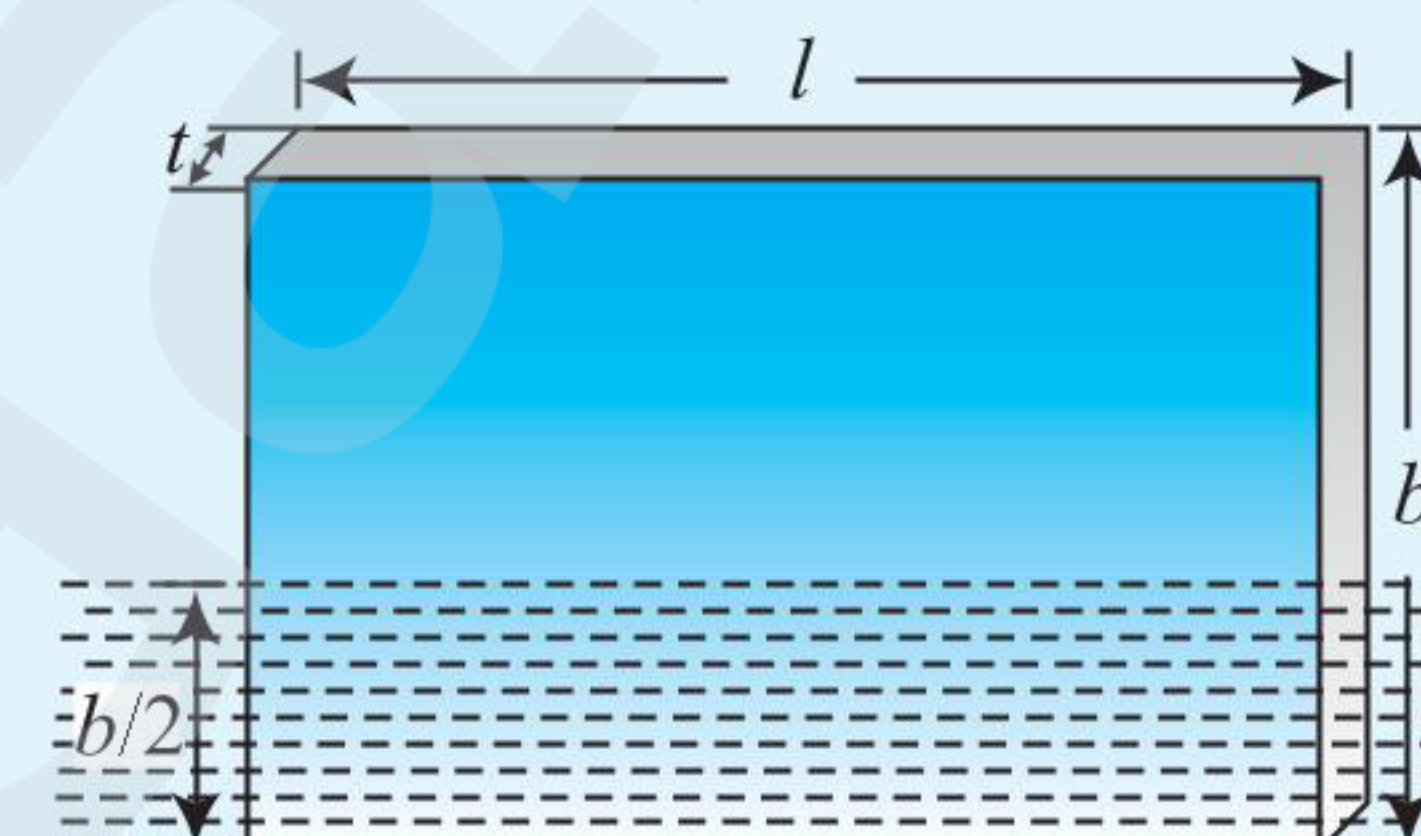
5. In the figure given below shows a thin liquid film supporting a small weight equal to $5.0 \times 10^{-2} \text{ N}$. What is the weight supported by a film of the same liquid at the same temperature in Figs. (b) and (c)?



6. A circular ring has inner and outer radii equal to r and R respectively. Mass of the ring is m . It gently pulled out vertically from a water surface by a sensitive spring. When the spring is stretched 3.4 cm from its equilibrium position the ring is on verge of being pulled out from the water surface. If spring constant is k . Find the surface tension of water.



7. A glass plate of length 50 cm, breadth 40 cm and thickness 5 cm weighs 3 kg in air. If it is held vertically with long side horizontal and the plate half immersed in a liquid, what will be its apparent weight? Surface tension of the liquid = $50 \times 10^{-2} \text{ N/m}$.



ANSWERS

1. (c) 2. 0.44 N 3. 0.88 N 4. 1.32 N
 6. $\frac{kx - mg}{2\pi(R - r)}$ 7. 5.55 N

SURFACE ENERGY

As we know the free surface of a liquid always has a tendency to contract and possess minimum surface area. If it is required to increase the surface area of the liquid, work has to be done. This work done is stored in the surface film of the liquid as its potential energy.

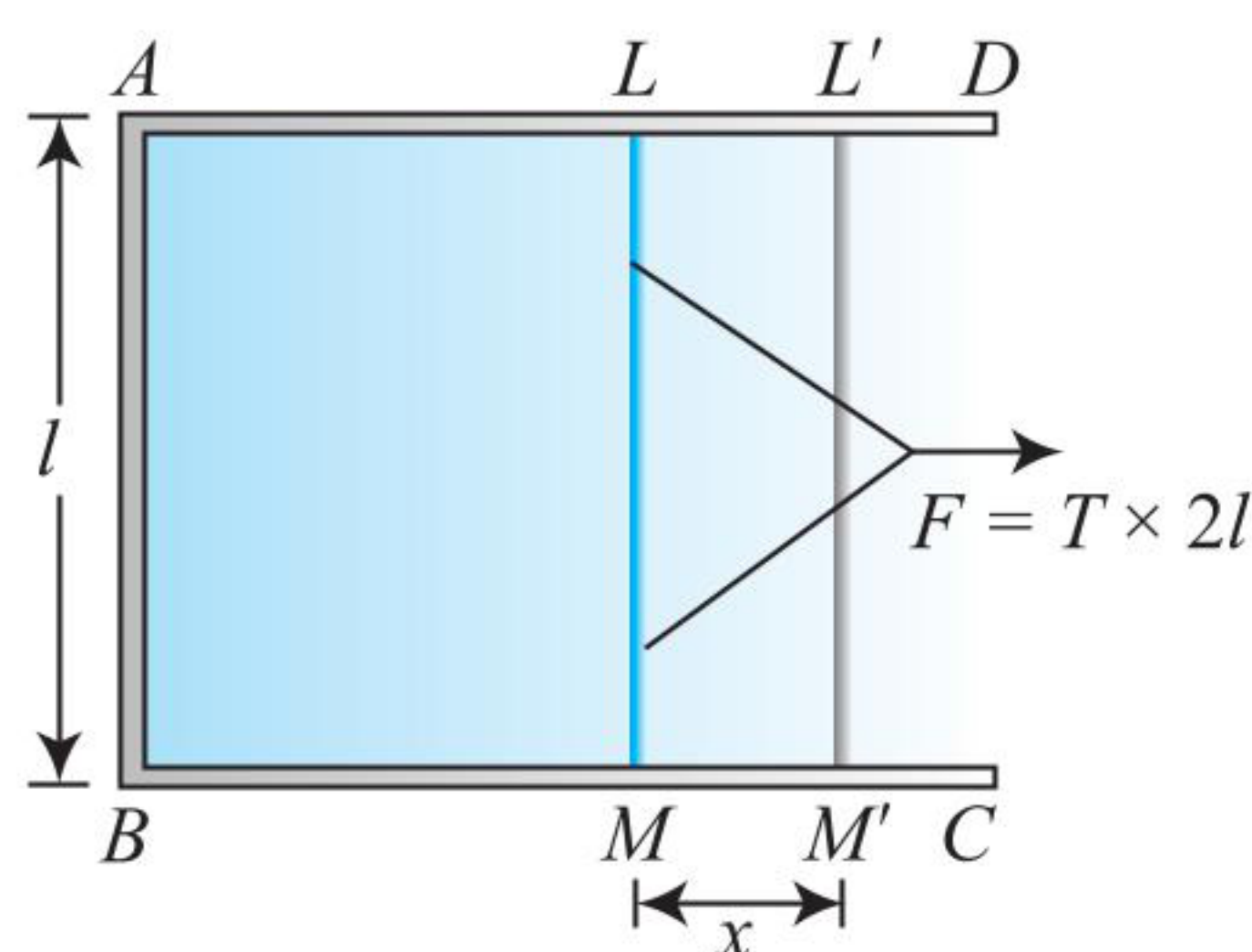
The potential energy per unit area of the surface film is called the surface energy.

It may also be defined as the amount of work done in increasing the area of the surface film through unity. Thus,

$$\text{Surface energy} = \frac{\text{work done in increasing the surface area}}{\text{increase in surface area}} \quad \dots(i)$$

RELATION BETWEEN SURFACE ENERGY AND SURFACE TENSION

Consider a rectangular frame *ABCD* made of a wire, such that the cross-piece *LM* can just slide. Dip the wire frame in the soap solution, so that a film is formed over the frame [see figure]. Due to the surface tension, the film will have a tendency to shrink and thereby, the cross-piece *LM* will be pulled in inward direction. However, the cross-piece can be held in this position under a force *F*, which is equal and opposite to the force acting on the cross piece *LM* all along its length due to surface tension in the soap film. If *T* is the force due to surface tension per unit length, then



$$F = T \times 2l$$

Here, l is length of the cross-piece LM . The length of the cross-piece has been taken as $2l$ for the reason that the film has got two free surfaces.

Suppose that the cross-piece LM is moved through a small distance x , so as to take the position $L'M'$. In this process, area of the film increases by $2l \times x$ (on the two sides) and to do so, the work done is given by

$$W = F \times x = (T \times 2l) \times x$$

This amount of work has been done in increasing area of surface film by an amount $2l \times x$. Therefore, by definition,

$$\begin{aligned} \text{Surface energy} &= \frac{\text{work done in increasing the surface area}}{\text{increase in surface area}} \\ &= \frac{T \times 2l \times x}{2l \times x} \end{aligned}$$

Or $T = \text{surface energy} \dots(i)$

Hence, the surface tension of a liquid is numerically equal to its surface energy.

ILLUSTRATION 8.6

Calculate the amount of energy evolved when 8 droplets of water (surface tension $= 72 \times 10^{-3} \text{ N/m}$) of radius 0.5 mm each combine into one.

Sol. Let R be the radius of the big drop formed by the combination of 8 small droplets.

As volume of the big drop = volume of 8 small droplets,

$$\frac{4}{3}\pi R^3 = 8 \left(\frac{4}{3}\pi r^3 \right)$$

$$\text{or } R^3 = 8r^3 \text{ or } R = 2r = 2(5 \times 10^{-4} \text{ m}) = 10^{-3} \text{ m}$$

$$\begin{aligned} \text{Surface area of the big drop} &= 4\pi R^2 \\ &= 4\pi (10^{-3} \text{ m})^2 = 4\pi \times 10^{-6} \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Surface area of 8 small droplets} &= 8 \times 4\pi r^2 \\ &= 8 \times 4\pi (5 \times 10^{-4} \text{ m})^2 = 8\pi \times 10^{-6} \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Decrease in surface area,} \\ &= 8\pi \times 10^{-6} \text{ m}^2 - 4\pi \times 10^{-6} \text{ m}^2 = 4\pi \times 10^{-6} \text{ m}^2 \end{aligned}$$

Energy evolved,

$$\begin{aligned} W &= T \times \Delta A = (72 \times 10^{-3} \text{ N/m}) \times (4\pi \times 10^{-6} \text{ m}^2) \\ &= 9 \times 10^{-7} \text{ J} \end{aligned}$$

ILLUSTRATION 8.7

Calculate the work done against surface tension in blowing a soap bubble from a radius of 10 cm to 20 cm, if the surface tension of soap solution is $25 \times 10^{-3} \text{ N/m}$.

Sol. Original total surface area $= 2 \times 4\pi r_1^2 = 2 \times 4\pi \times (0.1)^2 \text{ m}^2$ (as bubble has two surfaces).

$$\text{Final total surface area} = 2 \times 4\pi r_2^2 = 2 \times 4\pi \times (0.2)^2 \text{ m}^2$$

$$\text{Therefore, extension in area} = 2 \times 4\pi [(0.2)^2 - (0.1)^2] = 0.24\pi \text{ m}^2$$

$$\begin{aligned} \text{Now, work done } W &= \text{surface tension} \times \text{extension in area} \\ &= 25 \times 10^{-3} \times 0.24\pi = 6\pi \times 10^{-3} \text{ J} \end{aligned}$$

ILLUSTRATION 8.8

A film of water is formed between two straight parallel wires each 10 cm long and at separation 0.5 cm. Calculate the work required to increase 1 mm distance between the wires. Surface tension of water $= 72 \times 10^{-3} \text{ N/m}$.

$$\begin{aligned} \text{Sol. Initial surface area} &= 2 \times \text{length} \times \text{separation} \\ &= 2 \times 10 \text{ cm} \times 0.5 \text{ cm} = 10 \text{ cm}^2 = 10 \times 10^{-4} \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Final surface area} &= 2 \times 10 \text{ cm} \times (0.5 + 0.1) \text{ cm} \\ &= 2 \times 10 \times 0.6 \text{ cm}^2 = 12 \times 10^{-4} \text{ m}^2 \end{aligned}$$

The required work,

$$\begin{aligned} W &= T \Delta A = 72 \times 10^{-3} \times (12 \times 10^{-4} - 10 \times 10^{-4}) \text{ J} \\ &= 72 \times 10^{-3} \times 2 \times 10^{-4} = 144 \times 10^{-7} \text{ J} \end{aligned}$$

ILLUSTRATION 8.9

Two soap bubbles in vacuum having radii 3 cm and 4 cm respectively coalesce under isothermal conditions to form a single bubble. What is the radius of the new bubble?

Sol. Let r_1 and r_2 be the radii of two given soap bubbles and r be that of the coalesced bubble.

$$\text{Here, } r_1 = 30 \text{ cm and } r_2 = 4 \text{ cm.}$$

Since a soap bubble has two free surfaces, total surface area of the coalesced bubble $= 2 \times 4\pi r^2 = 8\pi r^2$

Potential energy of the coalesced bubble, i.e., $W = 8\pi r^2 T$

Similarly if W_1 and W_2 are the potential energies of the two given bubbles,

$$W_1 = 8\pi r_1^2 T \text{ and } W_2 = 8\pi r_2^2 T$$

Since the total energy of the system is conserved,

$$8\pi r^2 T = 8\pi r_1^2 T + 8\pi r_2^2 T$$

$$\text{or } r^2 = r_1^2 + r_2^2 = (3)^2 + (4)^2 = 25$$

$$\text{or } r = 5 \text{ cm}$$

CONCEPT APPLICATION EXERCISE 8.2

- Two droplets merge with each other and forms a large droplet. In this process
 - Energy is liberated
 - Energy is absorbed
 - Neither liberated nor absorbed
 - Some mass is converted into energy
- A soap bubble of radius R is blown. After heating the solution a second bubble of radius $2R$ is blown. The work required to blow the second bubble in comparison to that required for the first bubble is

- (a) Double
 (b) Slightly less than double
 (c) Slightly less than four times
 (d) Slightly more than four times
3. If the work done in blowing a bubble of volume V is W , then find the work done in blowing the bubble of volume $2V$ from the same soap solution.
4. Calculate the work done in blowing up a soap bubble from an initial surface area of 0.50 cm^2 to a final surface area of 1.10 cm^2 . The surface tension of soap solution is 30 dyne cm^{-1} .
5. A liquid drop of diameter D breaks into 27 tiny drops. Find the resultant change in energy. Take the surface tension of the liquid as T .

ANSWERS

1. (a) 2. (c) 3. $(4)^{1/3} W$ 4. 36 erg 5. $2\pi D^2 T$

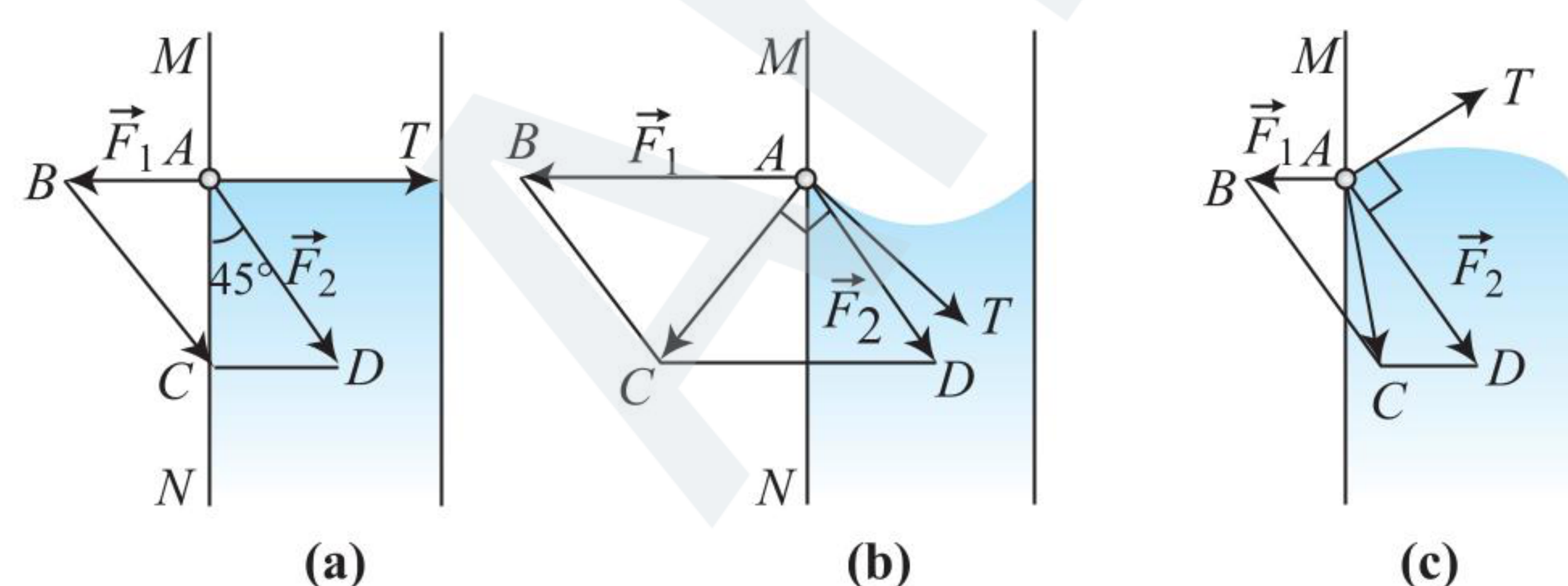
SHAPE OF LIQUID MENISCUS

If a liquid and a solid are in contact with each other, the liquid surface becomes curved near the point of contact. This curved surface is due to two forces, viz., cohesive force between the liquid molecules and the adhesive force between liquid and solid molecules. The nature of the curvature (i.e., whether it is plane, convex or concave) depends upon the relative magnitudes of these two forces.

Let us consider a thin glass capillary tube dipped in a liquid vertically in a liquid. Consider a molecule A on the liquid surface touching the wall MN of the tube. Neglecting the weight of the molecule, this liquid molecule is being acted upon by two forces:

1. Force F_1 due to force of adhesion because of the attraction of the glass molecules which acts outwards at right angle to the wall of the tube. This force is represented by AB .
2. Force F_2 due to force of cohesion because of the attraction of the liquid molecules which acts at an angle of 45° to the vertical. This force is represented by AD .

These two forces, AB and AD , are acting at an angle with each other and their resultant can be obtained by applying the parallelogram law of forces. It is represented in magnitude and direction by AC . The direction of the resultant force AC depends upon the relative magnitudes of AB and AD . Here three cases are possible:



- (a) If $AB = AD \sin 45^\circ = \frac{AD}{\sqrt{2}}$, the resultant AC acts vertically downwards as shown in Fig. (a).
- (b) $AB > AD \sin 45^\circ$, or if $AB > \frac{AD}{\sqrt{2}}$, the resultant AC lies outwards as shown in Fig. (b).

- (c) $AB < AD \sin 45^\circ$, i.e., $AB < \frac{AD}{\sqrt{2}}$, the resultant AC lies inwards as shown in Fig. (c).

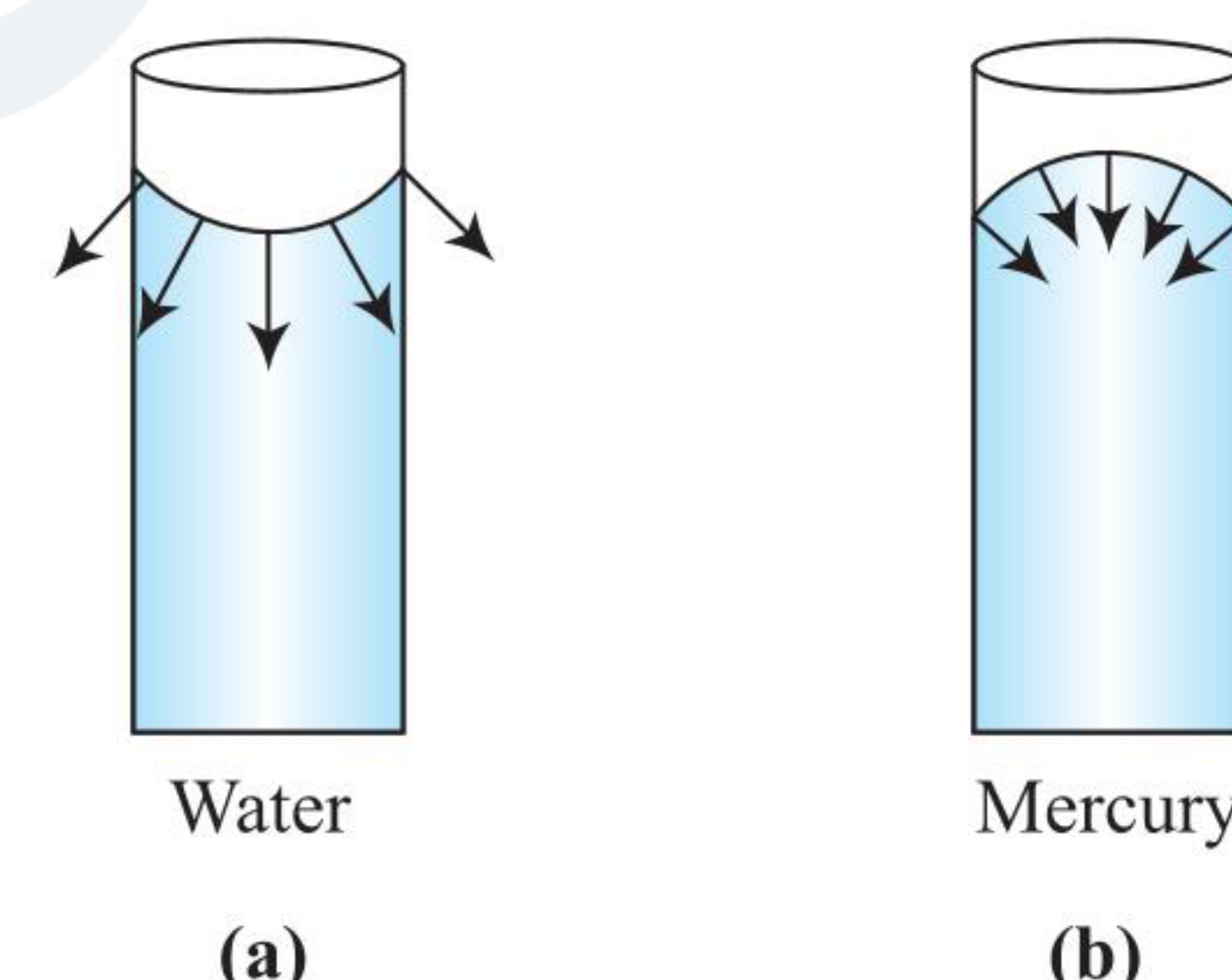
As liquid cannot permanently withstand any tangential force. As such when in equilibrium, its surface at every point sets itself perpendicular to the resultant force.

Special Cases

In case (a), $F_2 = \sqrt{2} F_1$, then the resultant force will act vertically downward and hence the meniscus will be plane or horizontal as shown in Fig. (a). For example, pure water contained in silver capillary tube.

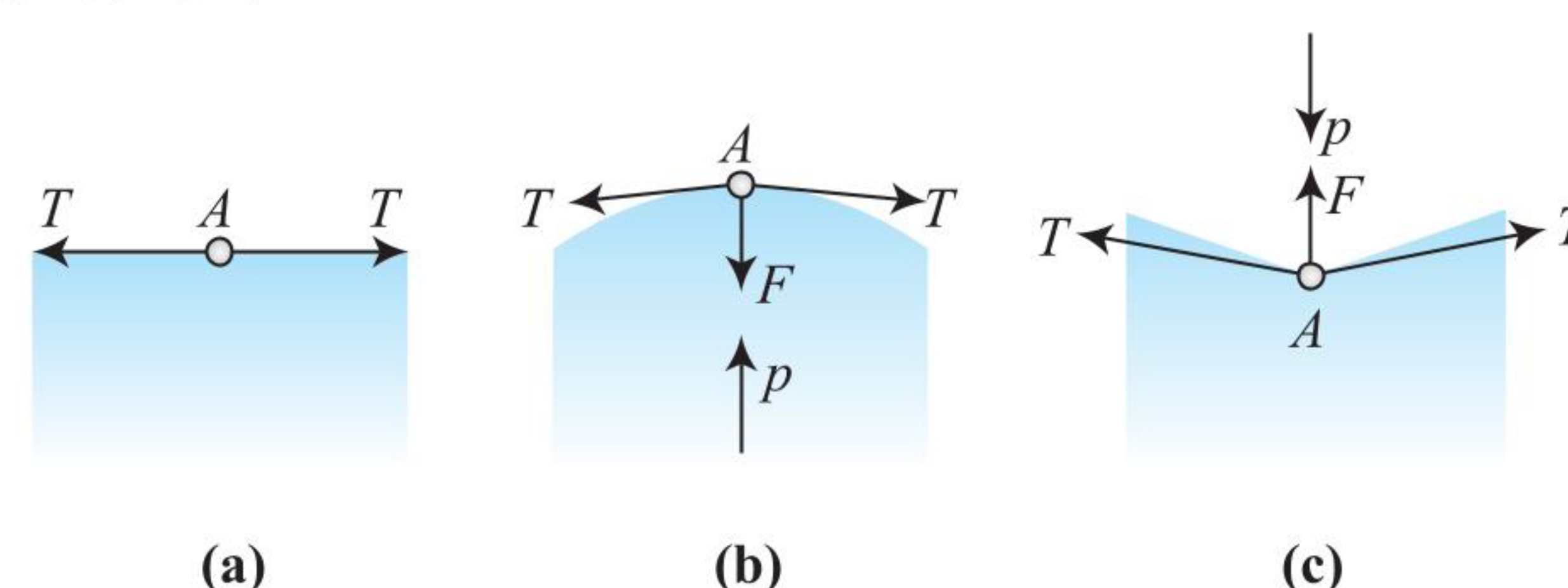
In case (b), $F_2 < \sqrt{2} F_1$, the resultant force AC is directed outwards, since the liquid surface has always to set itself perpendicular to this resultant force, it becomes concave upwards as in the case of water in a glass tube.

In case (c), $F_2 > \sqrt{2} F_1$, the resultant force AC is directed inwards, and here also the liquid surface has always to set itself perpendicular to the resultant force, it becomes convex upwards as in the case of mercury in a glass tube.



SURFACE OF A LIQUID

The surface of a liquid may be plane, convex or concave as shown in the figure. The force due to surface tension on molecule M in the surface of the liquid acts tangentially to the surface in all directions



Plane Surface

If the surface of the liquid is plane [as shown in Fig. (a)], the molecule on the liquid surface is attracted equally in all directions. The resultant force due to surface tension is zero. The pressure, therefore, on the liquid surface is normal.

Convex Surface

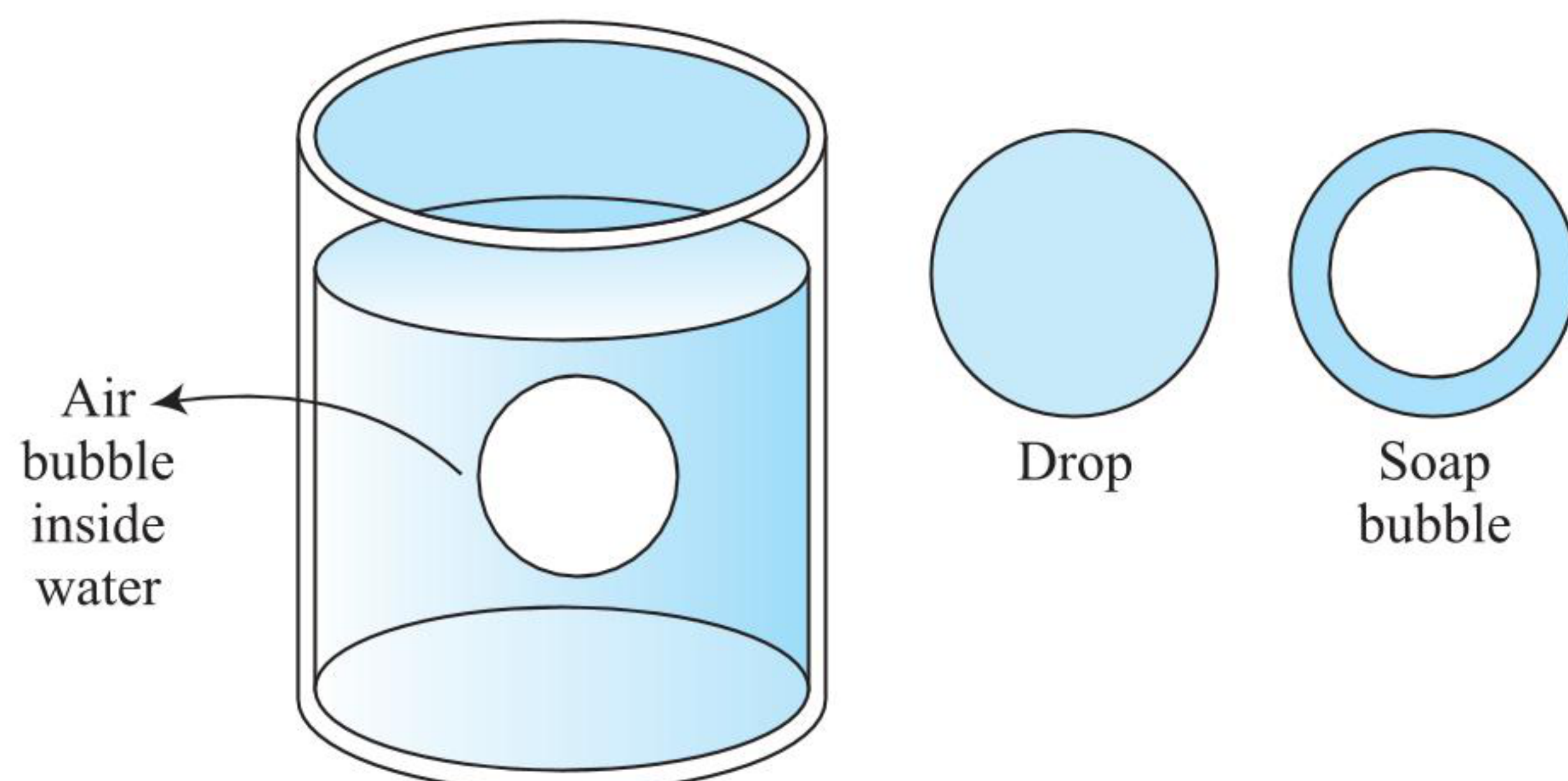
If the surface is convex [as shown in Fig. (b)], the resultant force due to surface tension acts in the downward direction. Since the molecules on the surface are in equilibrium, there must be an excess of pressure on the concave side of the surface acting in the upward direction to balance the downward resultant force of surface tension. Hence, there is always an excess of pressure on concave side of a curved surface over that on the convex side.

Concave Surface

If the surface is concave upwards [as shown in Fig. (c)], there will be upward resultant force due to surface tension acting on the molecule. Since the molecule on the surface is in equilibrium, there must be an excess of pressure on the concave side

EXCESS PRESSURE INSIDE A CURVED SURFACE

Some examples of plane and curved surfaces (cavities, drops and bubbles) are shown in figure.



The bubbles, such as soap bubbles, are like blown-up balloons, air inside and air outside with a thin liquid film in between them. This thin film naturally has two free surfaces, one inside and the other outside. Cavities generally have air inside and liquid outside. They have one interface or free surface. Drops generally have water or some other liquid inside and air or some other gas outside them. They too have one exposed surface.

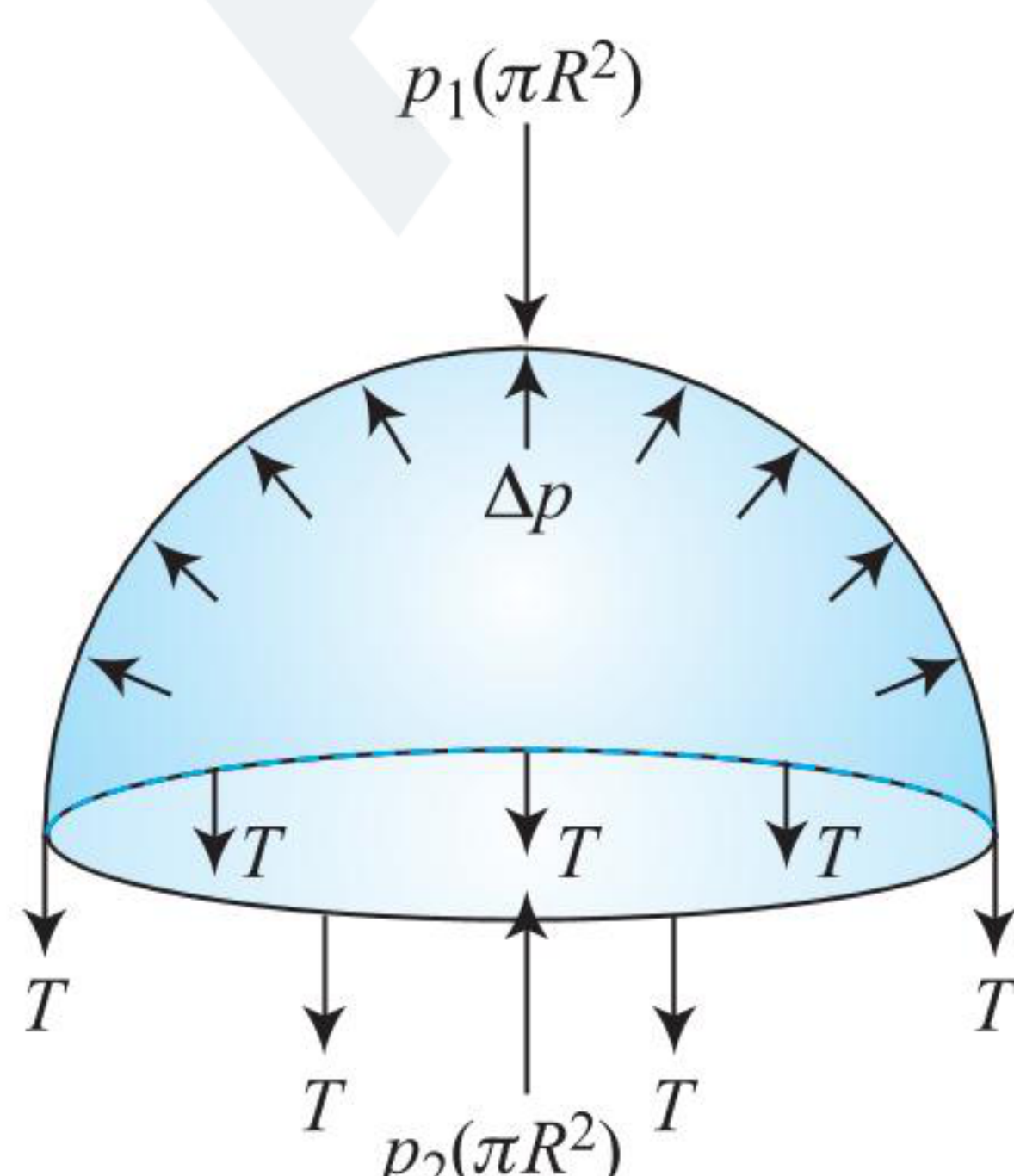
EXCESS PRESSURE INSIDE A LIQUID DROP AND A BUBBLE

Due to the property of the surface tension, a liquid surface always tends to have minimum surface area. The small drops and bubbles are found to possess spherical shapes. However, a big drop of liquid is not spherical in shape. The reason is that in case of a small drop of liquid, the effect of gravity is negligible. As the small drops and bubbles have spherical shapes, it implies that their spherical surfaces must be possessing minimum surface area. Further as a drop or a bubble does not collapse under the effect of the force of surface tension (which tends to minimise its surface area), it indicates that the pressure inside the drop or the bubble must be greater than that outside it.

EXCESS PRESSURE INSIDE A LIQUID DROP

Figure shows one-half cross section of an air bubble formed inside liquid. It is in equilibrium under the action of three forces:

1. due to external pressure p_1
2. due to internal pressure p_2
3. due to surface tension of the liquid σ



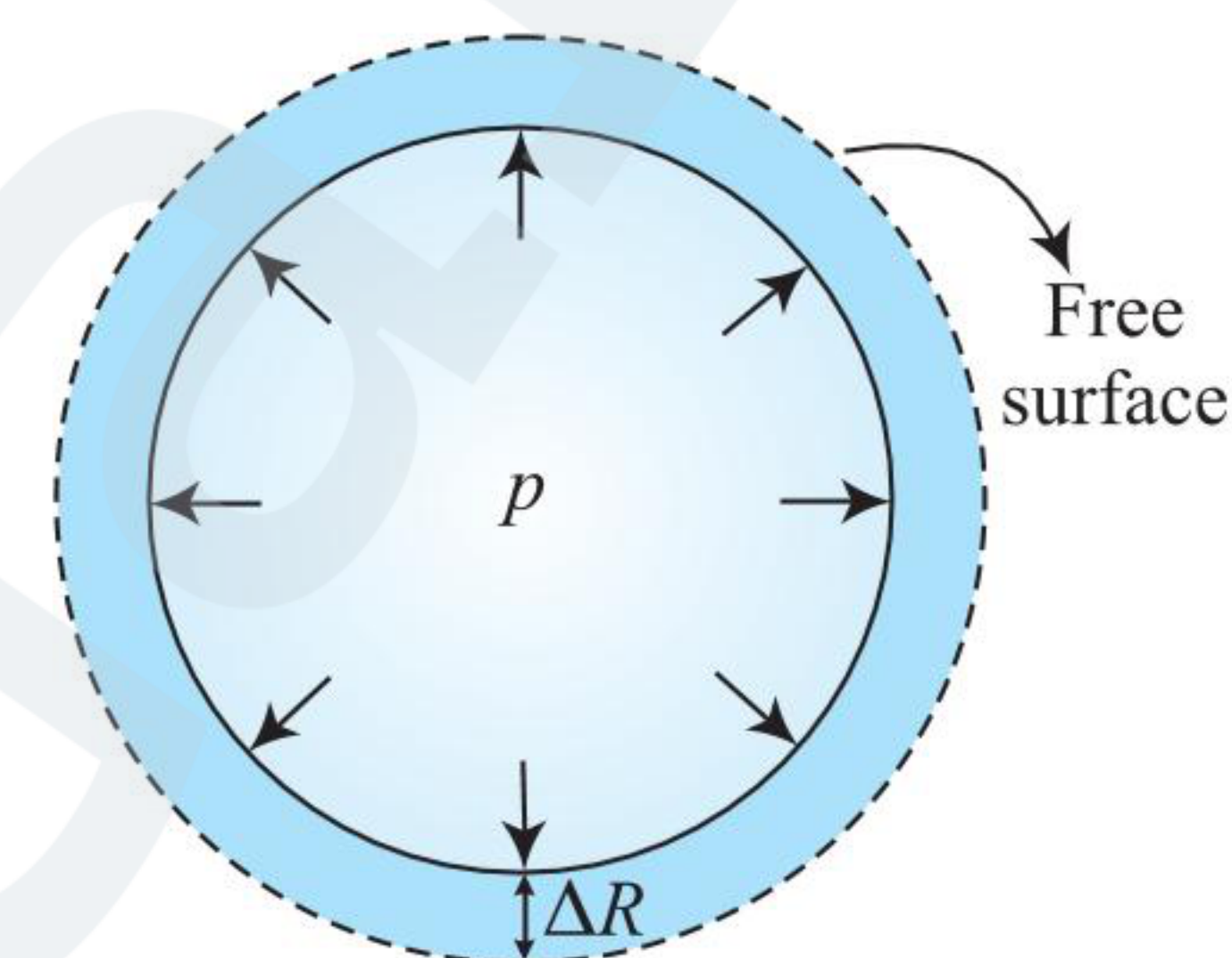
If R is the radius of the air bubble, then the forces due to external and internal pressures are $p_1(\pi R^2)$ and $p_2(\pi R^2)$, respectively. Since the surface tension acts around the circumference of the bubble, therefore, the force of surface tension is $T(2\pi R)$.

Thus, from the condition of equilibrium,

$$p_2(\pi R^2) = p_1(\pi R^2) + T(2\pi R)$$

$$\text{or } p_2 - p_1 = \frac{2T}{R}$$

Alternate approach: Let us consider a liquid drop of radius R and let T be surface tension of the liquid forming the drop. As already discussed, there is an excess pressure (p) inside the drop and it acts normally outwards as shown in the figure.



Let this excess pressure increase the radius of the drop from R to $(R + \Delta R)$.

Obviously, work done by the excess pressure, i.e.,

$$\begin{aligned} W &= \text{force} \times \text{distance} \\ &= \text{excess pressure} \times \text{surface area} \times \text{distance} \\ &= p \times 4\pi R^2 \times \Delta R \end{aligned}$$

$$\text{i.e., } W = 4\pi p R^2 \Delta R \quad \dots(i)$$

Increase in the surface area of the drop, i.e.,

$$\begin{aligned} \Delta A &= \text{final surface area} - \text{initial surface area} \\ &= 4\pi(R + \Delta R)^2 - 4\pi R^2 \\ &= 4\pi[R^2 + (\Delta R)^2 + 2R\Delta R - R^2] \\ &= 8\pi R\Delta R \quad [\text{neglecting } (\Delta R)^2 \text{ as it is very small}] \end{aligned} \quad \dots(ii)$$

$$\text{We know that } T = \frac{W}{\Delta A} \quad \dots(iii)$$

From Eqs. (i), (ii) and (iii),

$$T = \frac{4\pi p R^2 \Delta R}{8\pi R \Delta R} = \frac{pR}{2}$$

$$\text{Thus, } p = \frac{2T}{R} \quad \dots(iv)$$

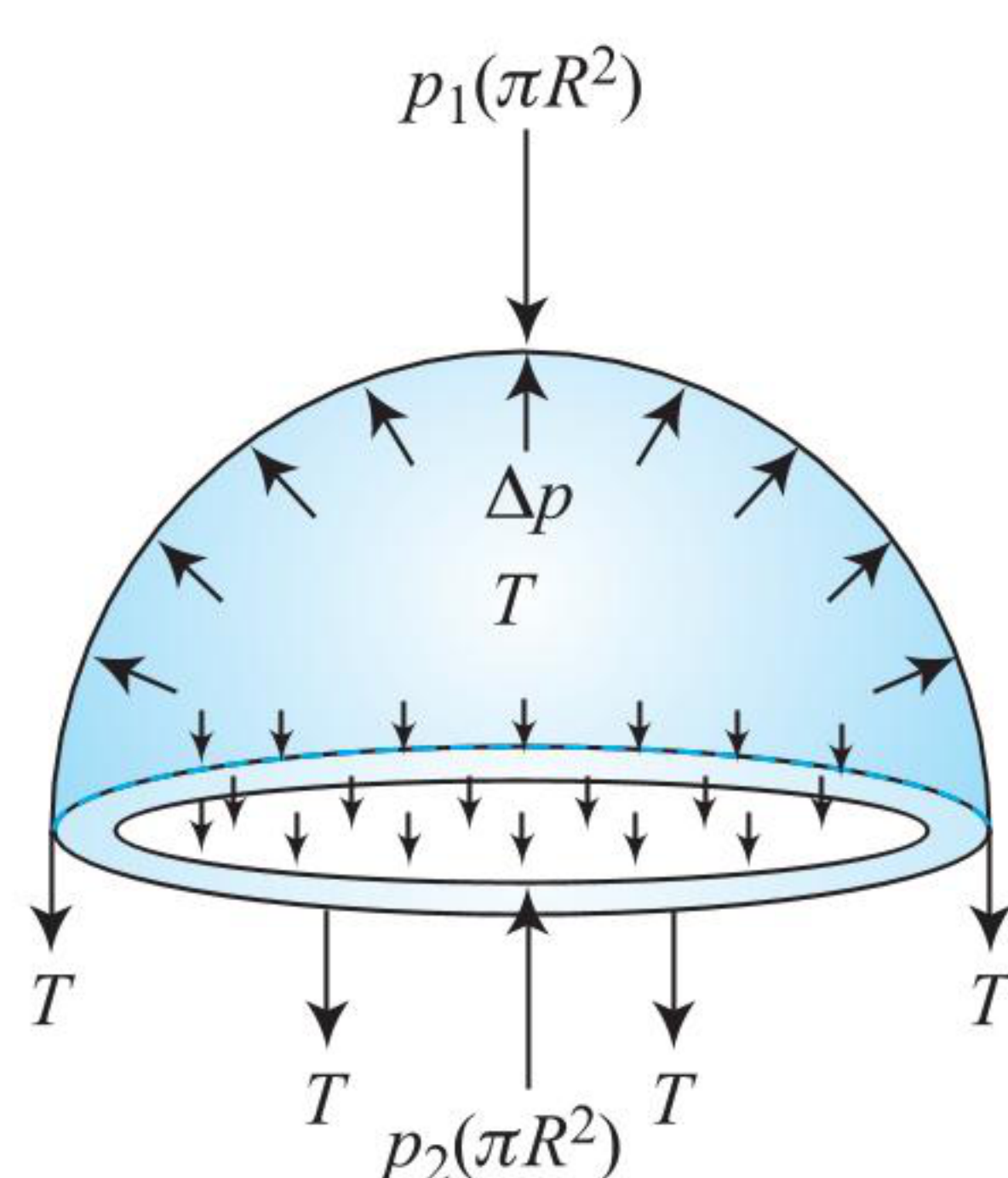
Obviously, excess pressure increases as the radius of drop decreases.

Note:

- A curved surface always maintains a pressure difference, which depends on the radius of the curved surface and surface tension of the liquid.
- The concave side of the surface always possesses greater pressure. If the angle of contact is zero, then the excess pressure is given by $\Delta p = \frac{2T}{R}$

EXCESS PRESSURE IN SOAP BUBBLE

A soap bubble forms two liquid surfaces in contact with air, one inside the bubble and the other outside the bubble. Figure shows one-half cross section of the soap bubble. By considering its equilibrium, we get



$$p_2(\pi R^2) = p_1(\pi R^2) + T[2(2\pi R)]$$

$$\Rightarrow p_2 - p_1 = \frac{4T}{R}$$

Alternate approach: Let us consider a liquid bubble (say a soap bubble) of radius R and let T be the surface tension of the liquid. We know that there is an excess pressure (p) inside the bubble and it acts normally outwards. Let the excess pressure increase the radius of the bubble from R to $(R + \Delta R)$ as shown in figure.

Work done by the excess pressure, i.e.,

$$\begin{aligned} W &= \text{force} \times \text{distance} \\ &= \text{excess pressure} \times \text{surface area} \times \text{distance} \\ &= p \times 4\pi R^2 \times \Delta R \end{aligned}$$

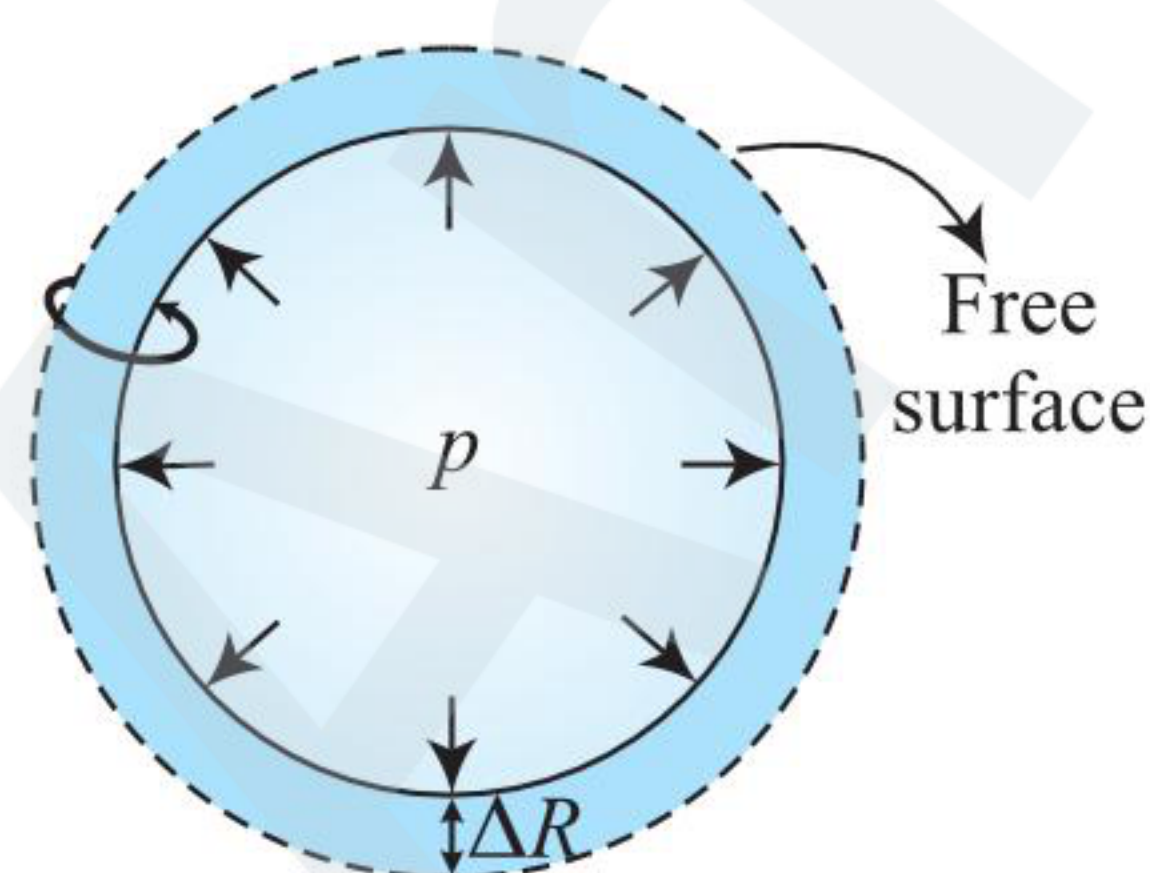
$$\text{i.e., } W = 4\pi p R^2 \Delta R \quad \dots(i)$$

The soap bubble has two free surfaces, one inside the bubble and the other outside it.

Total increase in the surface area of the bubble, i.e.,

$$\begin{aligned} \Delta A &= 2[4\pi(R + \Delta R)^2 - 4\pi R^2] \\ &= 8\pi[R^2 + (\Delta R)^2 + 2R \times \Delta R - R^2] \\ &= 16\pi R \Delta R \quad [\text{neglecting } (\Delta R)^2 \text{ as it is very small}] \quad \dots(ii) \end{aligned}$$

$$\text{We know that } T = \frac{W}{\Delta A} \quad \dots(iii)$$



$$\text{From Eqs. (i), (ii) and (iii), } T = \frac{4\pi p R^2 \Delta R}{16\pi R \Delta R} = \frac{pR}{4}$$

$$\text{Or } p = \frac{4T}{R} \quad \dots(iv)$$

Since the excess pressure is inversely proportional to the radius of the bubble, it means smaller the bubble, greater the excess pressure inside it

Inside an air bubble: In a drop of liquid, air is outside the drop and the liquid is inside it. In an air bubble, liquid is outside the spherical core of air. Thus, as in the case of a drop, an air bubble also has one free surface. As before, it can be easily proved that excess of pressure inside an air bubble,

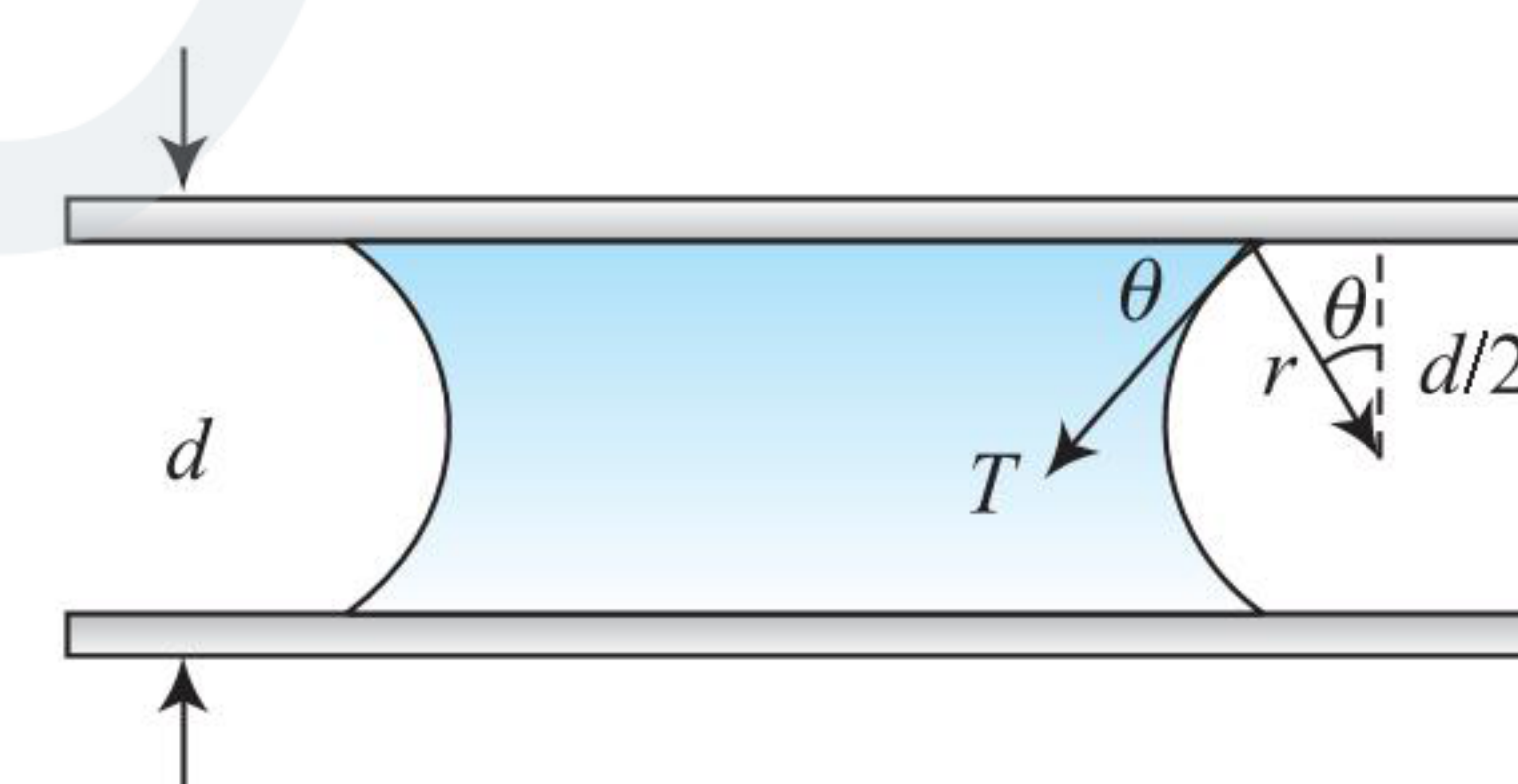
$$p_i - p_o = \frac{4T}{R} \quad \dots(i)$$

Note: The above discussion can be summarised in the form of a general result that there is an excess of pressure on the concave side of the liquid-air interface and the excess of pressure is given by

$$p_i - p_o = \frac{2T}{R}$$

where the letters have their usual meanings.

Liquid between two plates: When a small drop of water is placed between two glass plates put face to face, it forms a thin film which is concave outward along its boundary. Let ' R ' and ' r ' be the radii of curvature of the enclosed film in two perpendicular directions.



Hence, the pressure inside the film is less than the atmospheric pressure outside it by an amount p given by

$$p = T \left(\frac{1}{r} + \frac{1}{R = \infty} \right)$$

$$\text{and we have } p = \frac{T}{r} \quad \dots(i)$$

If d be the distance between the two plates and θ be the angle of contact for water and glass, then from the figure,

$$\cos \theta = \frac{\frac{1}{2}d}{r} \quad \text{or} \quad \frac{1}{r} = \frac{2 \cos \theta}{d}$$

$$\text{Substituting for } 1/r \text{ in Eq. (i), we get } p = \frac{2T}{d} \cos \theta$$

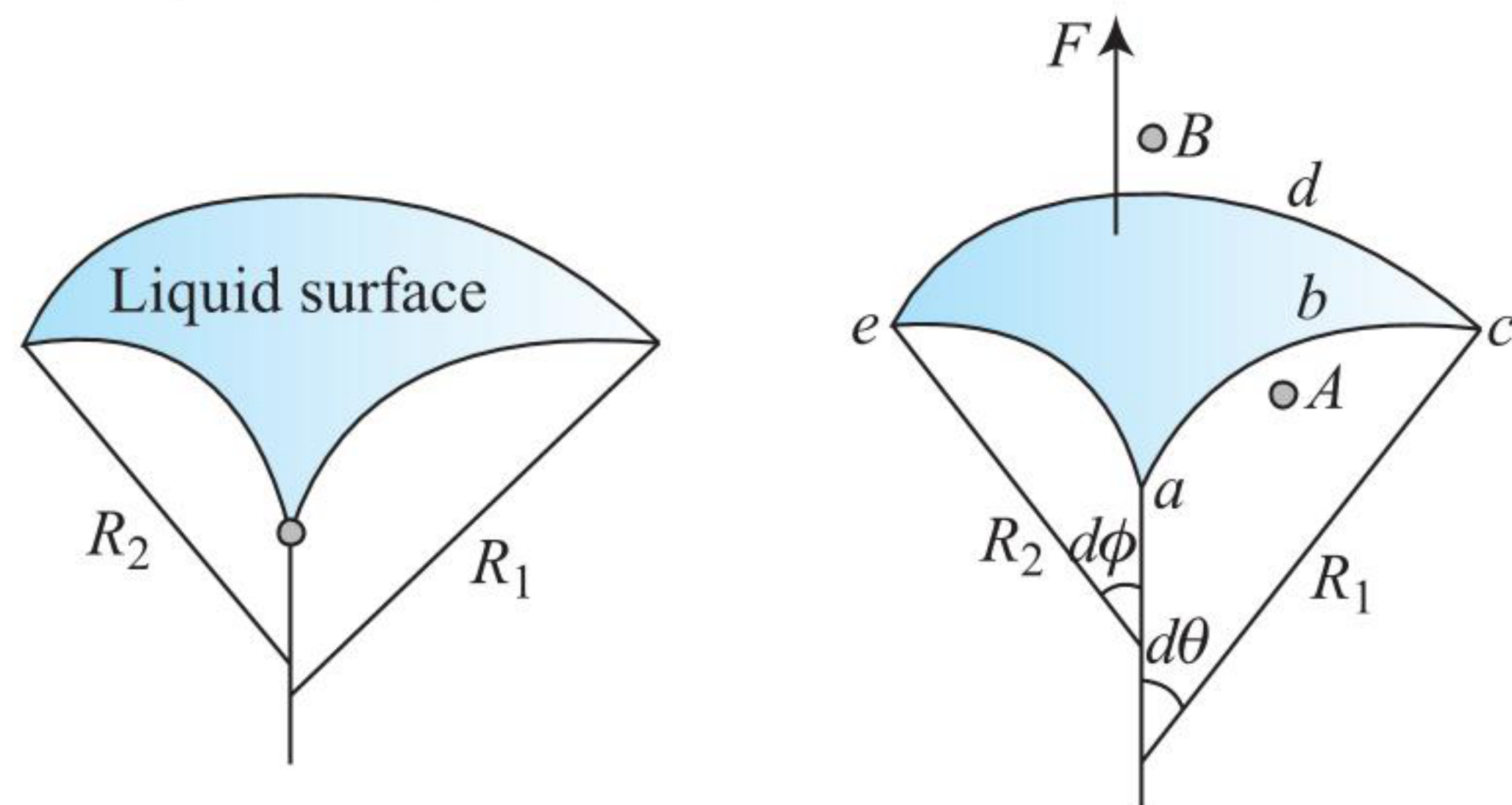
θ can be taken zero for water and glass, i.e., $\cos \theta = 1$. Thus, the upper plate is pressed downward by the atmospheric pressure minus $2T/d$. Hence, the resultant downward pressure acting on the upper plate is $2T/d$. If A be the area of the plate wetted by the film, the resultant force F pressing the upper plate downward is given by $F = \text{resultant pressure} \times \text{area} = 2TA/d$. For very nearly plane surface, d will be very small and hence the pressing force F is very large. Therefore, it will be difficult to separate the two plates normally.

RELATION BETWEEN SURFACE TENSION, RADII OF CURVATURE AND EXCESS PRESSURE ON A CURVED SURFACE

Let us consider a small element $ABCD$ (see figure) of a curved liquid surface which is convex on the upper side. R_1 and R_2 are

the maximum and minimum radii of curvature, respectively. They are called the ‘principal radii of curvature’ of the surface. Let p be the excess pressure on the concave side. Then

$$p = T \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$



If instead of a liquid surface, we have a liquid film, the above expression will be

$$p = 2T \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

because a film has two surfaces.

EXCESS PRESSURE ON CURVED SURFACES IN GENERAL

The pressure on the concave side (whatever be on its left or right) is always greater than the pressure on the convex side, such that

$$p_{\text{concave}} - p_{\text{convex}} = T \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

where σ = surface tension.

This formula is same as what we derived earlier, i.e.,

$$p_i - p_0 = \frac{2T}{R} = T \left(\frac{1}{R} + \frac{1}{R} \right)$$

1. For a single spherical surface, e.g. drops, cavities, etc.,

$$R_1 = R_2 = R \text{ (say)}$$

$$\text{Then } p_i - p_0 = T \left(\frac{1}{R} + \frac{1}{R} \right) = \frac{2T}{R}$$

2. For spherical film, e.g., soap bubble,

$$p_i - p_0 = (2T) \left(\frac{1}{R} + \frac{1}{R} \right) = \frac{4T}{R}$$

because it has two free surfaces.

3. For a single cylindrical surface, $R_1 = R, R_2 = \infty$

$$\therefore p_i - p_0 = (T) \left(\frac{1}{R} + \frac{1}{\infty} \right) = \frac{T}{R}$$

ILLUSTRATION 8.10

What should be the pressure inside a small air bubble of 0.1 mm radius situated just below the surface of water? Surface tension of water = 72×10^{-3} N/m and atmospheric pressure = 1.013×10^5 N/m².

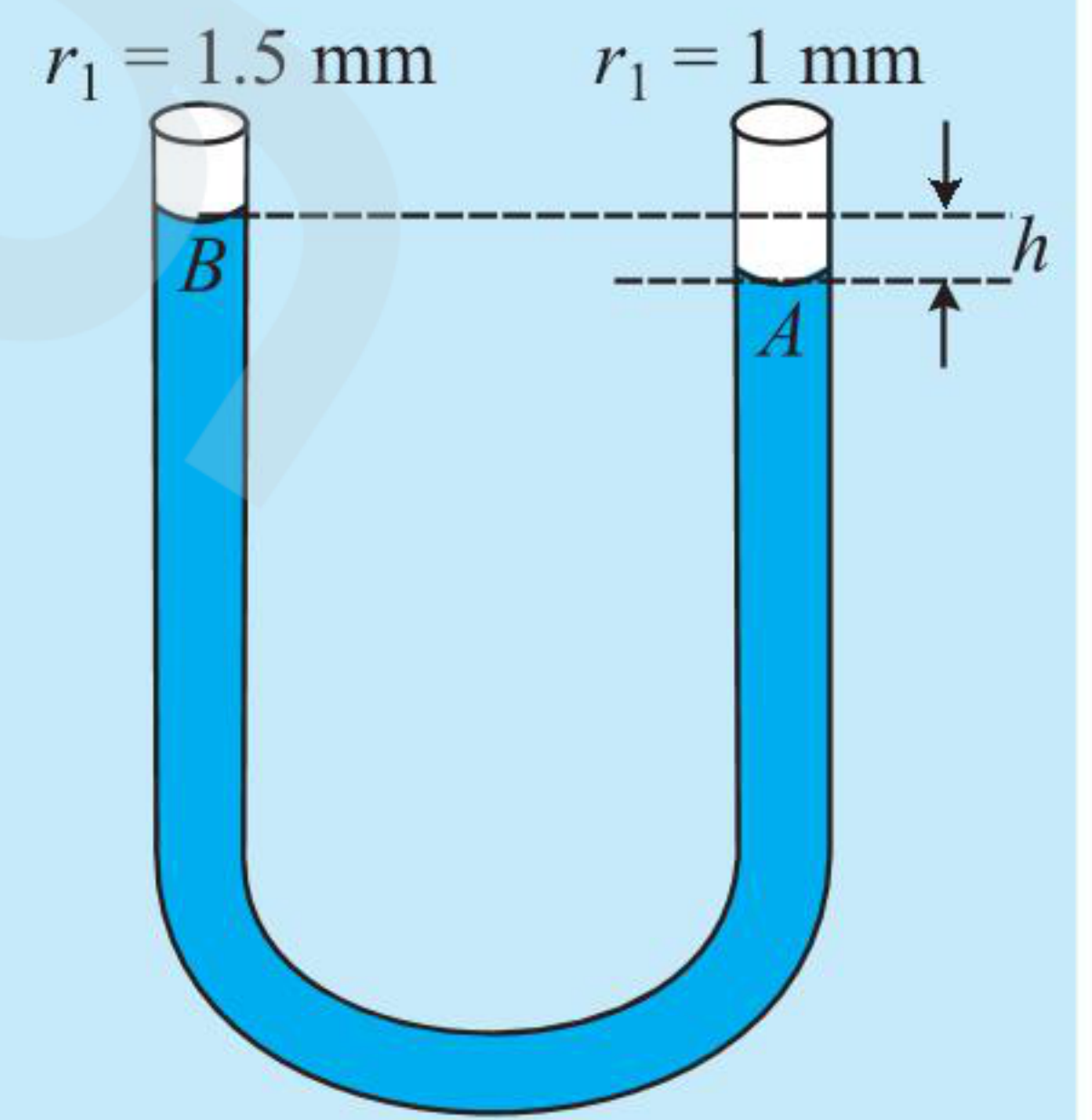
Sol. The excess pressure for air bubble is $p = \frac{2T}{r}$
or $p = \frac{2 \times 72 \times 10^{-3}}{0.1 \times 10^{-3}} = 1440$ N/m²

Since the bubble is just below the water surface, the external pressure on it is equal to the atmospheric pressure P . Hence the pressure inside the bubble is

$$\begin{aligned} P + p &= 1.013 \times 10^5 + 1440 \\ &= 1.013 \times 10^5 + 0.0144 \times 10^5 \\ &= 1.274 \times 10^5 \text{ N/m}^2 \end{aligned}$$

ILLUSTRATION 8.11

Calculate the difference (h) in water levels in two communicating capillary tubes of radius 1 mm and 1.5 mm. Surface tension of water = 0.07 N/m.



Sol. Pressure at $A = p_0 + \frac{2T}{r_1}$

(Since pressure on concave side is greater than that on convex side)

$$\text{Pressure at } B = p_0 - \frac{2T}{r_2}$$

$$\text{Therefore, pressure difference} = 2T \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

Let this pressure difference correspond to h units of the liquid. Then,

$$2T \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = \rho gh \Rightarrow h = \frac{2T}{\rho g} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$\therefore h = \frac{2 \times 0.07}{1000 \times 9.8} \left(\frac{1}{1 \times 10^{-3}} - \frac{1}{1.5 \times 10^{-3}} \right) = 4.76 \text{ mm}$$

ILLUSTRATION 8.12

A glass tube of 1 mm bore is dipped vertically into a container of mercury, with its lower end 2 cm below the mercury surface. What must be the gauge pressure of air in the tube to blow a hemispherical bubble at its lower end? Given that density of mercury = $13,600$ kg m⁻³ and surface tension of mercury = 0.465 N m⁻¹.

Sol. Here, surface tension of mercury, $T = 0.465$ N m⁻¹; density of mercury, $\rho = 13,600$ kg m⁻³

and radius of the tube, $R = \frac{1}{2} = 0.5 \text{ mm} = 5 \times 10^{-4} \text{ m}$

Now, the excess of pressure inside the air bubble blown at the lower end of the tube,

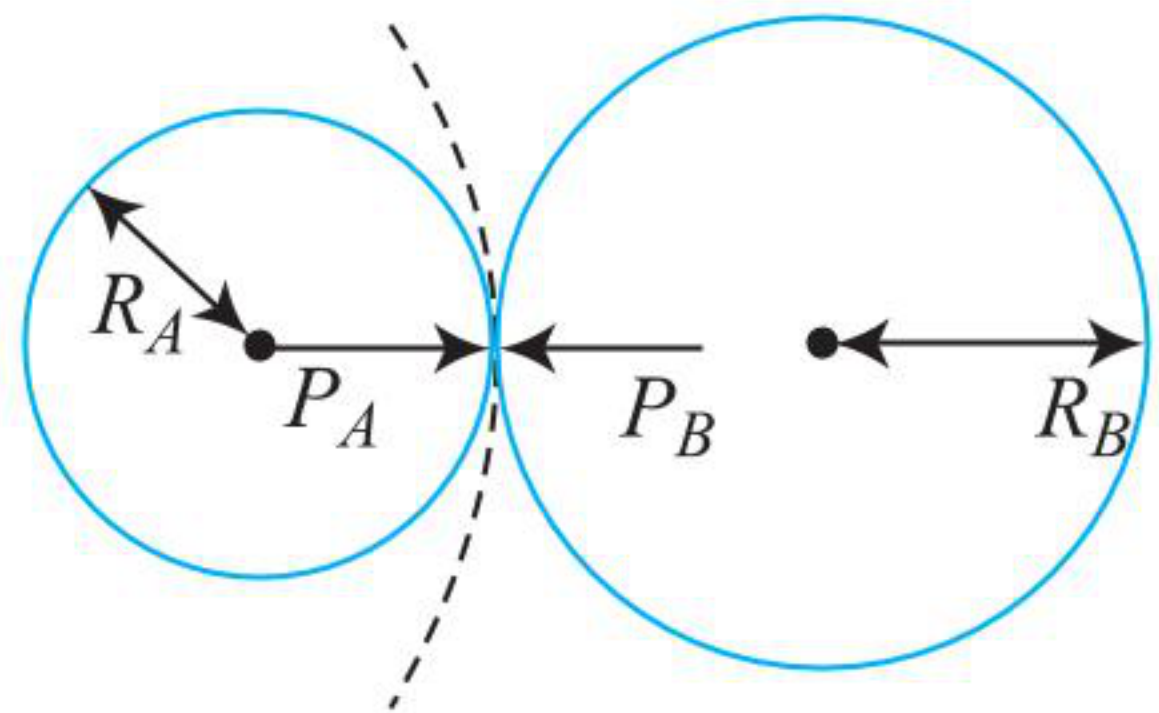
$$\begin{aligned} p_i - p_0 &= \frac{2T}{R} = \frac{2 \times 0.465}{5 \times 10^{-4}} = 1,860 \text{ N m}^{-2} \\ &= \frac{1,860}{13,600 \times 9.8} = 0.014 \text{ m of Hg} = 1.4 \text{ cm of Hg} \end{aligned}$$

ILLUSTRATION 8.13

Two separate air bubbles (radii 0.002 cm and 0.004 m) formed of the same liquid (surface tension 0.07 N/m) come together to form a double bubble. Find the radius and the sense of curvature of the internal film surface common to both the bubbles.

Sol. Let R be the radius of curvature of common surface when bubbles A and B of radii R_A and R_B coalesce. The excess pressure in A and B are $4TR_A$ and $4TR_B$, respectively.

$$\therefore p_A = \frac{4T}{R_A} \quad \text{and} \quad p_B = \frac{4T}{R_B}$$



If R is radius of common interface, then we must have

$$p_A - p_B = \frac{4T}{R} \quad \text{or} \quad \frac{4T}{R_A} - \frac{4T}{R_B} = \frac{4T}{R}$$

$$\text{This gives } R = \frac{R_A R_B}{R_B - R_A} = \frac{0.002 \times 0.004}{0.004 - 0.002} = 0.004 \text{ m}$$

As the excess pressure is always towards concave surface and pressure in smaller bubble is greater than that in larger bubble, the common surface is concave towards the centre of the smaller bubble.

ILLUSTRATION 8.14

A small hollow sphere which has a small hole in it is immersed in water to a depth 40 cm before any water penetrates into it. Find the radius of the hole. Surface tension of water = $75 \times 10^{-3} \text{ N m}^{-1}$ and density of water = 10^3 kg m^{-3} and $g = 10 \text{ m/s}^2$.

Sol. The depth of the water column, when the water just enters the hollow sphere through the hole,

$$h = 40 \text{ cm}$$

Let R be the radius of the hole in the hollow sphere. When the hollow sphere having hole is immersed in water, an air bubble is formed at the hole. The pressure due to water at the depth of 40 cm will try to force water into the hollow sphere, while the excess of pressure inside the air bubble will oppose it. The water will enter the hollow sphere just when the excess of pressure inside the air bubble becomes equal to the pressure exerted by 40 cm column of water.

$$\text{Now, } p_i - p_o = h\rho g = 0.40 \times 1000 \times 10$$

$$\text{or } p_i - p_o = \Delta p = 4.0 \times 10^3 \text{ N m}^{-2} \quad \dots(i)$$

As the radius of the air bubble formed will be equal to the radius of the hole,

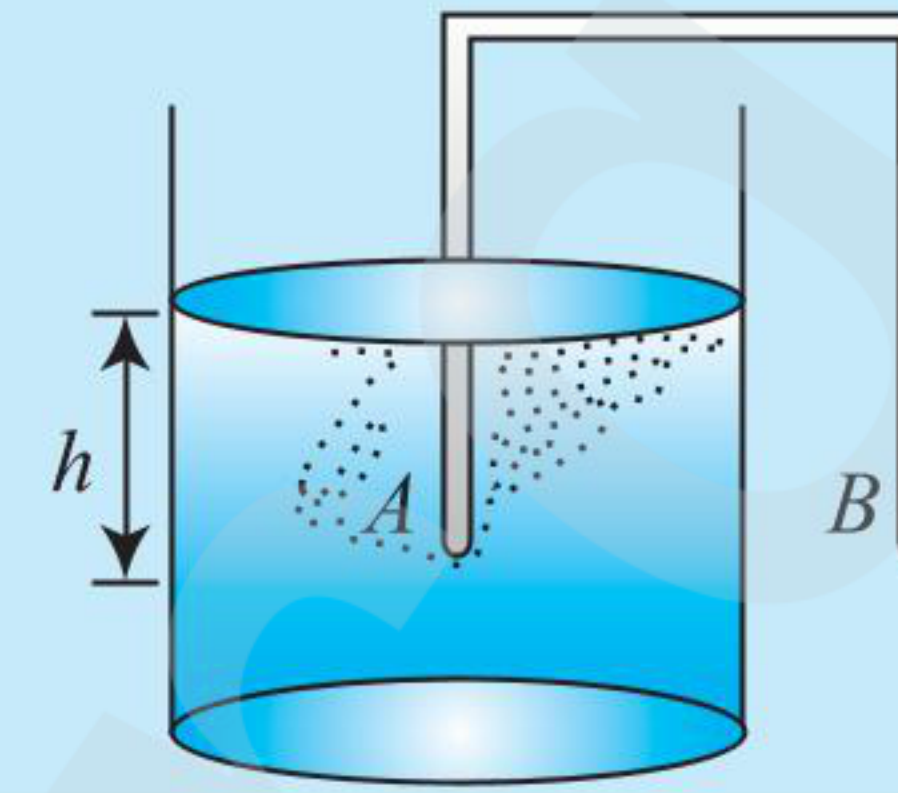
$$p_i - p_o = \Delta p = \frac{2T}{R} \quad \dots(ii)$$

From Eq. (ii), we have

$$R = \frac{2T}{\Delta p} = \frac{2 \times 75 \times 10^{-3} \text{ N/m}}{4.0 \times 10^3 \text{ N/m}^2} = 3.75 \times 10^{-6} \text{ m}$$

ILLUSTRATION 8.15

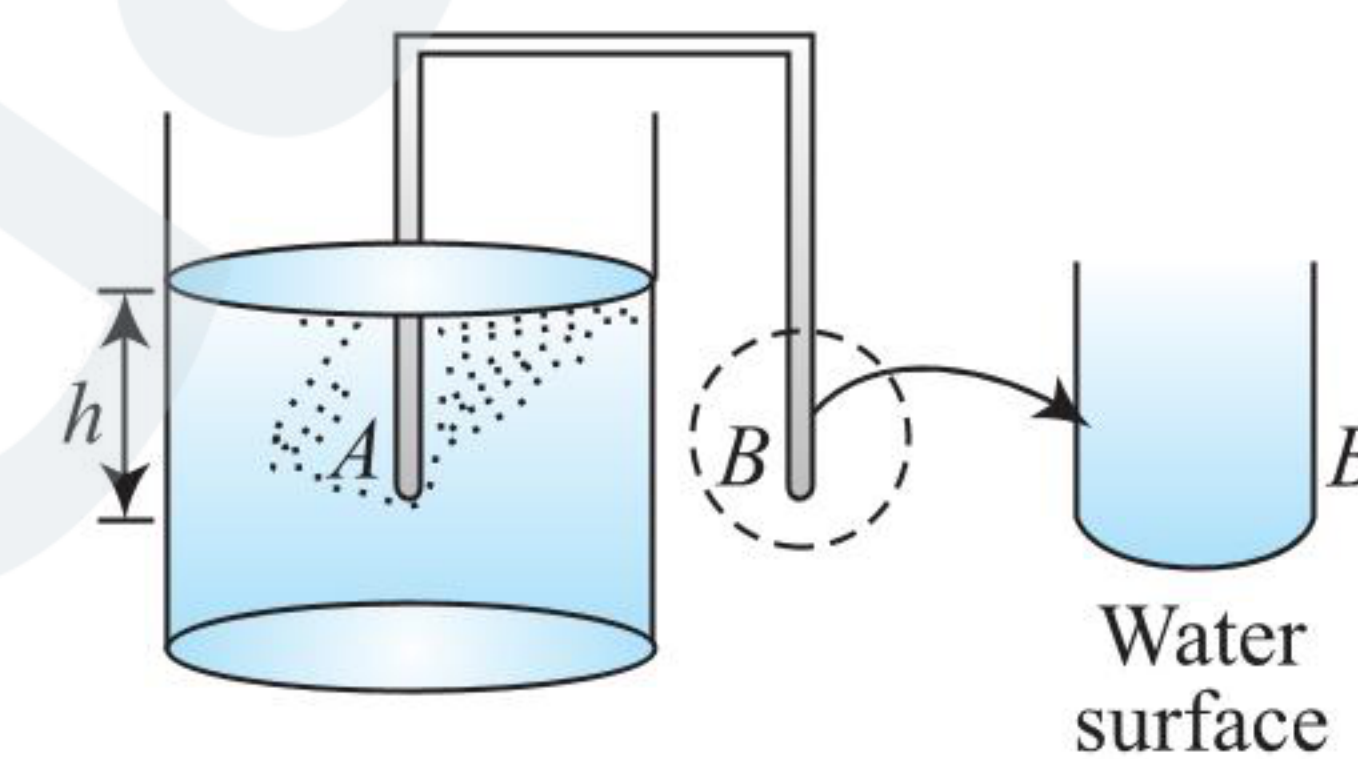
In the siphon shown in the figure the ends A and B of the tube are at same horizontal level. Water fills the entire tube but it does not flow out of the end B . With the help of a diagram show how the water surface at end B changes if the end B were slightly lower than the position shown.



Sol. The pressure at end B inside the siphon tube is

$$P_B = P_0 + \rho gh \quad [P_0 = \text{atmospheric pressure}]$$

$$\therefore \text{Gauge pressure } P_B - P_0 = \rho gh$$

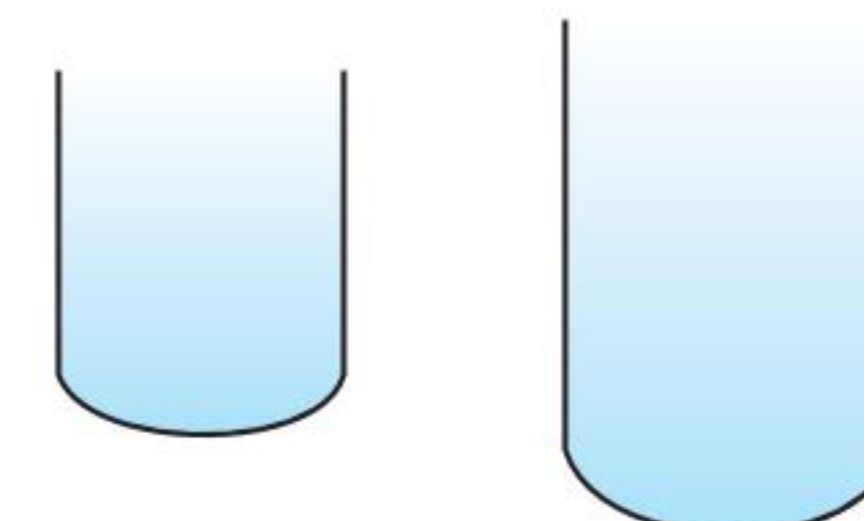


We know the pressure at concave side is always greater

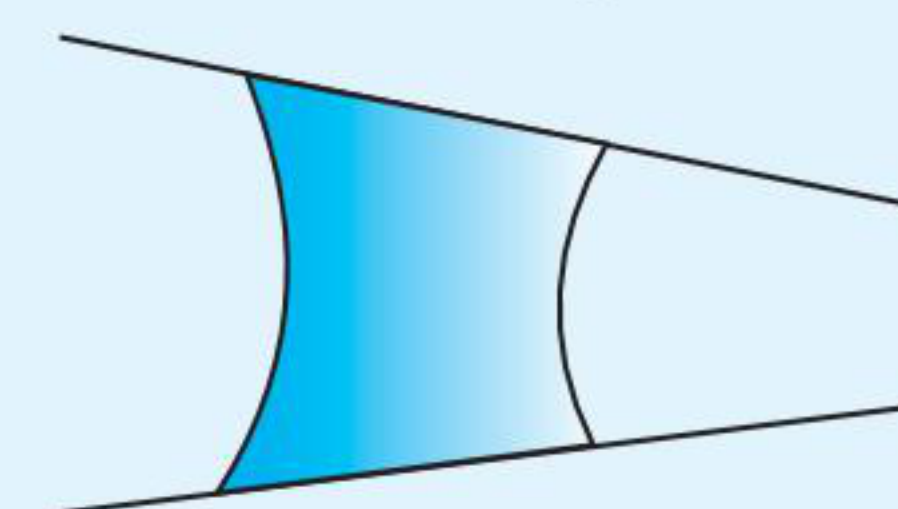
$$\therefore \frac{2T}{r} = \rho gh \quad [r = \text{radius of curvature of the surface}]$$

With increase in h , the curvature of water surface increases (i.e., r decreases) so as to prevent water from flowing out.

The figure shows the change in surface as B is lowered.

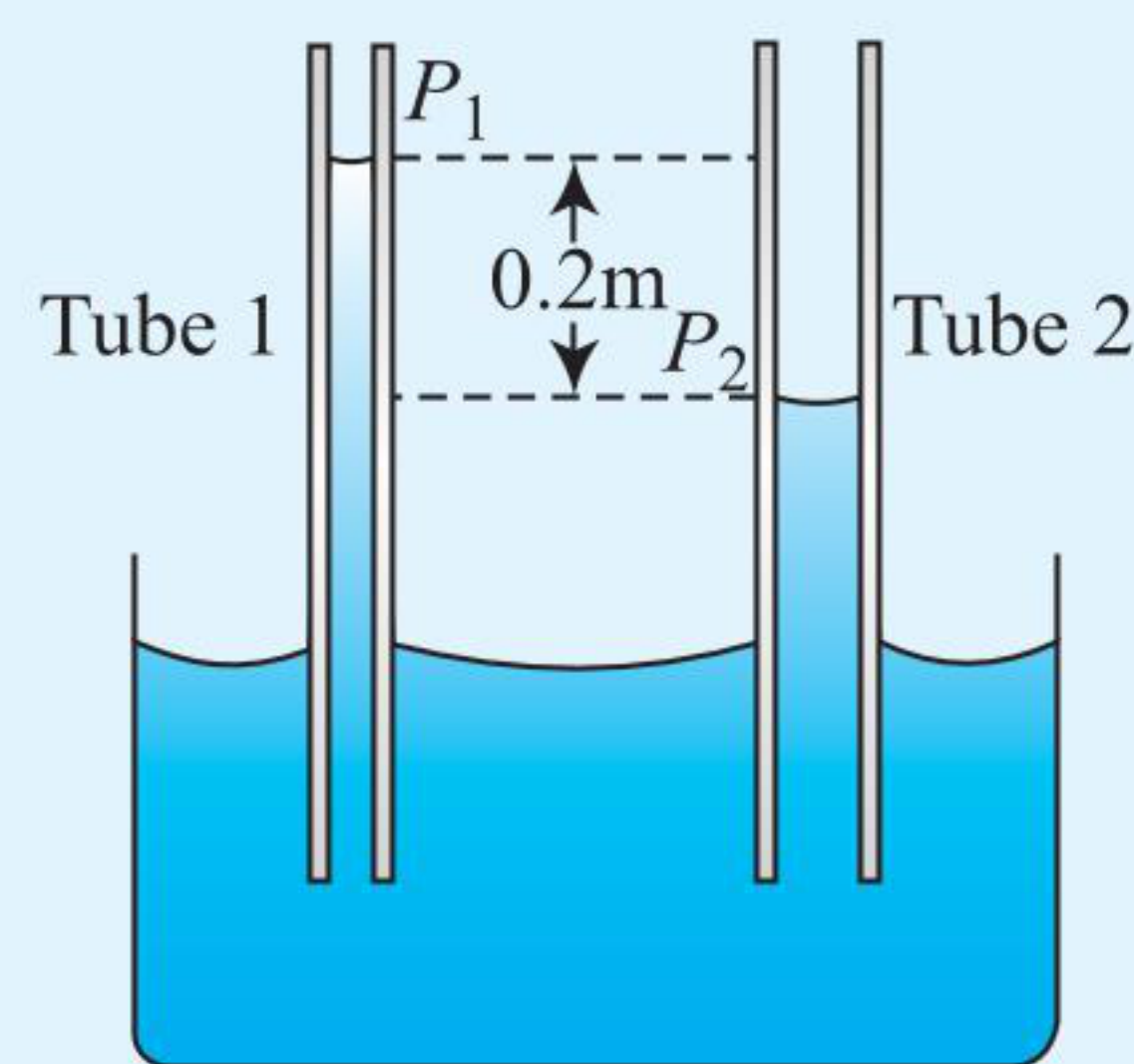
**CONCEPT APPLICATION EXERCISE 8.3**

1. A conical pipe shown in figure has a small water drop. In which direction does the drop will tend to move?

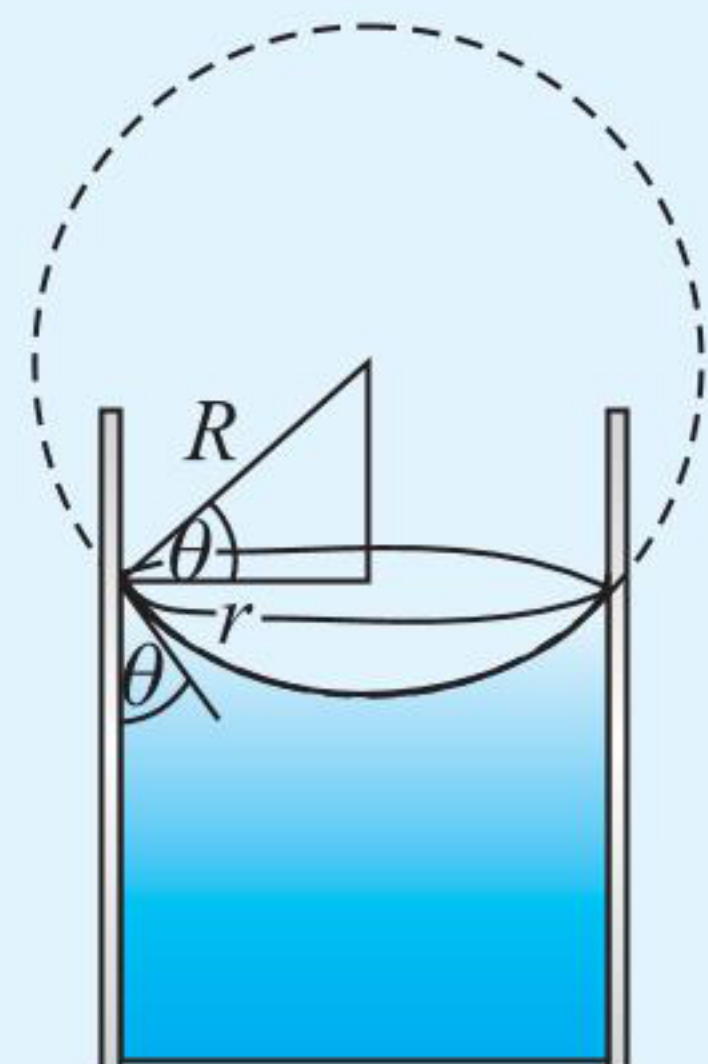


2. There is an air bubble of radius 1.0 mm in a liquid of surface tension 0.075 N/m and density 1000 kg/m^3 . The bubble is at a depth of 10 cm below the free surface. By what amount is the pressure inside the bubble greater than the atmospheric pressure? Take $g = 10 \text{ m/s}^2$.
3. Two soap bubbles have radii in the ratio 1 : 2. Compare the excess of pressure inside these bubbles. Also compare the work done in blowing these bubbles
4. Consider a rain drop falling at terminal speed. For what radius (R) of the drop can we disregard the influence of gravity on its shape? Surface tension and density of water are T and ρ respectively.

5. A barometer contains two uniform capillaries of radii $5.0 \times 10^{-4} \text{ m}$ and $1.0 \times 10^{-3} \text{ m}$. If the height of liquid in narrow tube is 0.2 m more than that in the wide tube, calculate the true pressure difference. (Given: Density of liquid = 10^3 kg m^{-3} , surface tension = $75 \times 10^{-3} \text{ N m}^{-1}$, angle of contact = 0° and $g = 10 \text{ m s}^{-2}$.)



(a)



(b)

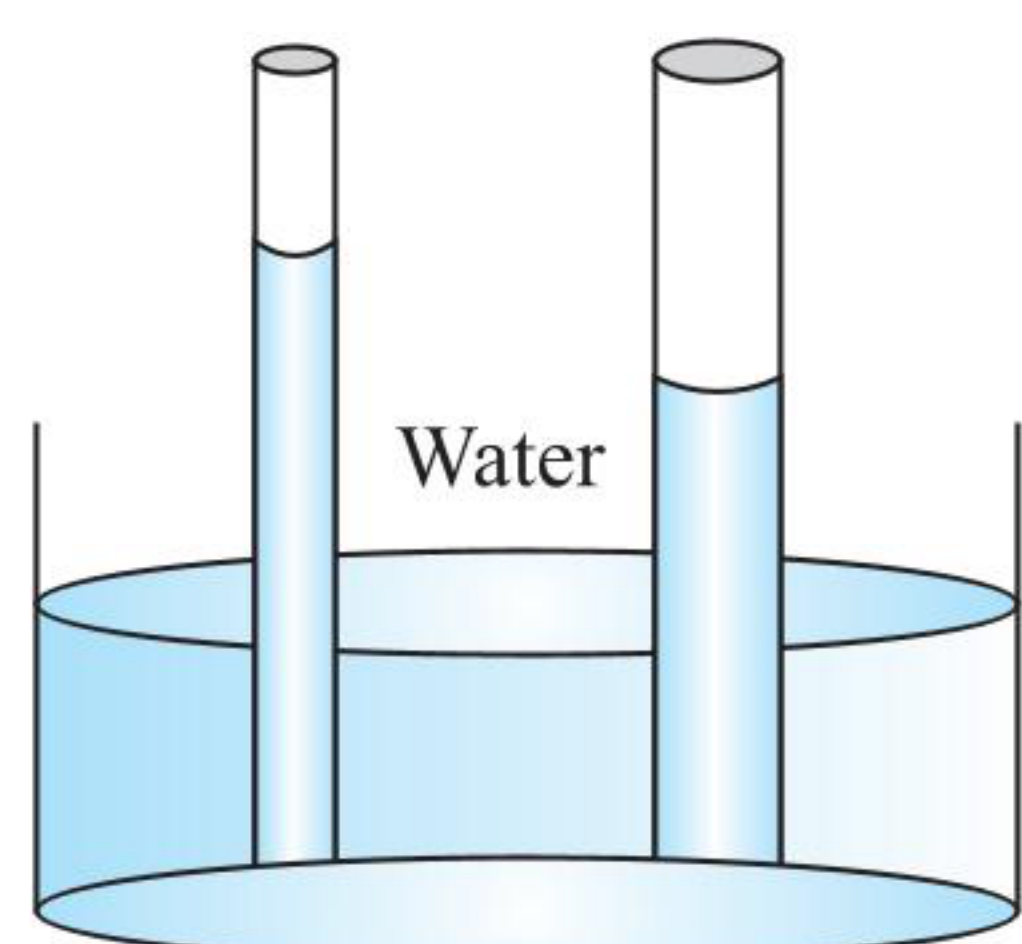
ANSWERS

1. Towards the tapered end. 2. 1150 Pa
3. 1 : 4 5. 1850 N/m²

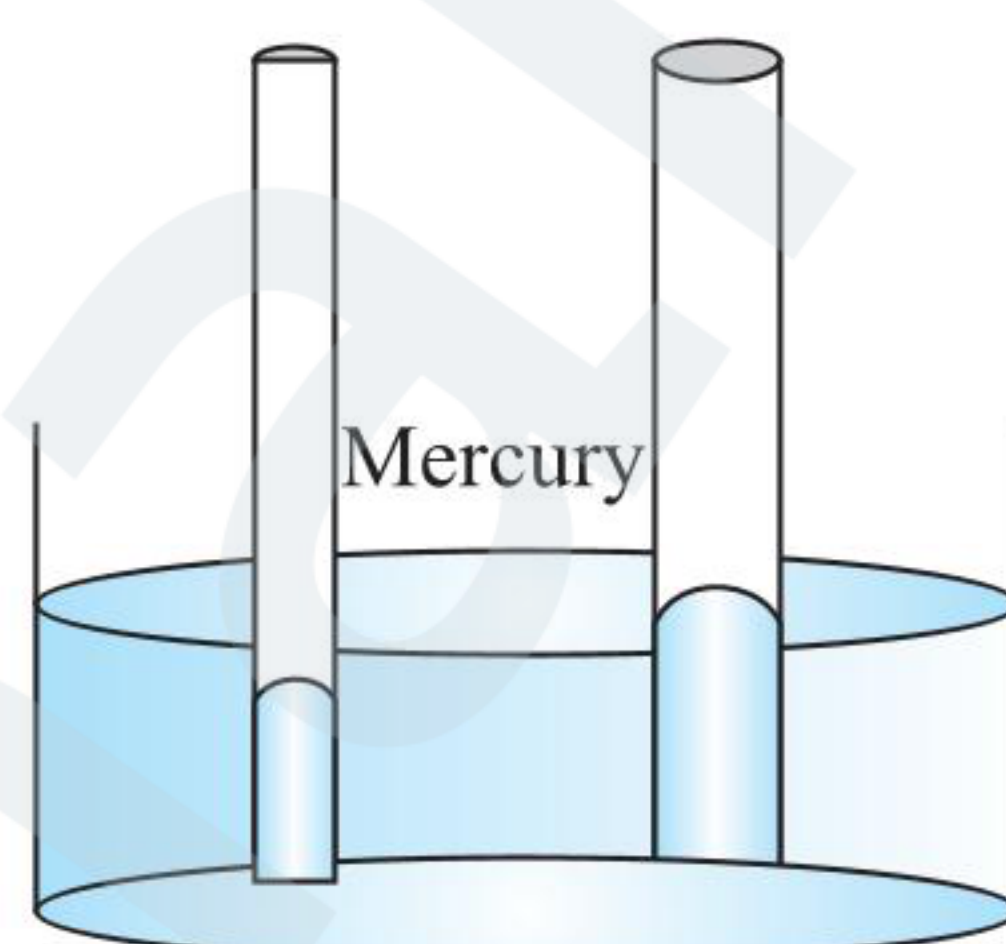
CAPILLARITY

If a narrow glass tube open at both ends is pushed in water as shown in the figure, water rises in the tube to a height above its surface. The narrower the tube, the greater is the height to which water rises. This phenomenon is known as capillarity. When the same capillary tubes are placed in mercury, the liquid is depressed below the outside level. The depression increases as the diameter of the capillary tube decreases. A close observation of the capillary phenomenon reveals the following facts.

The free surface (meniscus) of the liquid which rises in the capillary tube is concave upward. The meniscus of liquid which falls in the capillary tube is convex upward.



(a)



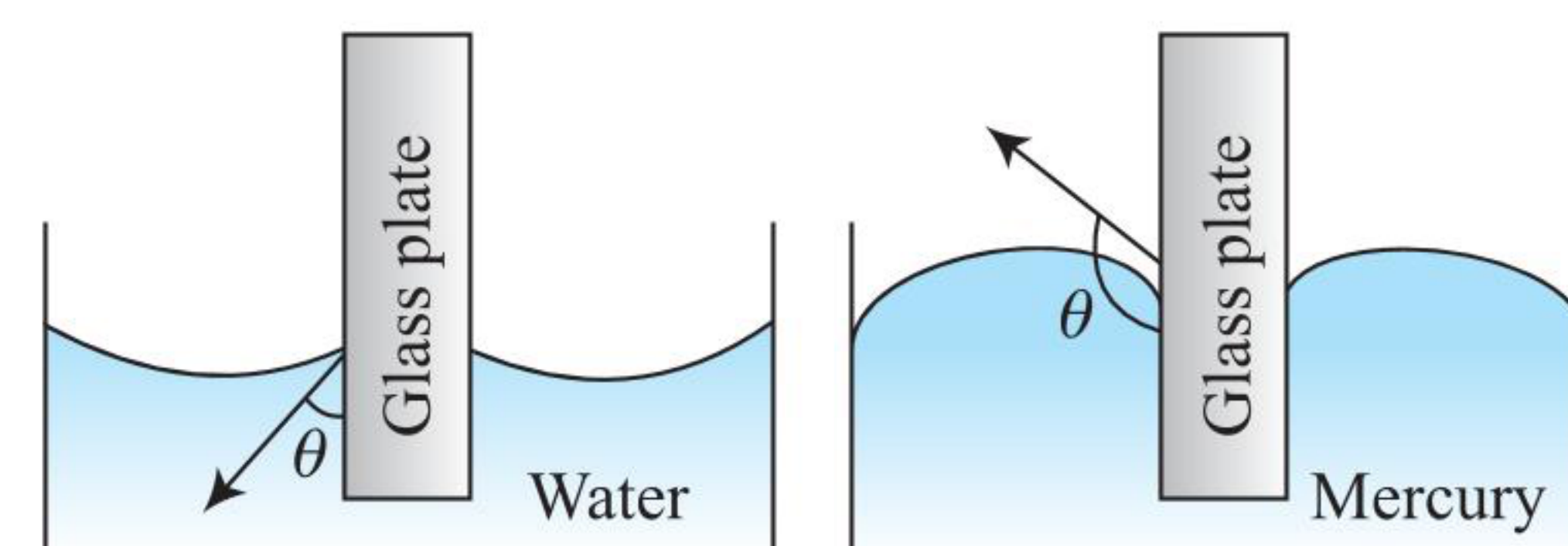
(b)

ANGLE OF CONTACT

It is seen that when a liquid comes in contact with a solid surface, it is generally curved.

The angle θ , which the tangent to the liquid surface at the point of contact makes with the solid surface inside the liquid, is called the angle of **contact** or the capillary angle.

The angle of contact is acute (less than 90°) in the case of liquids, which wet the walls of the container [Fig. (a)] and is obtuse (greater than 90°) for the liquids which do not wet the walls of the container as shown in Fig. (b).



(a)

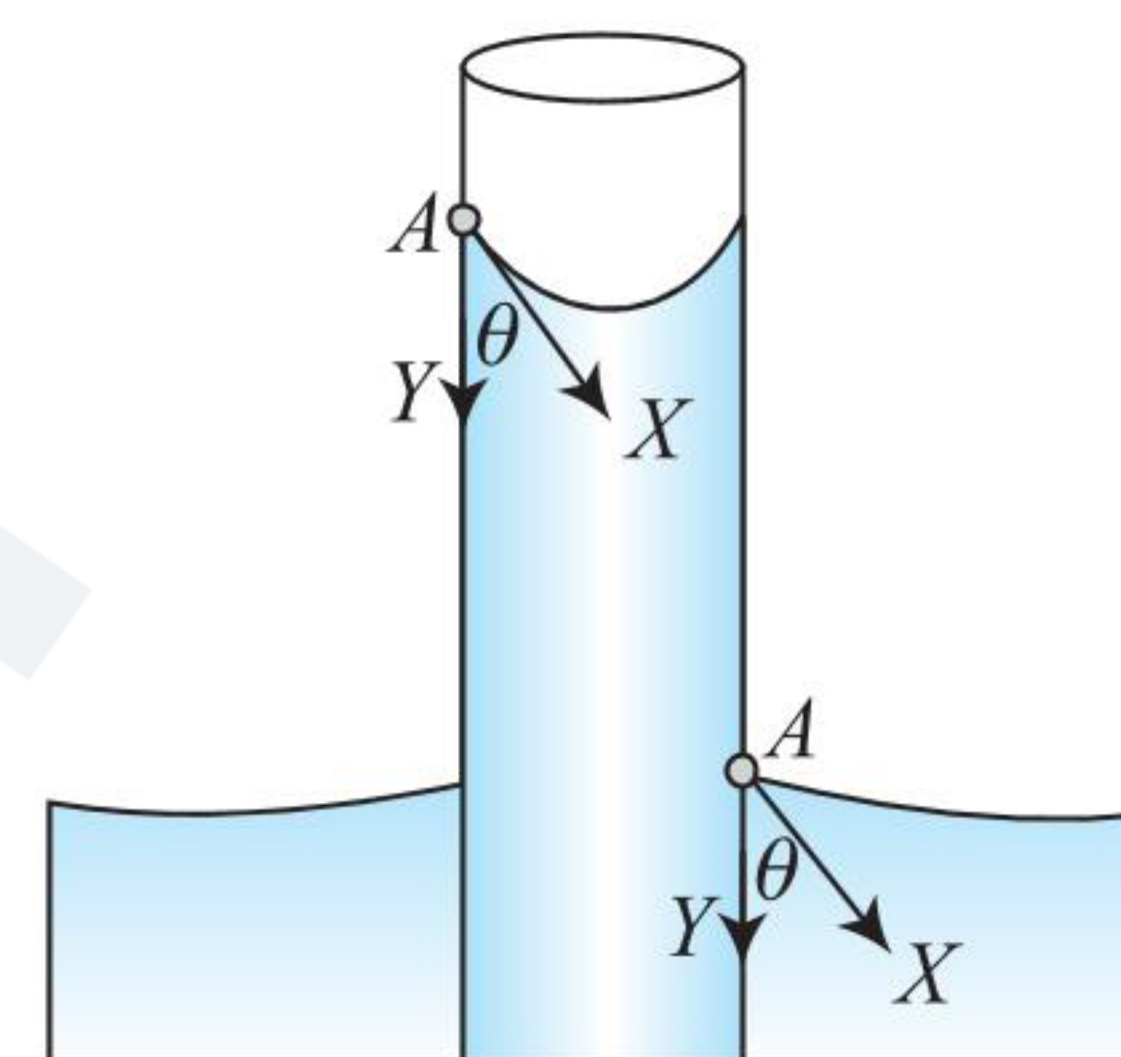
(b)

The angle of contact for pure water and clean glass surface is zero.

Those liquids, which wet the walls of the container (say in case of water and glass), have meniscus concave upwards and their value of angle of contact is less than 90° (also called acute angle). However, those liquids, which do not wet the walls of the container (say in case of mercury and glass), have meniscus convex upwards and their value of angle of contact is greater than 90° (also called obtuse angle).

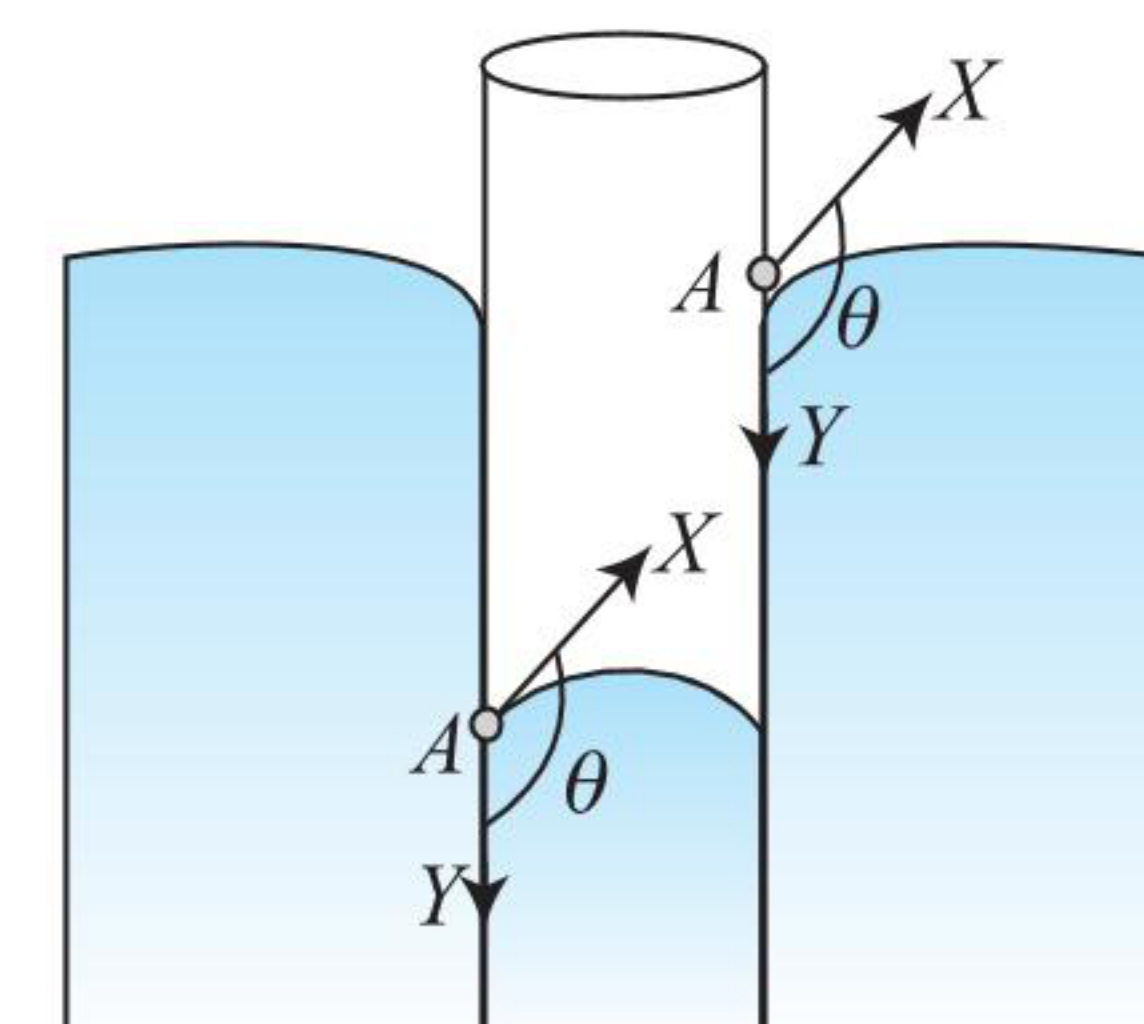
Consider two tubes dipping in two different liquids (water and mercury) as shown in Figs. (a) and (b).

Let A represent the point of contact of the liquid with the tube. Through A , draw the tangent AX to the liquid surface and another tangent AY to the solid surface inside the liquid. Then $\angle XAY = \theta$ (say) is called the angle of contact between the liquid and the given solid. Thus,



Water

(a)



Mercury

(b)

RISE OF A LIQUID IN A CAPILLARY TUBE (ASCENT FORMULA)

Consider a capillary tube of radius r open at both ends and dipped into a liquid, which wets the walls of the tube. Such a liquid rises in the tube forming a concave meniscus as already discussed.

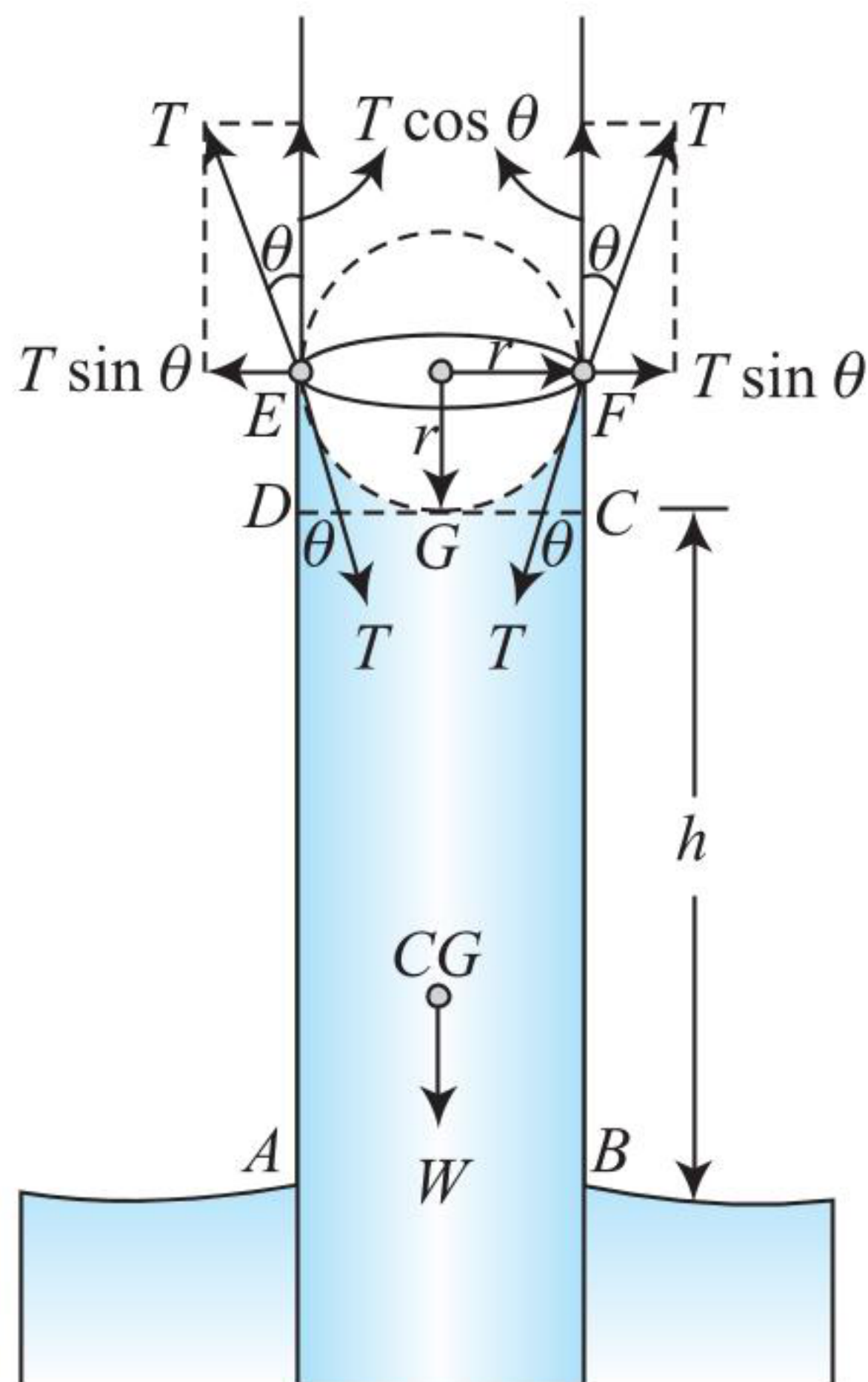
The liquid surface is in contact with the walls of the tube all along the circle of circumference $2\pi r$, where r is the radius of the tube. Due to surface tension of the liquid, force T per unit length acts along the tangent to the meniscus and in inward direction. Since the liquid surface has a tendency to contract, it pulls the tube inward all along the circumference of the circle of contact. Thus, the liquid exerts a force T per unit length on the glass tube and in accordance with Newton's third law of motion, the tube gives an equal and opposite reaction to the liquid in a direction as shown in the figure.

This reaction can be resolved into two rectangular components:

- $T \cos \theta$, acting upward along the walls of the tube,
- $T \sin \theta$, acting perpendicular to the walls of the tube. This component, therefore, plays no part in raising the liquid in the tube.

Here, θ = angle of contact between the liquid and the glass; Let h = height to which the liquid rises in the tube; and ρ = density of

the liquid at room temperature. The only component responsible for raising the liquid in the tube is $T \cos \theta$.



The component $T \cos \theta$ acts all along the circumference of the circle of contact of the liquid with the tube.

$$\text{The total upward force on the liquid} = 2\pi r \times T \cos \theta \quad \dots(i)$$

Under the effect of this upward force, the liquid rises in the tube to a height, say h ; till the weight of the liquid above the free surface becomes just equal to the vertical upward force.

Volume of the liquid in the cylindrical column $ABCD$ of height h and radius r , $V = \pi r^2 h$.

If the tube is narrow, the radius of the curvature of the meniscus is approximately equal to the radius (r) of the capillary tube.

Volume of the liquid under the meniscus

= volume of the cylinder $CDEF$ of height r and radius r
 – volume of the hemisphere EGF of radius r

$$= \pi r^2 \times r - \frac{1}{2} \times \frac{4}{3} \pi r^3 = \frac{1}{3} \pi r^3$$

Total volume of the liquid in the capillary tube

$$= \pi r^2 h + \frac{1}{3} \pi r^3 = \pi r^2 (h + r/3)$$

Hence, mass of liquid in the tube.

$$m = \text{volume} \times \text{density} = \pi r^2 \left(h + \frac{r}{3} \right) \rho \quad \dots(ii)$$

The weight of the liquid in the tube.

$$W = mg = \pi r^2 \left(h + \frac{r}{3} \right) \rho g$$

This weight acts at the CG (centre of gravity) of liquid column in the tube.

In equilibrium, total upward force, F = weight of the liquid in the tube, W . From Eqs. (i) and (ii), we have

$$2\pi r T \cos \theta = \pi r^2 \left(h + \frac{r}{3} \right) \rho g$$

$$\text{or } 2T \cos \theta = r \left(h + \frac{r}{3} \right) \rho g \quad \dots(iii)$$

$$\text{or } h + \frac{r}{3} = \frac{2T \cos \theta}{r \rho g}$$

$$\text{or } h = \frac{2T \cos \theta}{r \rho g} - \frac{r}{3} \quad \dots(iv)$$

This relation is known as the ascent formula or Jurin's equation.

When the tube is of a very fine bore, $r/3$ may be neglected as compared to h and the value of surface tension T is given by [From Eq. (iv)]

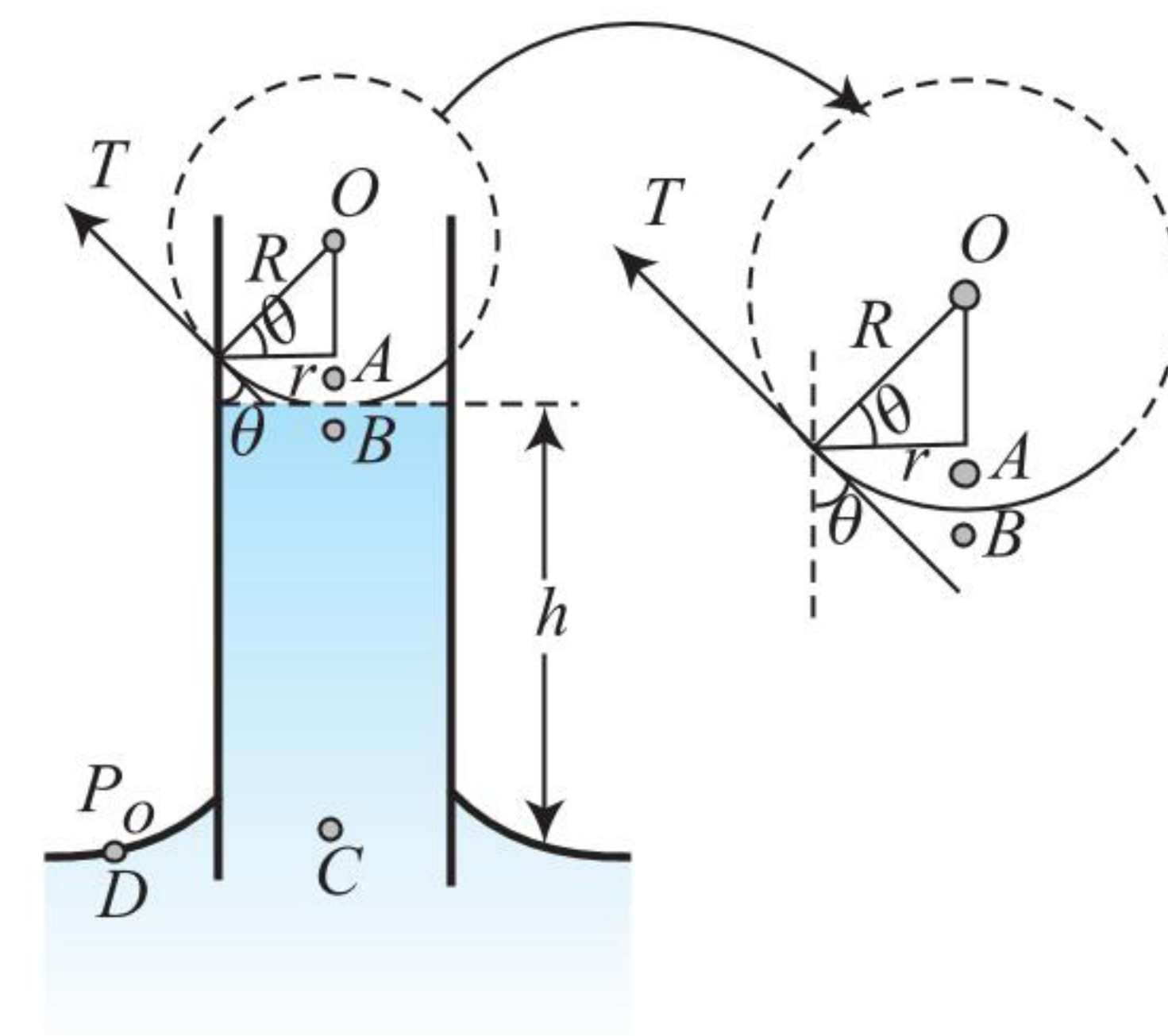
$$h = \frac{2T \cos \theta}{r \rho g} \quad \dots(v)$$

This relation is also called the **ascent formula**.

As, r , θ , ρ and g are constants, therefore, for tubes of different radii, $h \propto \frac{1}{r}$. Thus, the liquid rises more in a narrow tube and less in a wider tube. When pure water is contained in a glass tube, $\theta = 0^\circ$, $\cos \theta = \cos 0^\circ = 1$.

$$\text{From Eq. (v), } h = \frac{2T}{r \rho g} \quad \dots(vi)$$

Alternative method: Let us take four points A , B , C and D as shown in the figure. A and B are the points just above and just below the meniscus, respectively.



$$\text{Now, we have } P_A = P_B + \frac{2T}{R}$$

where R = radius of curvature of liquid meniscus. Substituting $P_A = P_0$, we have

$$P_B = P_0 - \frac{2T}{R} \quad \dots(vii)$$

If C is the point at the bottom of the excess liquid column, we have

$$P_C = P_B + \rho g h$$

where h = height of the excess liquid column.

Substituting $P_C = P_D = P_0$, we have

$$P_B = P_0 - \rho g h \quad \dots(viii)$$

Using Eqs. (vii) and (viii), we have

$$P_0 - \rho g h = P_0 - \frac{2T}{R}$$

This gives $h = \frac{2T}{\rho g R}$ where $R = \frac{r}{\cos \theta}$

$$\text{Then } h = \frac{2T \cos \theta}{\rho g r}$$

Important Points:

- In case of liquids which do not wet the container, θ is obtuse (i.e. $\theta > 90^\circ$) and $\cos \theta$ is negative. From Eq. (v), h is negative, i.e., the level of the liquid in the tube is below the liquid level outside the tube.
- Equation (v) can be obtained in a much simpler way as given below. Let us neglect the volume of the liquid below the meniscus. Weight of the liquid in the cylinder $ABCD$, i.e., $W = \text{volume of cylinder } ABCD \times \text{density of the liquid} \times g$ or $W = \pi r^2 h \rho g$... (ix)

As we have already derived in Eq. (i),
total upward force due to surface tension, i.e.,

$$F = 2\pi r T \cos \theta \quad \dots (x)$$

In equilibrium, from Eqs. (ix) and (x),

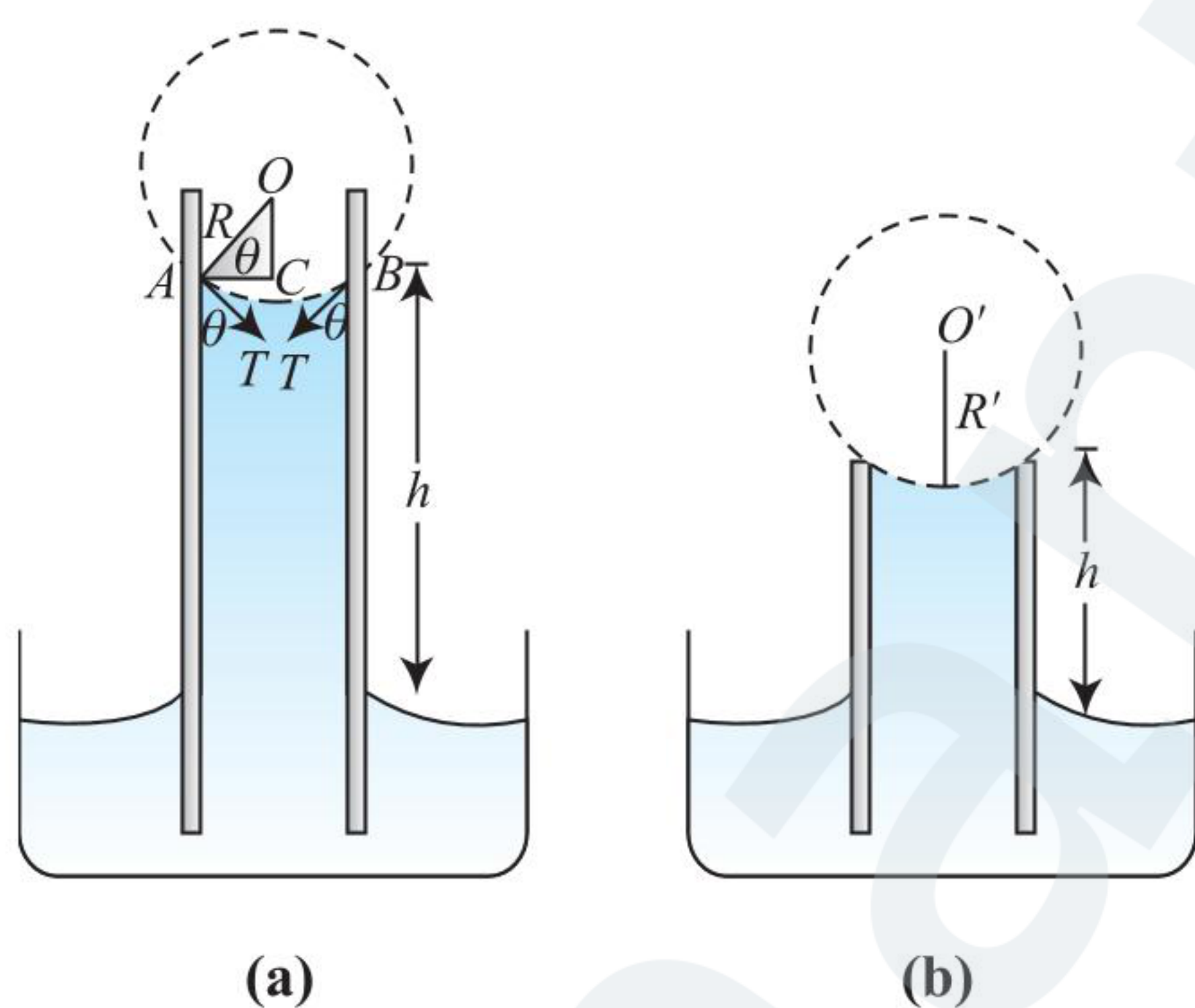
$$\pi r^2 h \rho g = 2\pi r T \cos \theta \quad \text{or} \quad h = \frac{2T \cos \theta}{\rho g r}$$

RISE OF LIQUID IN A CAPILLARY TUBE OF INSUFFICIENT LENGTH

Consider that a capillary tube of radius r is dipped in a liquid of surface tension T . The height to which the liquid will rise in the tube is given by

$$h = \frac{2T \cos \theta}{r \rho g} \quad \dots (i)$$

Let O be the centre of curvature and R , the radius of curvature of the meniscus of the liquid [Fig. (a)].



From the right angled ΔACO , we have

$$\cos \theta = \frac{r}{R} \quad \text{or} \quad r = R \cos \theta$$

Substituting for r in the Eq. (i), we have

$$h = \frac{2T \cos \theta}{(R \cos \theta) \rho g} = \frac{2T}{R \rho g}$$

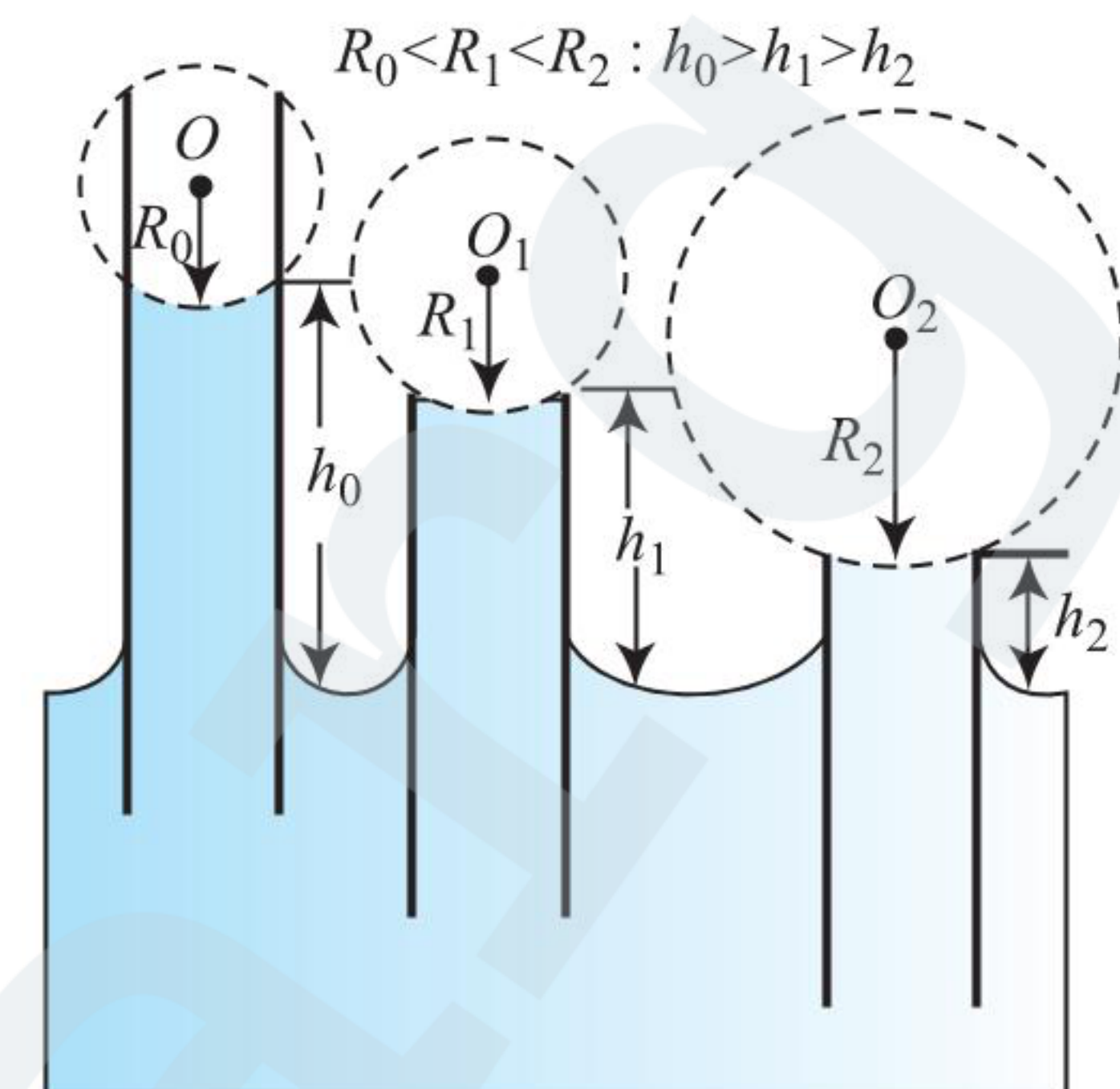
$$\text{or} \quad Rh = \frac{2T}{\rho g} = a \text{ (constant)} \quad \dots (ii)$$

When tube is of insufficient length. If the length of the tube h' is less than h [Fig. (b)], it is found that the liquid does not overflow. In a tube of insufficient length, the liquid rises up to the top of the tube and increases the radius of curvature of its meniscus to a value R' , so that

$$R'h' = Rh \quad \dots (iii)$$

i.e., smaller the length (h') of the tube, greater will be the radius of curvature (R') of the meniscus, but the liquid will never overflow.

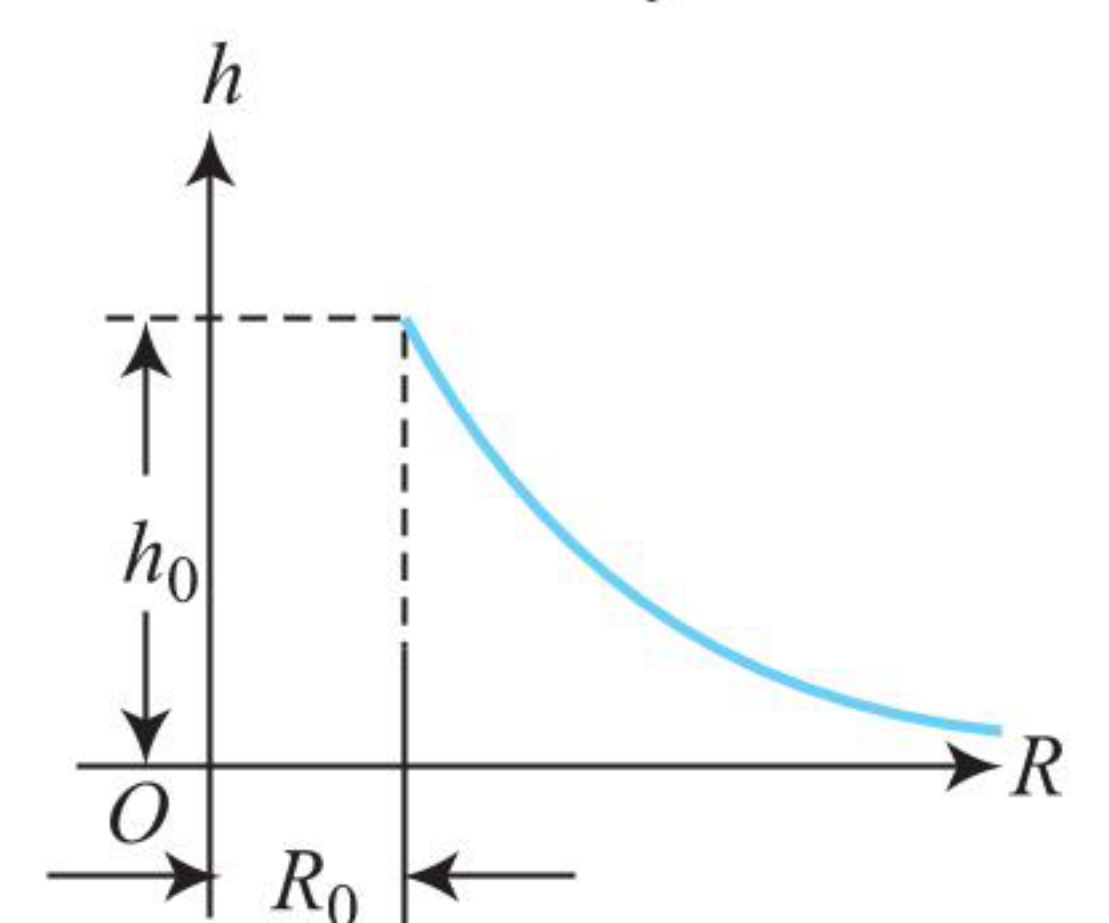
It should be clearly noted that whereas the radius of curvature of the meniscus changes, angle of contact remains constant as it depends upon the nature of the liquid and solid.



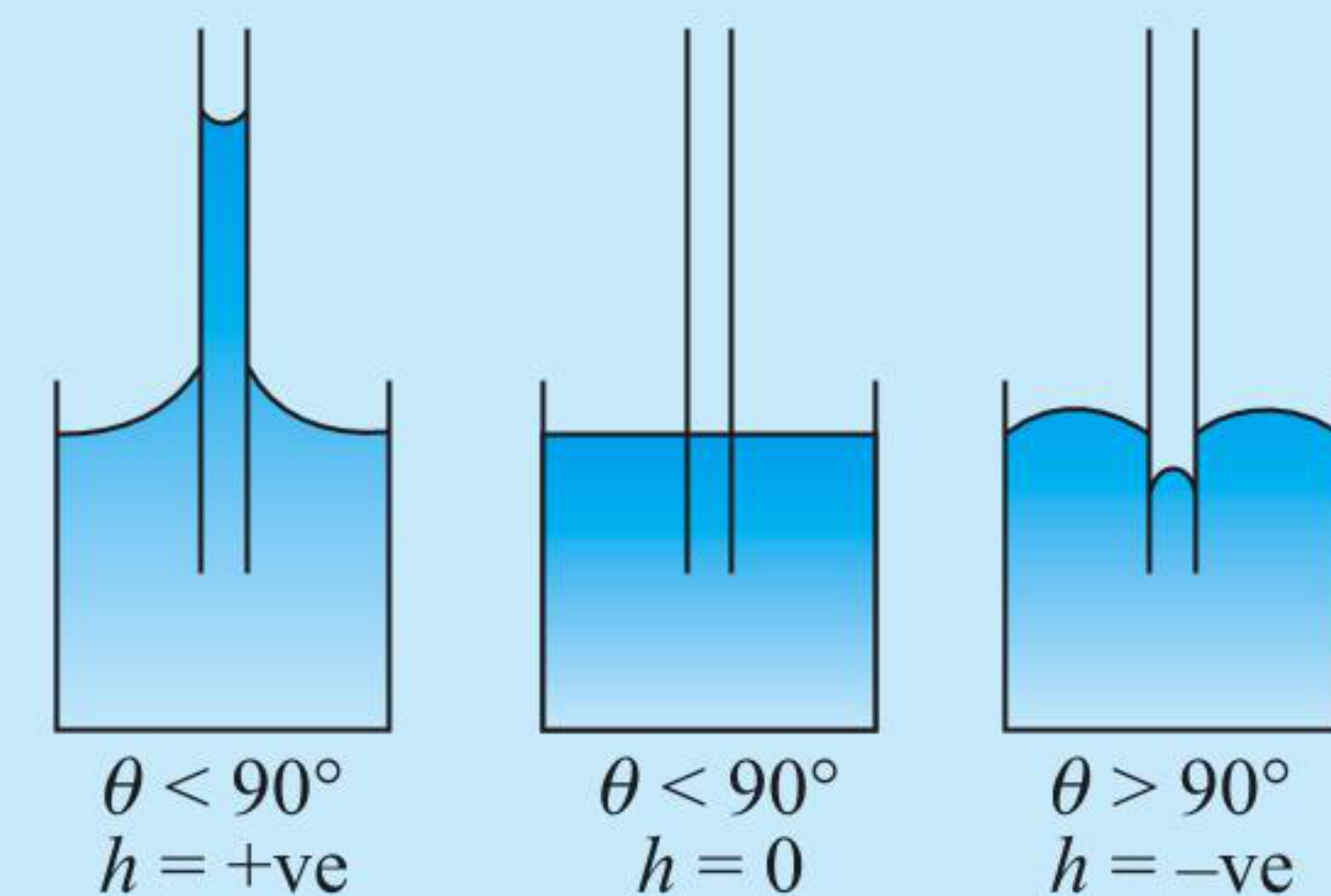
If we slowly push the capillary tube vertically down (into) the liquid, the height of the liquid column remains the same till the length of the tube above the liquid surface is equal to $2T \cos \theta / (\rho g r) (= h)$. Therefore, the liquid meniscus gets flattened gradually by increasing its radius of curvature to obey the law $hR = \text{constant}$.

It means that $hr = h_1 r_1 = h_2 r_2 = \dots$

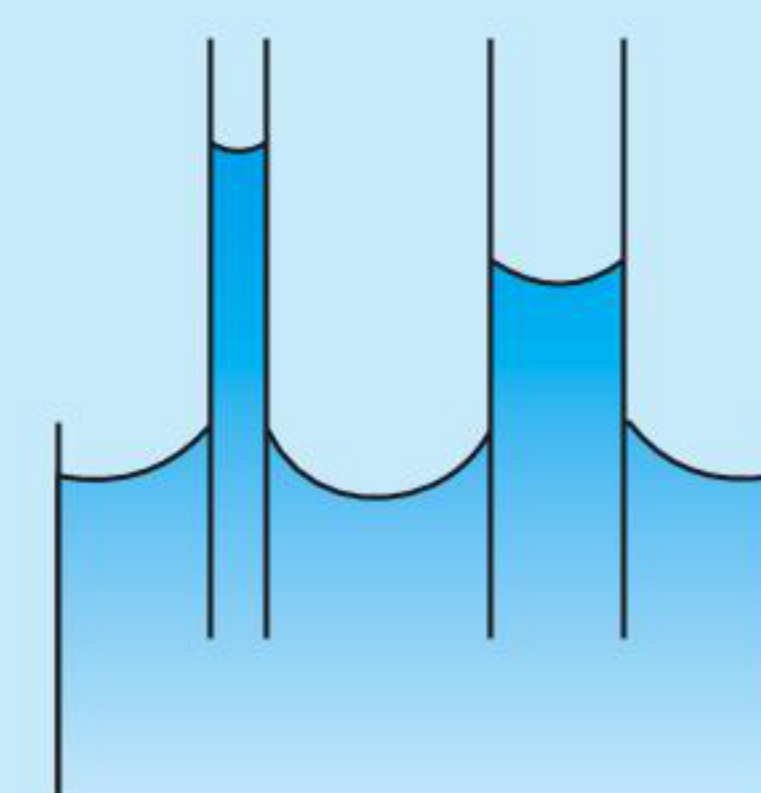
When the capillary tube just sinks, i.e., $h = 0$, we can see that the meniscus becomes flat, radius of curvature $r \rightarrow \infty$.

**Important Points**

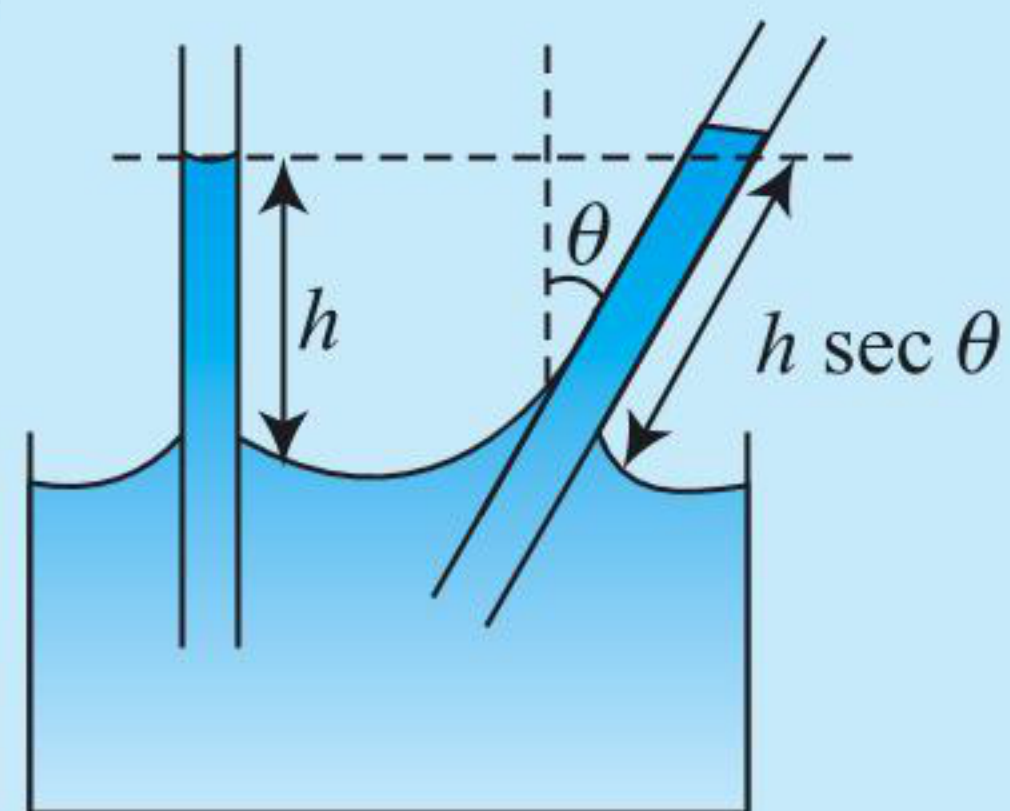
- h depends on surface tension T , angle of contact θ , radius of the tube, density ρ of liquid and acceleration due to gravity (or effective g). Hence the capillary action depends on the nature of liquid and solid (capillary tube) in contact, temperature of the surroundings.
- If $\theta < 90^\circ$, h is positive (wetting) which signifies capillary rise and concave meniscus; if $\theta = 90^\circ$, $h = 0$ (neither wetting nor non-wetting): plane meniscus; if $\theta > 90^\circ$, $h = \text{negative}$ (non-wetting) which signifies capillary fall and convex meniscus.



- Since T , ρ and θ are constant at a given temperature, for any given liquid and capillary tube, $hr = c$; this tells us that capillary rise (or fall) will be greater in thinner tubes and vice versa.



- Since T, ρ, θ, g and R are constant for a given set of capillary and liquid, we have $h = \text{constant}$. It means that if we change the angle β of orientation from vertical, the capillary rise h (or capillary fall) remains constant even though the length of the excess liquid in the tube increases from h to $h \sec \beta$.



- If the radius of the liquid meniscus remains constant, the weight (but not length) of the liquid column remains the same in capillary tubes of different shapes and sizes.

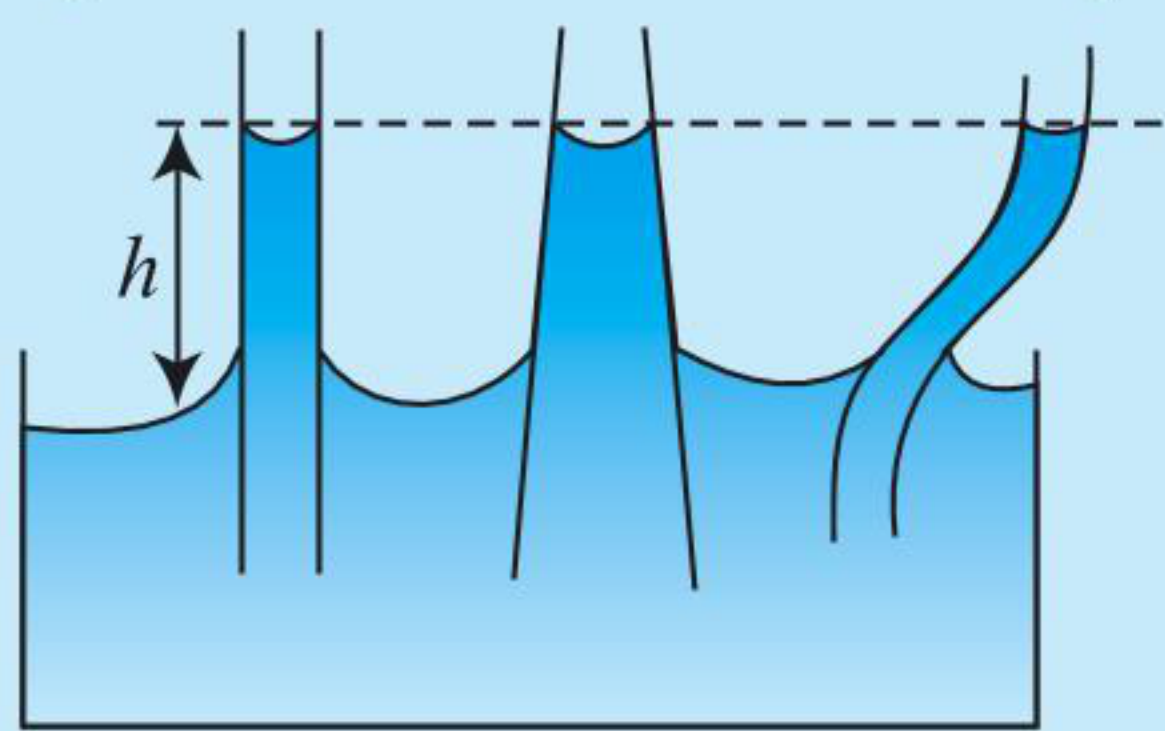


ILLUSTRATION 8.16

A vertical capillary with inside diameter 0.50 mm is submerged into water so that the length of its part emerging outside the water surface is equal to 25 mm. Find the radius of curvature of the meniscus. Surface tension of water is 73×10^{-3} N/m.

Sol. In the capillary tube, the water should rise to a height

$$h = \frac{2T}{r\rho g}$$

$$\text{Here } T = 73 \times 10^{-3} \text{ Nm}, r = \frac{0.50 \text{ mm}}{2} = 0.25 \times 10^{-3} \text{ m},$$

$$\rho = 10^3 \text{ kg/m}^3$$

$$\therefore h = \frac{2 \times 73 \times 10^{-3}}{0.25 \times 10^{-3} \times 10^3 \times 9.8} = 59 \times 10^{-3} \text{ m} = 59 \text{ mm}$$

Now $h > h'$, i.e., length is outside water surface.

Therefore, radius of meniscus $>$ radius of capillary r .

If R is the radius of meniscus, then we have

$$\frac{2T}{R} = h'\rho g \quad \text{or} \quad R = \frac{2T}{h'\rho g}$$

$$\text{Here } h' = 25 \text{ mm} = 25 \times 10^{-3} \text{ m}$$

$$\therefore R = \frac{2 \times 73 \times 10^{-3}}{25 \times 10^{-3} \times 10^3 \times 9.8} = 0.6 \times 10^{-3} \text{ m} = 0.6 \text{ mm}$$

ILLUSTRATION 8.17

A glass rod of diameter $d_1 = 1.5$ mm is inserted symmetrically into a glass capillary with inside diameter $d_2 = 2.0$ mm. Then the whole arrangement is vertically oriented and brought in contact with the surface of water. To what height will the liquid rise in the capillary? Surface tension of water = 73×10^{-3} N/m.

Sol. If R is radius of meniscus, then $\frac{2T}{R} = h\rho g$

$$\text{Here } R = \frac{r_2 - r_1}{\cos \theta}$$

θ being angle of contact, r_1 = radius of glass rod, r_2 = radius of capillary.

$$\frac{2T \cos \theta}{r_2 - r_1} = h\rho g \quad \text{or} \quad h = \frac{2T \cos \theta}{(r_2 - r_1)\rho g}$$

$$\text{Here } r_1 = d_1/2, r_2 = \frac{d_2}{2}$$

$$\therefore h = \frac{4T \cos \theta}{(d_2 - d_1)\rho g}$$

Substituting given values and $\theta \cong 0^\circ$ for water-glass interface, we have

$$h = \frac{4 \times 73 \times 10^{-3} \cos 0^\circ}{(2.0 - 1.5) \times 10^{-3} \times 10^3 \times 9.8} = 60 \times 10^{-3} \text{ m} = 6 \text{ cm}$$

ILLUSTRATION 8.18

The end of a capillary tube with a radius r is immersed in water. Is mechanical energy conserved when the water rises in the tube? The tube is sufficiently long. If not, calculate the energy change.

Sol. In the equilibrium position ($\theta = 0^\circ$ for water and glass)

$$2\pi r T \cos 0^\circ = \pi r^2 h \rho g$$

$$\text{or} \quad h = \frac{2T}{\rho g r}$$

$$\text{Work done by surface tension} = (2\pi r T) \times h = \frac{4\pi T^2}{\rho g}$$

The potential energy of water in the tube, $U = (\pi r^2 h \rho)gh/2$; it is multiplied by $h/2$ because the centre of gravity of the water and the capillary tube is at a height $h/2$

$$\therefore U = \frac{2\pi T^2}{\rho g}$$

Thus, it is seen that the mechanical energy is not conserved.

$$\text{Therefore, mechanical energy loss} = \frac{4\pi T^2}{\rho g} - \frac{2\pi T^2}{\rho g} = \frac{2\pi T^2}{\rho g}$$

This energy is converted into heat.

ILLUSTRATION 8.19

A glass capillary tube of internal radius $r = 0.25$ mm is immersed in water. The top end of the tube projects by 2 cm above the surface of water. At what angle does the liquid meet the tube? Surface tension of water = 0.7 N m^{-1} .

Sol. Water wets glass and so the angle of contact is zero. Neglecting the small mass in the meniscus, for full rise,

$$2\pi r T = \pi r^2 h \rho g$$

$$\text{or} \quad h = \frac{2T}{r\rho g} = \frac{2 \times 0.7}{0.25 \times 10^{-3} \times 1000 \times 9.8} = 0.057 \text{ m} = 5.7 \text{ cm}$$

But here the tube is only 2 cm above the water and so water will rise by 2 cm and meet the tube at an angle such that

$$2\pi rT \cos \theta = \pi r^2 h' \rho g$$

$$\Rightarrow 2T \cos \theta = h' r \rho g \Rightarrow \cos \theta = \frac{h' r \rho g}{2T}$$

$$\therefore \cos \theta = \frac{2 \times 10^{-2} \times 0.25 \times 10^{-3} \times 1000 \times 9.8}{2 \times 0.07} \Rightarrow \theta = 70^\circ$$

EFFECT OF DETERGENTS ON SURFACE TENSION

Detergent molecules have a very clever property, with one end hydrophilic, or water-loving, and the other hydrophobic, or repelled by water. This dual nature allows detergent to attract both water and oil, which gives it ability to clean laundry. It is also very effective at reducing the surface tension of water by pushing apart water molecules with the hydrophobic end of the detergent molecule.

WATER MOLECULES AND SURFACE TENSION

Soap molecules consist of a long chain of hydrogen and carbon atoms. This is the infrastructure of atoms that enjoy inside water. The other terminal is easily connected with the soap molecule. The molecules of soap surround the water particle.

Surface tension of water breaks with the help of the detergent molecules attached to the water molecule at both its ends. The grease particle is moved apart and connected with soap molecules, to be accomplished by a water jet spray.

The two terminals of detergent molecules help to break the surface tension of water. This is called hydrophobic, meaning “water fearing”. The hydrophobic terminals of the detergent particle are being pushed up to the water surface is an attempt to draw out from the water molecule.

This phenomenon helps to weaken the hydrogen bonding of water molecules available at the surface. This destroys the surface tension of water.

The soap molecule does not desire to convert into liquid form in a soap and water solution. The molecules which have found their path to the surface destroy their path between the molecules of surface water by raising the hydrophobic ends on the water surface. This phenomenon breaks the water molecules apart from each other.

SURFACE TENSION AND SURFACTANTS

The mixture of detergent and water decreases water's surface tension. The compounds which help to reduce water's surface tension are known as surfactants. This word “surfactant” is the composite of a surface-active agent. The surfactants decrease the surface tension and increase the sticking capacity of the things.

In this whole phenomenon, the lowered surface tension area can be seen where the surfactant is combined with water. These surfactants can be illustrated as detergents, soaps, emulsifiers, foaming agents and dispersants.

CONCEPT APPLICATION EXERCISE 8.4

- A false statement is
 - Angle of contact $\theta < 90^\circ$, if cohesive force < adhesive force
 - Angle of contact $\theta > 90^\circ$, if cohesive force > adhesive force
 - Angle of contact $\theta = 90^\circ$, if cohesive force = adhesive force
 - If the radius of capillary is reduced to half, the rise of liquid column becomes four times
- When a capillary tube is dipped in water it rises up to 8 cm in the tube. What happens when the tube is pushed down such that its end is only 5 cm above the outside water level
 - The radius of the meniscus increases and therefore water does not overflow
 - The radius of the meniscus decreases and therefore water does not overflow
 - The water forms a droplet on top of the tube but does not overflow
 - The water start overflowing
- A glass capillary tube of internal diameter 0.6 mm is held vertically with the lower end in water and with 80 mm of the tube above the surface of water. How much high does the water rise? If the tube is now lowered until 30 mm of its length is above the surface of water, what happens? Surface tension of water is 75 dyne cm^{-1} and $g = 1000 \text{ cm/s}^2$
- A long capillary tube of radius 0.1 cm open at both ends is filled with water and placed vertically. What will be the height of the column of water left in the capillary? Given, surface tension of water = 72 dyne cm^{-1} and density of water = 1 g cm^{-3} .
- Water rises in a capillary tube to a height 2 cm. In an other capillary tube, whose radius is one third of it, how much water will rise? If the first capillary tube is inclined at an angle of 60° with the vertical, then what will be the position of water in the tube?

ANSWERS

1. (d) 2. (a) 3. 5.0 cm, 0.05 cm
4. 2.94 cm 5. 6 cm, 12 cm

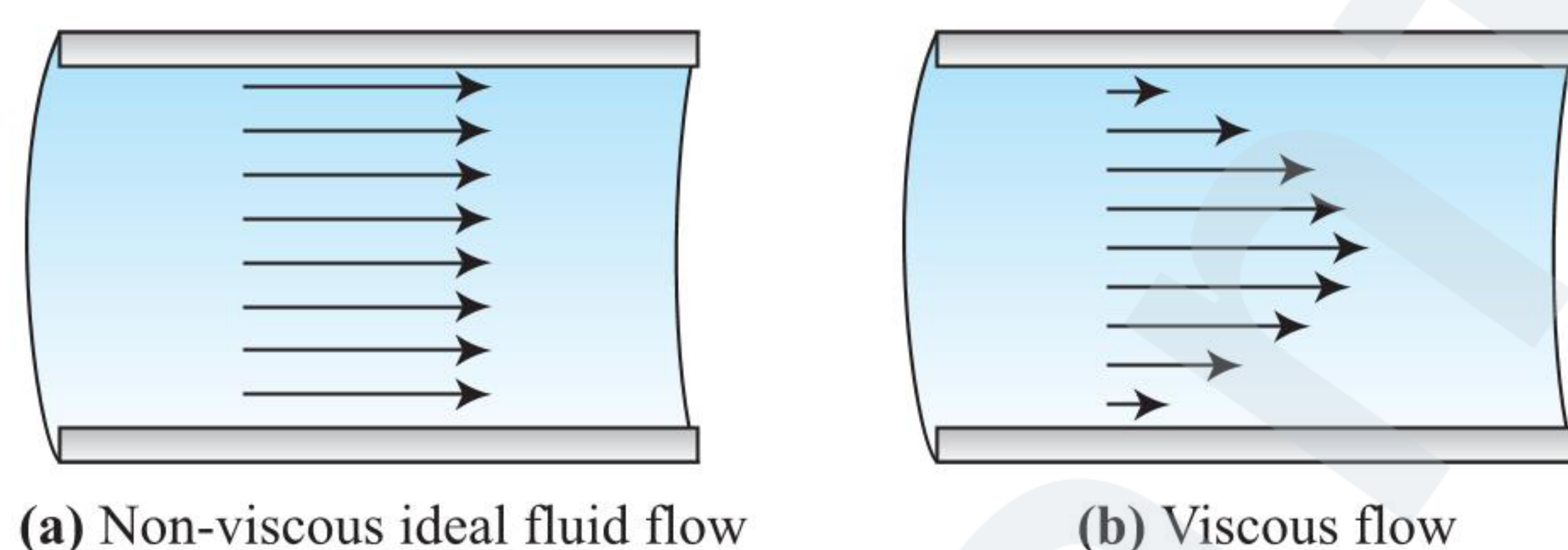
VISCOSITY

We are familiar with the property of friction in case of solids. When a solid body slides over another solid body, a frictional force begins to act between them. This force opposes the relative motion of the bodies. Similarly, when a layer of a liquid slides over another layer of the same liquid, a frictional force acts between them which opposes the relative motion between the layers. This force is called ‘internal frictional force’ or ‘viscosity’. Because of this property a backward dragging force (viscous drag) acts tangentially on the layers of the liquid in motion and it tries to destroy the motion of the liquid. Let us consider the following examples.

- (i) If you move in a boat on a gentle river, you might have noticed that your boat moved faster in the middle of the river than very close to the banks. Why would this happen?
- (ii) Let us pour equal quantities of water, glycerine and honey into three similar funnels. We observe that whereas water flows down very rapidly, glycerine trickles down drop by drop and honey requires maximum time to trickle down. This difference in behaviour of various liquids is on account of the difference in their internal frictional forces. This internal frictional force opposing the flow of liquid is maximum in case of honey, lesser in case of glycerine and least in the case of water.
- (iii) When we stir a liquid contained in a beaker with a glass rod, it starts rotating in coaxial cylindrical layers. If the stirring is stopped, the liquid continues rotating for some time and then comes to rest. According to the Newton's first law of motion, the liquid can stop only if an opposing force acts on it. Since no force has been applied from outside, this opposing force, called the internal frictional force, must be arising from within the liquid.

In an ideal fluid there is no viscosity to hinder the fluid layers as they slide past one another. Within a pipe of uniform cross section, every layer of an ideal fluid moves with the same velocity, even the layer next to the wall, as Fig. (a) shows. When viscosity is present, the fluid layers have different velocities, as Fig. (b) of the drawing illustrates. The velocity profile for the streamlines in viscous flow in a tube is sketched in Fig. (b). The profile is parabolic, with the velocity approaching zero at the walls and reaching its maximum value in the centre. This flow is still laminar, with the streamlines all flowing parallel to one another.

Velocity profiles in a cylindrical tube

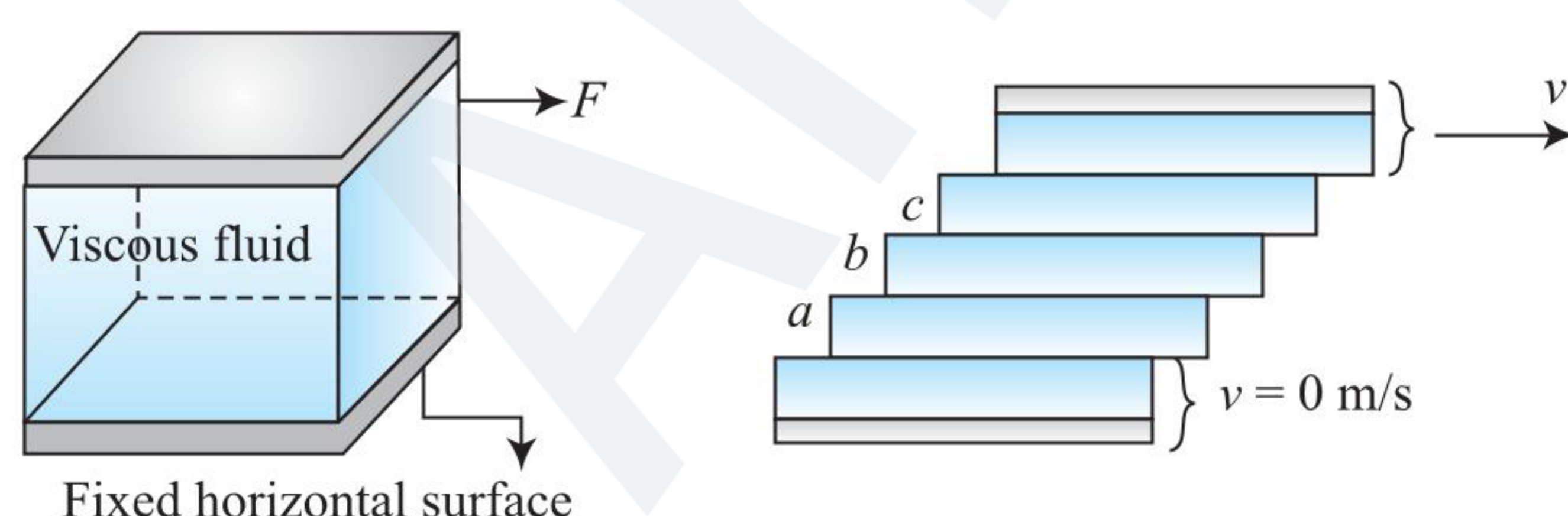


(a) Non-viscous ideal fluid flow

(b) Viscous flow

HOW VISCOUS FORCE ACTS

Suppose a liquid is flowing in a streamlined motion on a fixed horizontal surface.



(a) A force F is applied to the top plate, which is in contact with a viscous fluid

(b) Because of the force, the top plate and the adjacent layer of fluid move with a constant velocity

The layer of the liquid which is in contact with the surface is at rest, while the velocity of other layers increases with a distance from the fixed surface. Thus, there is a relative motion between adjacent layers of the liquid. Let us consider three parallel

layers— a , b and c . Their velocities are in an increasing order. The layer a tends to retard the layer b , while b tends to retard c .

Thus, each layer tends to decrease the velocity of the layer above it. Similarly, each layer tends to increase the velocity of the layer below it. This means that in between any two layers of the liquid, internal tangential forces act which try to destroy the relative motion between the layers. These forces are called 'viscous forces'. If the flow of the liquid is to be maintained, an external force must be applied to overcome the dragging viscous forces. In the absence of the external force, the viscous forces would soon bring the liquid to rest. The property of the liquid by virtue of which it opposes the relative motion between its adjacent layers is known as 'viscosity'.

Viscosity comes into play only when there is a relative motion between the layers of the same material. This is why it does not act in solids.

SIMILARITIES AND DIFFERENCES BETWEEN VISCOSITY AND SLIDING FRICTION

Similarities:

- Both viscosity and sliding friction oppose relative motion. Whereas viscosity opposes the relative motion between two adjacent liquid layers, sliding friction opposes the relative motion between two solid layers.
- Both come into play whenever there is a relative motion between layers of liquid or solid surfaces as the case may be.
- Both are due to molecular attractions.

Dissimilarities:

	Viscosity	Sliding friction
1.	Viscosity (or viscous drag) between layers of liquid is directly proportional to the area of the liquid layers.	Friction between two solids is independent of the area of solid surfaces in contact.
2.	Viscous drag is proportional to the relative velocity between two layers of liquid.	Friction is independent of the relative velocity between two surfaces.
3.	Viscous drag is independent of normal reaction between two layers of liquid.	Friction is directly proportional to the normal reaction between two surfaces in contact.

SOME APPLICATIONS OF VISCOSITY

Knowledge of viscosity of various liquids and gases has been put to use in daily life. Some applications of its knowledge are discussed as follows:

- As the viscosity of liquids varies with temperature, proper choice of lubricant is made depending upon season.
- Liquids of high viscosity are used in shock absorbers and buffers at railway stations.
- The phenomenon of viscosity of air and liquid is used to damp the motion of some instruments.
- The knowledge of the coefficient of viscosity of organic liquids is used in determining the molecular weight and shape of the organic molecules.
- It finds an important use in the circulation of blood through the arteries and veins of human body.

COMPARING THE VISCOSITY OF TWO FLUIDS

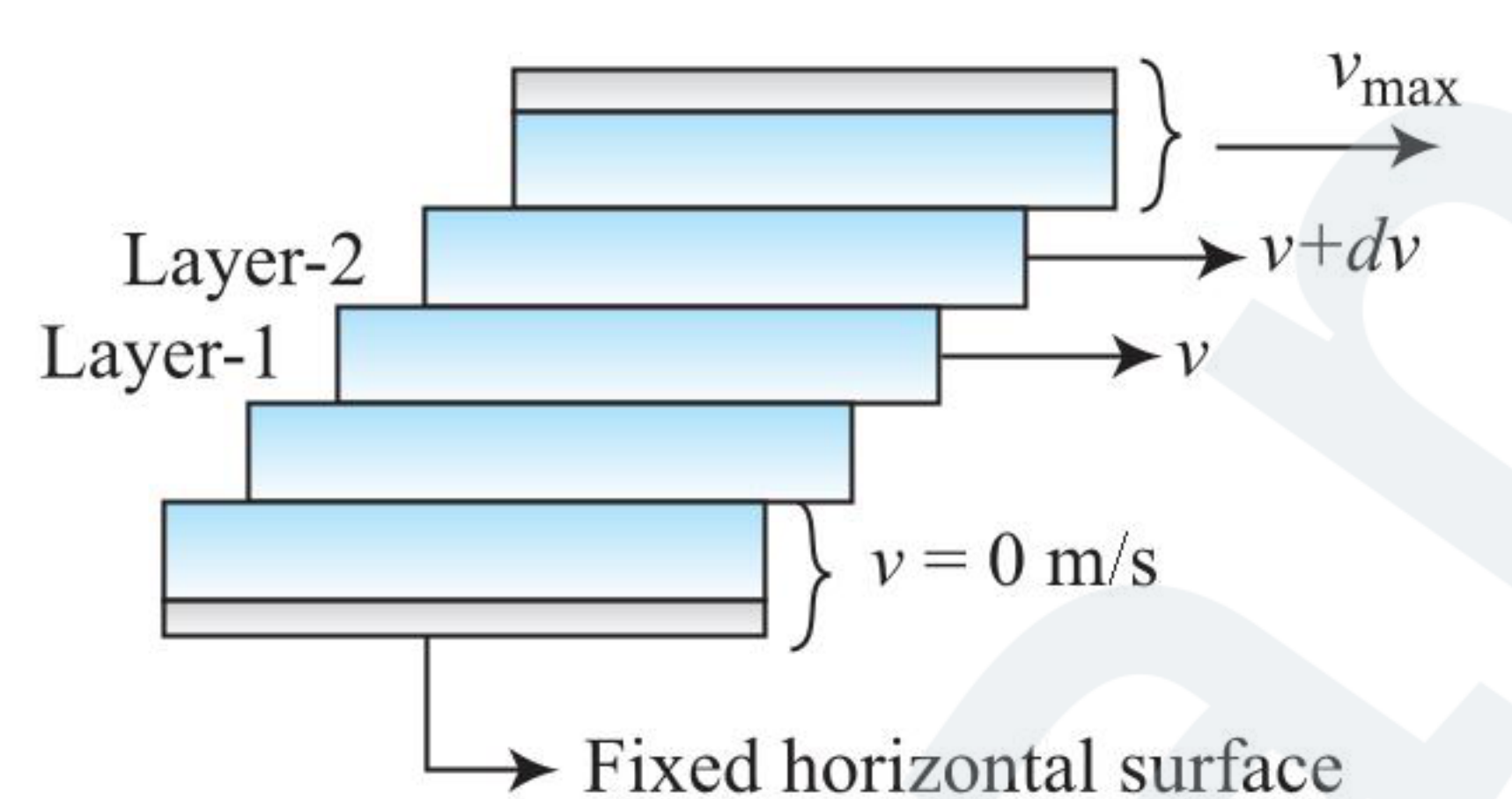
Viscosity is a measure of frictional forces between the adjacent layers of a fluid with which it opposes the relative motion between them. The frictional resistance of fluids is also observed when a solid body moves within the body of a fluid. Viscous forces are present both in liquids and gases. Obviously, the liquids are more viscous than gases.

The viscosity of two liquids can be compared in the following two ways:

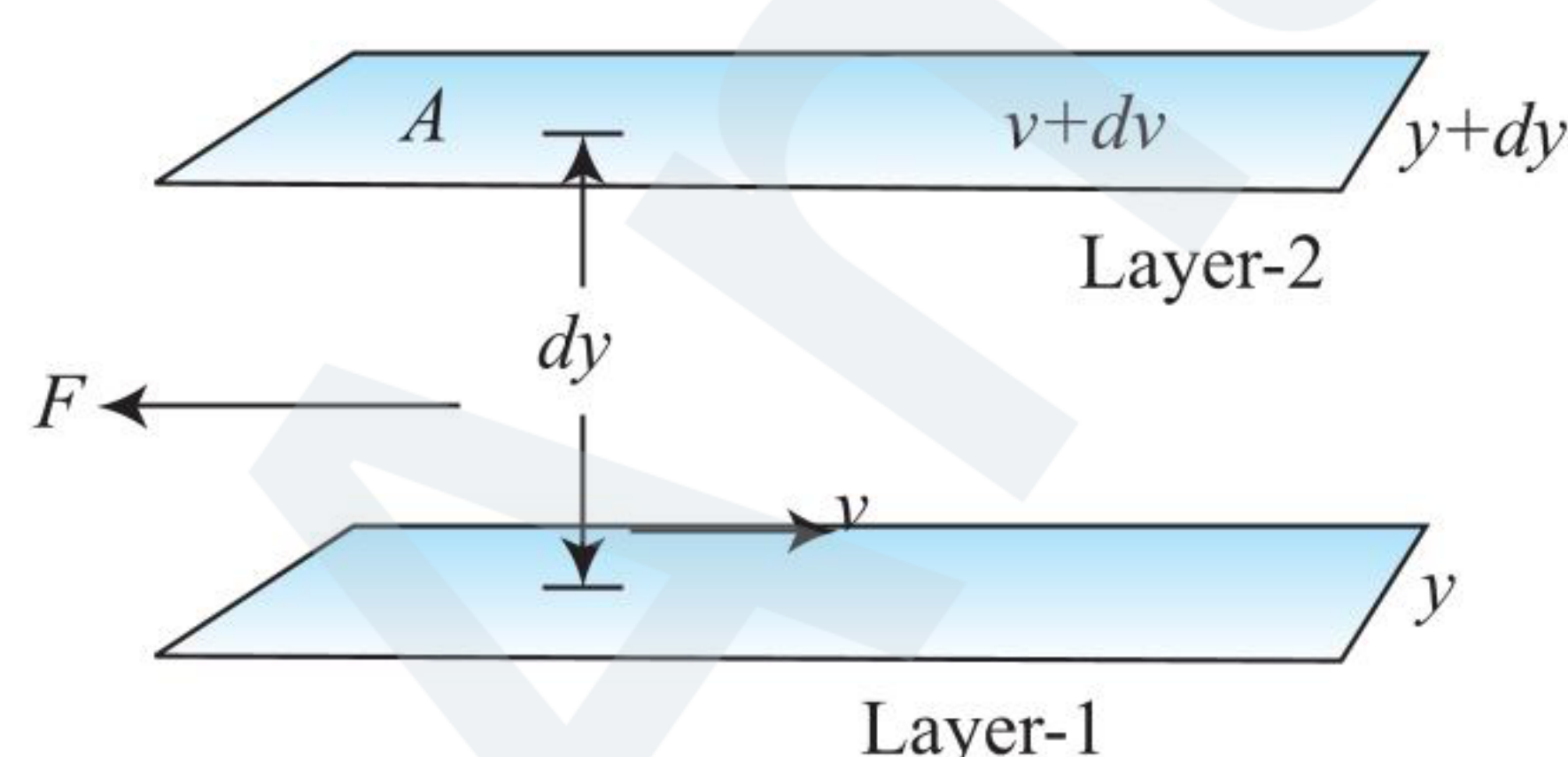
1. By allowing them to flow out of identical vessels. For example, a liquid of low viscosity such as water flows rapidly out of it, and on the contrary, a highly viscous liquid such as glue or glycerine makes a slow movement out of it.
2. By dropping steel ball bearings through each liquid. The balls fall more slowly in a liquid of high viscosity.

MEASUREMENT OF VISCOSITY

Consider a liquid flowing over a fixed horizontal solid surface in the form of parallel layers. The layer in contact with the solid surface is found to be at rest and as we move up, the velocity of the layers goes on increasing and the layer at the top possesses maximum velocity. Let us consider two parallel layers '1' and '2' at distances $y, y + dy$ from the fixed surface and moving with the velocities v and $v + dv$ respectively. Then, dv/dy denotes the rate of change of velocity with the distance in the direction of increasing distance and it is called the velocity gradient. The relative motion between the two layers can take place only, if some external force acts between them. Due to viscosity, a force F acts in opposite direction to destroy the relative motion.



(a)



(b)

According to Newton, the viscous force F depends upon the following factors:

- It is directly proportional to the area of the layers in contact i.e. $F \propto A$
- It is directly proportional to the velocity gradient between the layers i.e. $F \propto \frac{dv}{dy}$

Combining these two factors, we have

$$F \propto A \frac{dv}{dy} \text{ or } F = -\eta A \frac{dv}{dy} \quad \dots(i)$$

where η is a constant of proportionality and is called the **coefficient of viscosity** or simply the viscosity of the liquid. Its value depends upon the nature and physical conditions of the liquid. The negative sign in Eq. (i) indicates that the viscous drag acts in a direction opposite to the liquid flow.

Definition of η

Let the area of the layer $A = 1$ and velocity gradient $dv/dy = 1$, then from Eq. (i), $\eta = F$ (neglecting negative sign). Thus, 'coefficient of viscosity of a liquid is defined as the tangential viscous drag acting per unit area between two liquid layers moving with unit velocity gradient.'

From Eq. (i), neglecting negative sign,

$$\eta = \frac{(F/A)}{(dv/dy)} = \frac{\text{tangential stress}}{\text{velocity gradient}} \quad \dots(ii)$$

Also, we know that modulus of rigidity,

$$\eta = \frac{F/A}{\Delta x/L} = \frac{\text{tangential stress}}{\text{displacement gradient}} \quad \dots(iii)$$

Comparing Eqs. (ii) and (iii), we can observe that coefficient of viscosity in liquids is analogous to the modulus of rigidity in solids. Thus viscosity of a liquid is the limiting case of rigidity in solids when they (i.e., solids) break down under the applied stress. A liquid develops a shearing strain (i.e., shear) for a short time after which it breaks down. This shear is, however, formed again and the process of breaking down and reformation of shear goes on till the shearing stress is removed. Thus, a liquid offers resistance to shearing stress only for a short time and as such it is said to possess fugitive (i.e., transient, short lived) rigidity i.e., fluids possess rigidity only for a short time. It is due to this reason that viscosity is termed fugitive rigidity.

Note: The coefficient of viscosity η is analogous to the modulus of rigidity of solids. The shear stress of fluids is proportional to the rate of shear strain and that in case of solids is proportional to the shear strain.

Dimensional Formula for η

From Eq. (i), $\eta = \frac{F}{A(dv/dy)}$ (neglecting -ve sign)

$$[\eta] = \frac{[MLT^{-2}]}{[L^2][LT^{-1}/L]} = [ML^{-1}T^{-1}]$$

UNITS OF COEFFICIENT OF VISCOSITY

- In cgs system**, the unit of η is dyne s/cm² and is called poise (P). The coefficient of viscosity of a liquid is said to be one poise if a tangential viscous drag of 1 dyne/cm acts between two liquid layers moving with a velocity gradient of 1 (cm/s) per cm (i.e., s⁻¹).
- SI unit** of viscosity is Ns / m² and is called a **pascal second** (Pa s) as

$$1 \frac{\text{N} \cdot \text{s}}{\text{m}^2} = 1 \left(\frac{\text{N}}{\text{m}^2} \right) \cdot \text{s} = 1 \text{ Pa} \cdot \text{s}$$

The coefficient of viscosity of a liquid is said to be one Pa s if a tangential drag of 1 N/m acts between two liquid layers moving with a velocity gradient of 1 (m/s) per m (i.e., s^{-1}). Also,

$$1 \text{ Pa} \cdot \text{s} = 1 \frac{\text{N} \cdot \text{s}}{\text{m}^2} = 1 \left(\frac{10^5 \text{ dyne}}{10^4 \text{ cm}^2} \right) \cdot \text{s} = 10 \left(\frac{\text{dyne} \cdot \text{s}}{\text{cm}^2} \right)$$

$$\Rightarrow 1 \text{ Pa} \cdot \text{s} = 10 \text{ poise} = 1 \text{ decapoise}$$

EFFECT OF TEMPERATURE ON VISCOSITY

Experiments show that η of a liquid decreases sharply with increase in its temperature and becomes zero at its boiling point. On the other hand, η of gases increase with temperature.

The viscosity of liquids decreases with increase in temperature and increases with the decrease in temperature. That is, $\eta \propto 1/\sqrt{T}$. On the other hand, the value of viscosity of gases increases with the increase in temperature and vice versa. That is, $\eta \propto \sqrt{T}$.

ILLUSTRATION 8.20

A plate of area 100 cm^2 is placed on the upper surface of castor oil, 2 mm thick. Taking the coefficient of viscosity to be 15.5 poise, calculate the horizontal force necessary to move the plate with a velocity 3 cm s^{-1} .

Sol. The (horizontal tangential) viscous force is given by

$$F = -\eta A \frac{dv}{dy}$$

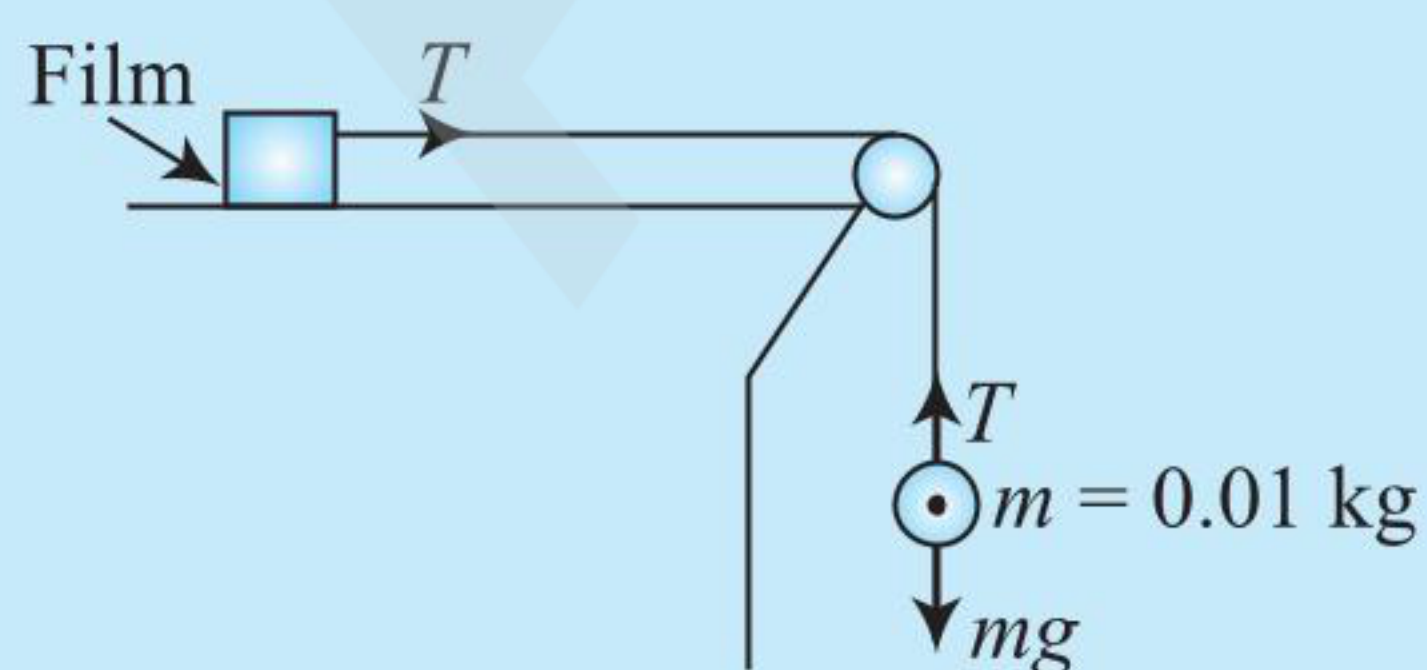
Given $\eta = 15.5 \text{ poise}$, $A = 100 \text{ cm}^2$

$$\frac{dv}{dx} = \frac{v_2 - v_1}{x_2 - x_1} = \frac{(0 - 3) \text{ cm/s}}{(2 - 0) \text{ mm}} = -\frac{3}{0.2} \text{ s}^{-1} = -15 \text{ s}^{-1}$$

$$F = -15.5 \times 100 \times (-15) = 2.235 \times 10^4 \text{ dyn} = 0.2325 \text{ N}$$

ILLUSTRATION 8.21

A metal plate of area 0.10 m^2 is connected to a 0.01 kg mass via a string that passes over an ideal pulley (considered to be frictionless), as shown in the figure. A liquid with a film thickness of 3.0 mm is placed between the plate and the table. When released, the plate moves to the right with a constant speed of 0.085 m s^{-1} . Find the coefficient of viscosity of the liquid.



Sol. As the metal plate moves with constant velocity,

$$mg - T = 0 \Rightarrow T = mg$$

If F is the viscous force on the plate, then

$$F = T = mg = 0.01 \times 9.8 = 9.8 \times 10^{-2} \text{ N}$$

By Newton's law of viscosity, $F = \eta A \frac{dv}{dy}$

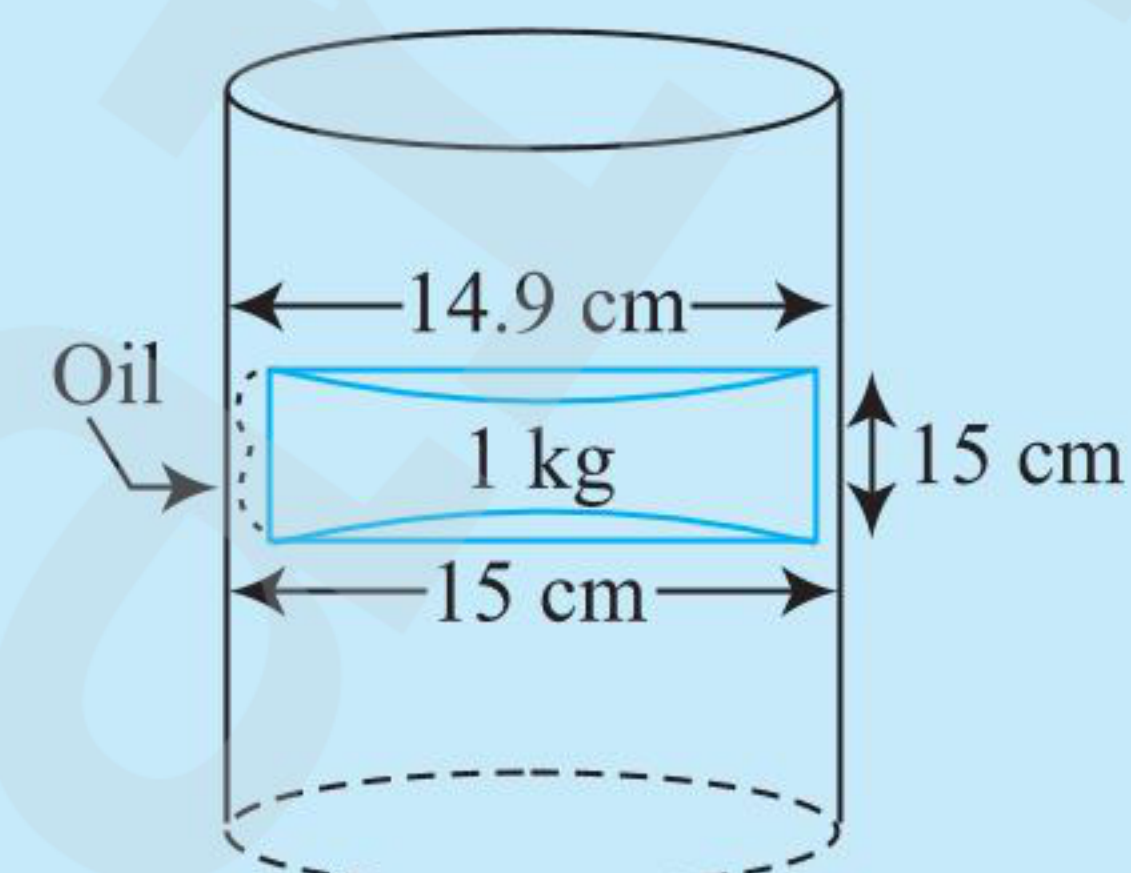
$$\text{where } \frac{dv}{dy} = \frac{0.085}{0.3 \times 10^{-3}} = 28.3 \text{ s}^{-1}$$

$$\Rightarrow 9.8 \times 10^{-2} = \eta \times 0.10 \times 28.3$$

$$\Rightarrow \eta = 3.4 \times 10^{-3} \text{ N s/m}^2$$

ILLUSTRATION 8.22

A sliding fit cylindrical body of mass of 1 kg drops vertically down at a constant velocity of 5 cm s^{-1} . Find the viscosity of the oil.



Sol. As the body moves with constant velocity,

$$mg = F_v \Rightarrow 1g = \eta A \frac{dv}{dy}$$

$$\Rightarrow 1g = \eta (2\pi l) \frac{dv}{dy}$$

$$\Rightarrow 1g = \eta (2\pi \times 7.5 \times 10^{-2} \times 15 \times 10^{-2}) \left(\frac{0.05 - 0}{0.05 \times 10^{-2}} \right)$$

After solving, we get $\eta = 1.4 \text{ N s/m}^2$.

REYNOLDS NUMBER

Reynolds was able to show that flow of a liquid in a tube is laminar or turbulent depending upon the values of four physical parameters:

- average velocity of the liquid, v
- viscosity of the liquid, η
- density of the liquid, ρ and
- diameter of the tube, d

In order to find a relation between them, let

$$v \propto \eta^a \rho^b d^c \text{ or } v = k \eta^a \rho^b d^c \quad \dots(i)$$

Writing the dimensions of various quantities on both sides, we get,

$$[M^0 L^1 T^{-1}] = [ML^{-1} T^{-1}]^a [ML^{-3}]^b [L]^c$$

Equating the powers of M, L and T on both sides,

$$a + b = 0, -a - 3b + c = 1 \text{ and } -a = -1$$

Solving these equations, we get

$$a = 1, b = -1 \text{ and } c = -1$$

From Eq. (i),

$$v = k \eta \rho^{-1} d^{-1} \text{ or } v = k \frac{\eta}{\rho d} \quad \dots(ii)$$

Equation (ii) is called **Reynolds formula** and the dimensionless number k is called the **Reynolds number** and is denoted by R_e .

Replacing k by R_e , Eq. (ii) becomes,

$$v = R_e \frac{\eta}{\rho d} \text{ or } R_e = \frac{\rho v d}{\eta} \quad \dots(iii)$$

Experiments have shown that if

- (i) $R_e < 2000$, liquid flow is laminar.
- (ii) $R_e > 3000$, liquid flow is turbulent.
- (iii) For values of R_e between 2000 and 3000, the flow is unstable and may change back and forth from one type to the other.

Equation (iii) is a general relation and is valid for all velocities. For fixed values of ρ , d and η , R_e is constant for a fixed velocity. Its value changes with the velocity of liquid flow.

The value of R_e when $v = v_c$ (critical velocity, the velocity of flow above which the flow becomes turbulent), is called the critical Reynolds number and is denoted by $(R_e)_c$. From Eq. (iii),

$$v_c = (R_e)_c \frac{\eta}{\rho d} \quad \text{or} \quad (R_e)_c = \frac{v_c \rho d}{\eta} \quad \dots(\text{iv})$$

ILLUSTRATION 8.23

Calculate the Reynolds number for blood flowing at 30 cm/s through an aorta of radius 1.0 cm. Assume that the blood has a viscosity of 4 mPa s and a density of 1060 kg/m³. What is the nature of flow?

Sol. Here, $v = 30 \text{ cm/s} = 3 \times 10^{-1} \text{ m/s}$, $\rho = 1060 \text{ kg/m}^3$,

$$d = 2 \times 1.0 \text{ cm} = 20 \times 10^{-2} \text{ m}$$

$$\eta = 4 \text{ mPa s} = 4 \times 10^{-3} \text{ Pa s} = 4 \times 10^{-3} \text{ Ns/m}^2$$

$$\text{Thus, } R_e = \frac{v \rho d}{\eta}$$

$$= \frac{(3 \times 10^{-1} \text{ m/s})(1060 \text{ kg/m}^3)(2.0 \times 10^{-2} \text{ m})}{(4 \times 10^{-3} \text{ Ns/m}^2)} = 1590$$

Since R_e is less than 2000, this flow will be laminar.

ILLUSTRATION 8.24

What should be the maximum average velocity of water in a tube of diameter 2 cm so that the flow is laminar? The viscosity of water is 10^{-3} Ns/m^2 .

Sol. Here, $d = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$, $\eta = 10^{-3} \text{ Ns/m}^2$,

$$\rho = 10^3 \text{ kg/m}^3$$

For laminar flow through the tube, the maximum value for R_e is 2000. If v is the maximum average velocity of water, then

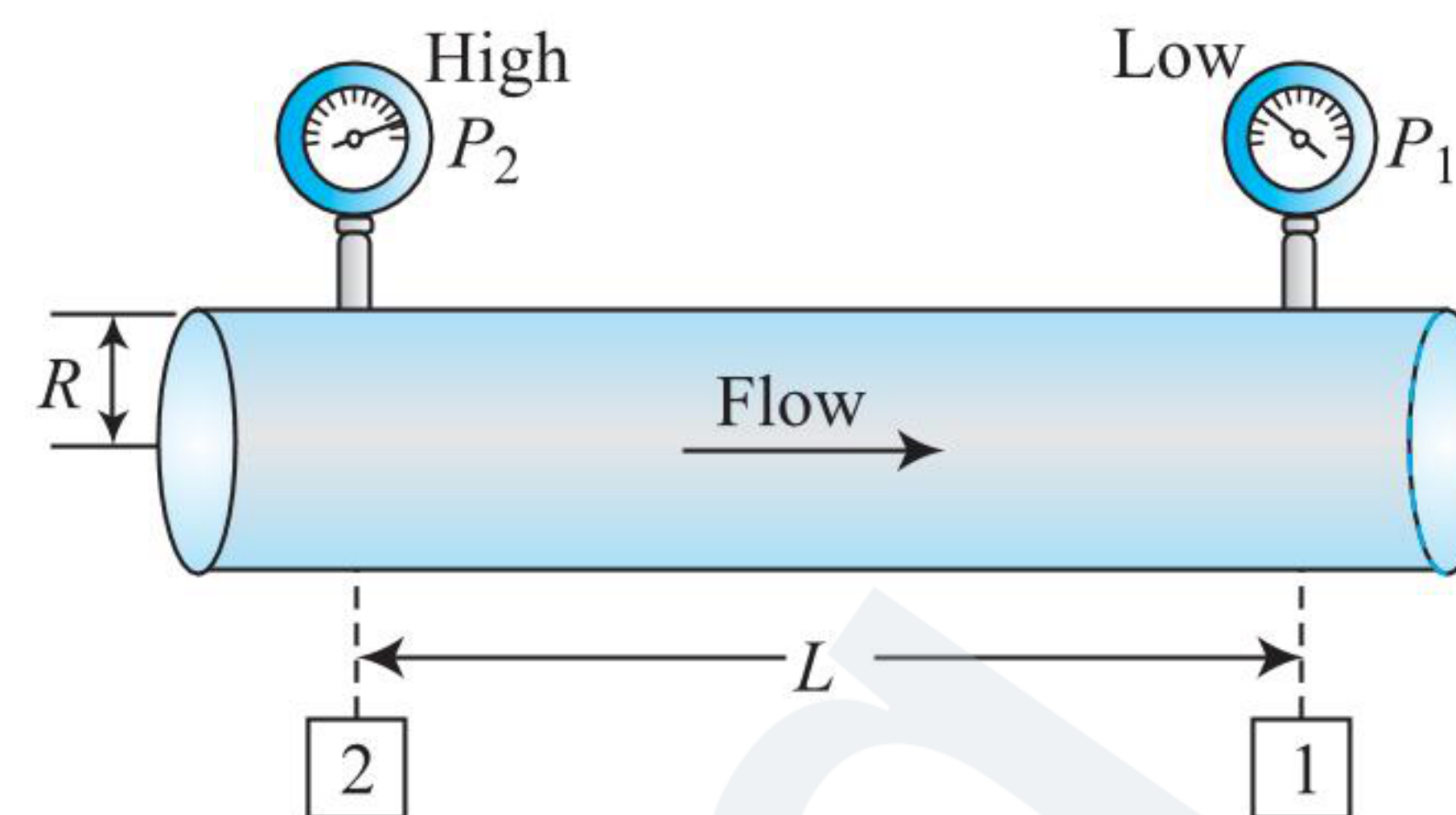
$$R_e = \frac{v \rho d}{\eta} \quad \text{or} \quad v = \frac{R_e \eta}{\rho d}$$

$$\text{or } v = \frac{(2000)(10^{-3} \text{ Ns/m}^2)}{(10^3 \text{ kg/m}^3)(2 \times 10^{-2} \text{ m})} = 0.1 \text{ m/s}$$

POISEUILLE'S FORMULA FOR FLOW OF LIQUID THROUGH A CAPILLARY TUBE

When a liquid flows 'in a tube', the viscous forces oppose the flow of the liquid. Hence a pressure difference is applied between

the ends of the tube which maintains the flow of the liquid.



Poiseuille derived a formula for the volume of liquid flowing out per second through a narrow horizontal tube by assuming the flow of liquid to be streamlined and parallel to axis of the tube.

Poiseuille found that volume of liquid flowing per second (Q) is as follows:

- Directly proportional to $P_2 - P_1 = \Delta P$, a greater pressure difference leading to a larger flow rate, i.e. $Q \propto \Delta P$.

- A long pipe offers greater resistance to the flow than a short pipe does, and Q is inversely proportional to the length L , i.e. $Q \propto \frac{1}{L}$

- High-viscosity fluids flow less readily than low-viscosity fluids, and Q is inversely proportional to the viscosity η , i.e.

$$Q \propto \frac{1}{\eta}$$

- Finally, the volume flow rate is larger in a pipe of larger radius, other things being equal. The dependence on the radius R is a surprising one, Q being proportional to the fourth power of the radius, or R^4 , i.e. $Q \propto R^4$.

If, for instance, the pipe radius is reduced to one-half of its original value, the volume flow rate is reduced to one-sixteenth of its original value.

Combining all these factors, we have

$$Q \propto \frac{R^4 \Delta p}{\eta L} \quad \text{or} \quad Q = \frac{k R^4 \Delta p}{\eta L}$$

Here, k is constant of proportionality and is found to be equal to $\pi/8$. Therefore, above relation becomes

$$Q = \frac{\pi R^4 \Delta p}{8 \eta L}$$

This expression is known as **Poiseuille's equation** for the flow of a liquid through a narrow tube. The volume of liquid flowing out per second from a tube is also known as rate of flow or rate of discharge of the liquid. Poiseuille's equation holds good only under the following assumptions:

1. The liquid flow is steady and the streamlines are parallel to the axis of the tube.
2. As there is no radial flow of liquid, the pressure is constant over any cross section of the tube.
3. The liquid in contact with the walls of the tube is at rest.
4. The tube is narrow and of uniform area of cross section.
5. The tube is held horizontally, so that there is no effect of gravitational force on the liquid flow.
6. The liquid is capable of sustaining a small shearing stress.

ILLUSTRATION 8.25

An engineer wants to have the same flow rate of water and light machine oil from the pipes of the same length and with the same pressure head. What should be ratio of the radii of the two pipes? Given that viscosity of water = 0.01 poise and that of light machine oil = 11 poise.

Sol. Let L and R be the length and radius of the pipe. Let Δp be the pressure difference between the two ends of the pipe. If η is viscosity of the water, then the rate of flow of the water through the pipe,

$$Q = \frac{\pi \Delta p R^4}{8 \eta L} = \frac{\pi \Delta p R^4}{8 \times 0.01 \times L} \quad \dots(i)$$

Suppose that the radius of the pipe through which light machine oil flows has to be taken as R' , so as to keep the rate of flow same in the two cases. If η' is viscosity of the light machine oil, then the rate of flow of the oil through the pipe,

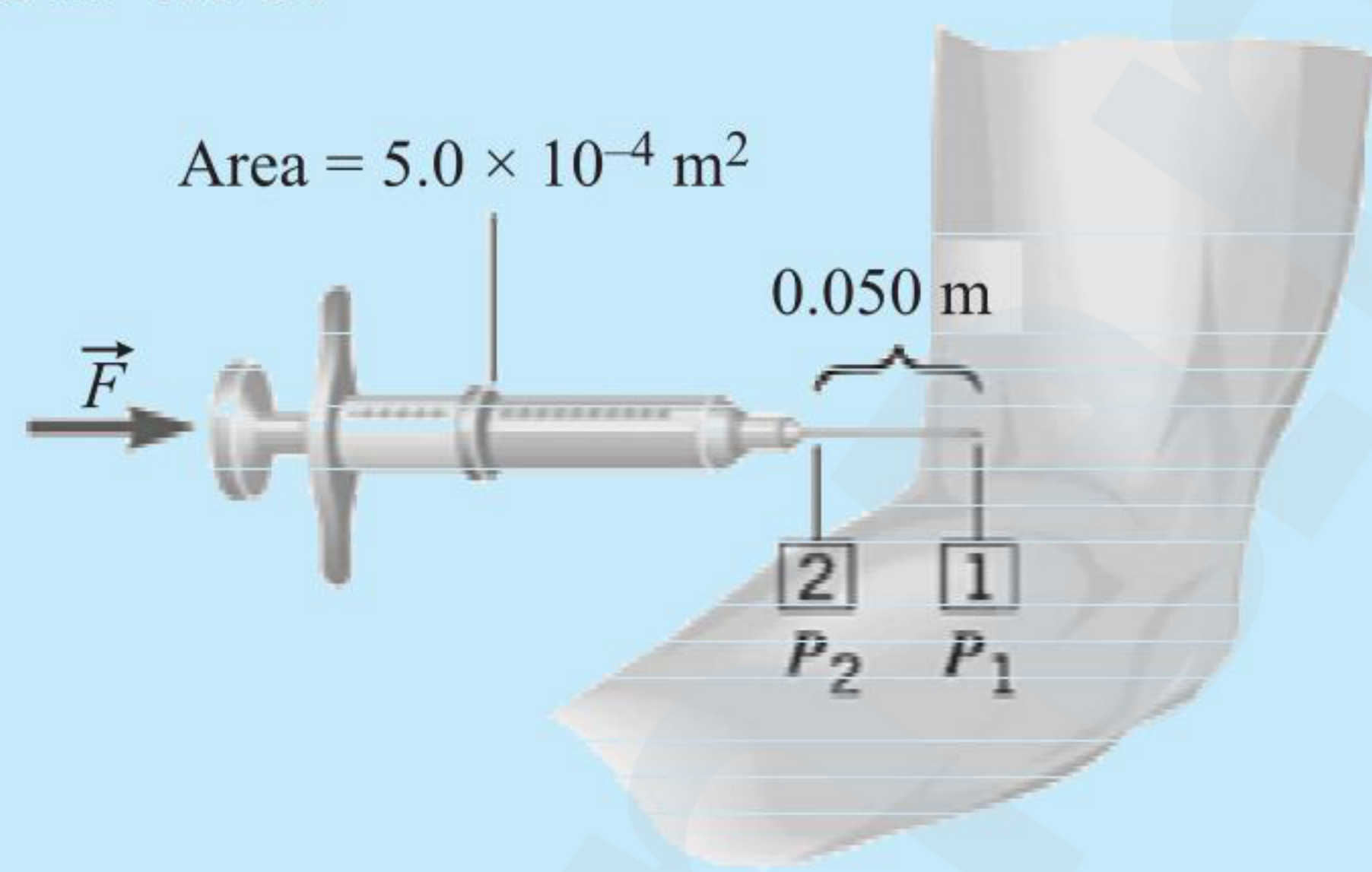
$$Q = \frac{\pi p R'^4}{8 \eta' L} \quad \text{or} \quad Q = \frac{\pi \Delta p R'^4}{8 \times 11 \times L} \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$\frac{\pi \Delta p R^4}{8 \times 11 \times L} = \frac{\pi \Delta p R^4}{8 \times 0.01 \times L} \quad \text{or} \quad \frac{R'}{R} = \left(\frac{11}{0.01} \right)^{1/4} = (1100)^{1/4} = 5.76$$

ILLUSTRATION 8.26

A hypodermic syringe is filled with a solution whose viscosity is $3.14 \times 10^{-5} \text{ Pa} \cdot \text{s}$. As figure shows, the plunger area of the syringe is $5.0 \times 10^{-4} \text{ m}^2$, and the length of the needle is 5.0 cm. The internal radius of the needle is $2.0 \times 10^{-4} \text{ m}$. The gauge pressure in the vein that is injected is 2000 Pa. What force must be applied to the plunger, so that $8.0 \times 10^{-6} \text{ m}^3$ of solution can be injected in 4.0 s?



Sol. The force required here is the pressure applied to the plunger times the area of the plunger. In this case the viscous flow is occurring, the pressure is different at different points along the syringe. The barrel of the syringe is wide enough that little pressure difference is required to sustain the flow up to point 2, where the fluid encounters the narrow needle. It means the pressure applied to the plunger is nearly equal to the pressure P_2 at point 2.

To find this pressure, we apply Poiseuille's law to the needle. According to Poiseuille's law

$$P_2 - P_1 = \frac{8 \eta L Q}{\pi R^4} \quad \dots(i)$$

Here pressure P_1 is given as a gauge pressure, which, in this case, is the amount of pressure in excess of atmospheric pressure. And we are interested to find the amount of force in excess of the force applied to the plunger by the atmosphere.

$$\text{The volume flow rate } Q = \frac{8.0 \times 10^{-6} \text{ m}^3}{4.0 \text{ s}} = 2.0 \times 10^{-6} \text{ m}^3/\text{s}$$

Hence from (i)

$$P_2 - P_1 = \frac{8(3.14 \times 10^{-5} \text{ Pa} \cdot \text{s})(0.05 \text{ m})(2.0 \times 10^{-6} \text{ m}^3/\text{s})}{\pi(2.0 \times 10^{-4} \text{ m})^4} = 5000 \text{ Pa}$$

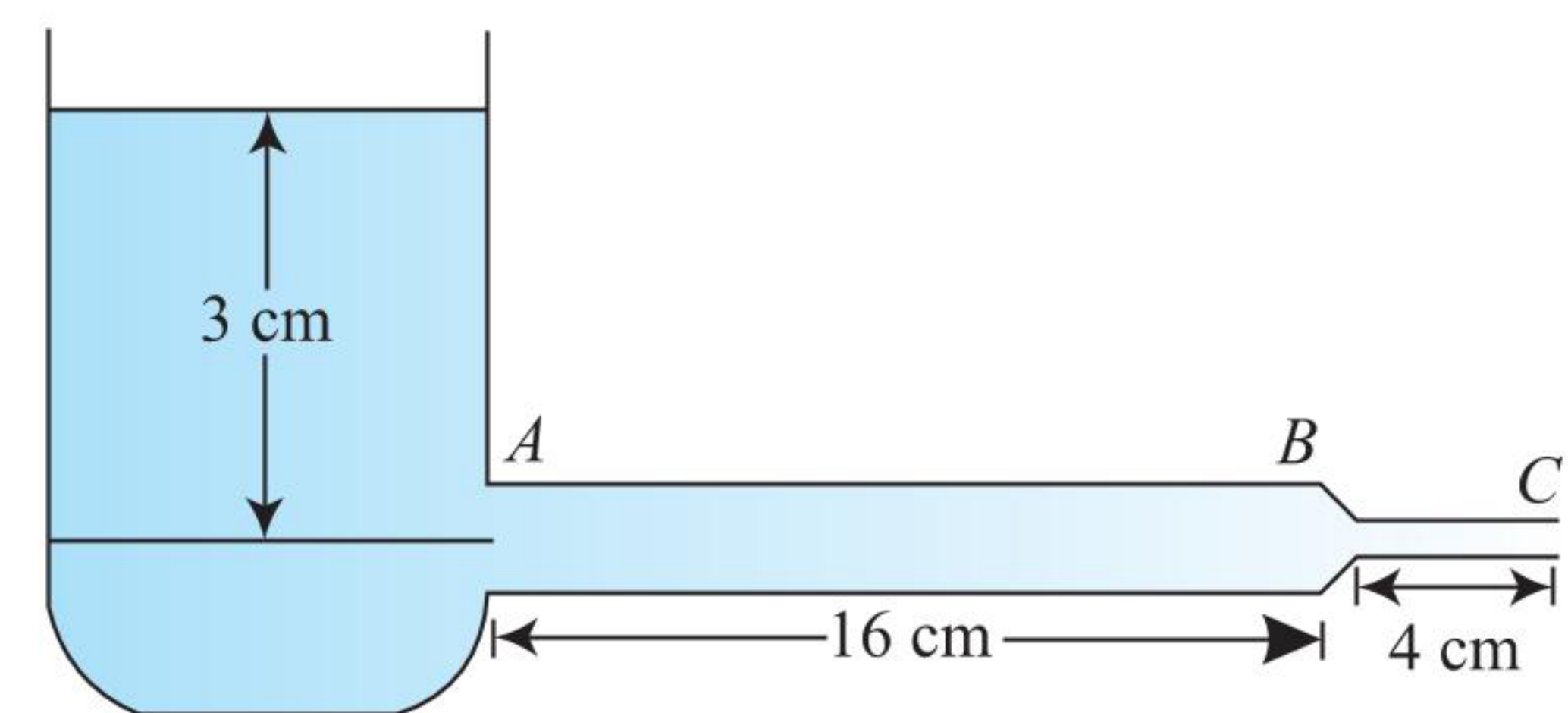
Since $P_1 = 2000 \text{ Pa}$, the pressure $P_2 = 5000 \text{ Pa} + 2000 \text{ Pa} = 7000 \text{ Pa}$. The force that must be applied to the plunger is this pressure times the plunger area:

$$F = (7000 \text{ Pa})(5.0 \times 10^{-4} \text{ m}^2) = 3.5 \text{ N}$$

ILLUSTRATION 8.27

Two capillary tubes AB and BC are joined end to end at B . AB is 16 cm long of diameter 4 mm, tube BC is 4 cm long and of diameter 2 mm. The composite tube is held horizontally as in Poiseuille's experiment with A connected to a vessel of water giving a constant head of 3 cm and C is open to the air. Calculate the pressure difference between B and C .

Sol. AB and BC are two capillary tubes which are joined end to end at B as shown in the figure.



Let the atmospheric pressure be h cm of water column. Pressure at $A = (h + 3)$ and pressure at $C = h$

Further, let h' be the pressure at B .

Clearly, pressure difference across AB , i.e., $P_1 = (h + 3 - h')$ and pressure difference across BC , i.e., $P_2 = (h' - h)$

Since the volume of the liquid flowing (per second) through $AB =$ volume of the liquid flowing (per second) through BC ,

$$\frac{\pi P_1 r_1^4}{8 \eta l_1} = \frac{\pi P_2 r_2^4}{8 \eta l_2} \quad \text{or} \quad \frac{P_1 r_1^4}{l_1} = \frac{P_2 r_2^4}{l_2}$$

$$\text{Or} \quad \frac{P_1}{P_2} = \frac{l_1}{l_2} \left(\frac{r_2}{r_1} \right)^4$$

$$\text{Or} \quad \frac{(h + 3 - h')}{(h' - h)} = \frac{16}{4} \times \left(\frac{0.1}{0.2} \right)^4$$

$$(l_1 = 16 \text{ cm}, l_2 = 4 \text{ cm}, r_1 = 0.2 \text{ cm}, \text{ and } r_2 = 0.1 \text{ cm})$$

$$\text{Or} \quad \frac{h + 3 - h'}{h' - h} = \frac{16}{4} \times \frac{1}{16} = \frac{1}{4}$$

$$\text{Or} \quad h' - h = 4h + 12 - 4h'$$

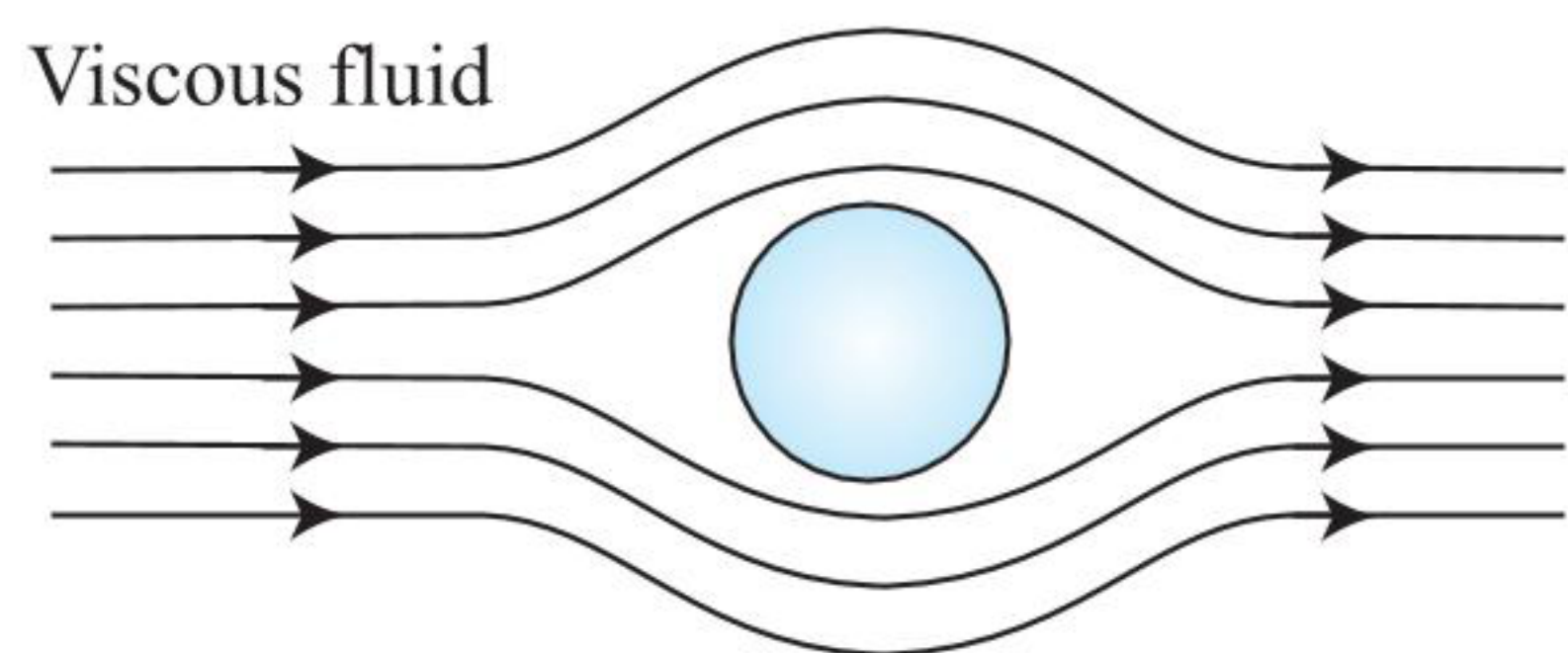
$$\text{Or} \quad 5h' = 5h + 12$$

$$\text{Or} \quad h' = h + 2.4$$

Pressure difference across $BC = h' - h = 2.4 \text{ cm}$ of water column.

STOKES' LAW

If you move a spherical body through a fluid, it experiences a viscous force. An expression for the viscous force experienced by the spherical object was derived by Sir George Gabriel Stokes and after his name, this expression is called Stokes' law. This law deals with the motion of a body through a viscous medium. When a solid body moves through a fluid, the layer of the fluid immediately in contact with the body moves with it. This causes a relative motion amongst different layers of the fluid, giving rise to a viscous drag which opposes the motion of the body. The streamlines for a fluid flowing slowly past a stationary solid sphere are shown in the figure.



When the sphere moves slowly as compared to the fluid, the pattern is similar but the streamlines then flow in such a way that the apparent motion of the fluid particles is as seen by someone on the moving sphere. In this latter case, it is known that the layer of fluid in contact with the sphere moves with it, thus creating a velocity gradient between this layer and other layers of fluid. Viscous forces are thereby brought into play and constitute the resistance experienced by the moving sphere. If we make the plausible assumption that the viscous retarding force F depends on the size of the body, the velocity with which it moves, the viscosity of the fluid and mass density of the fluid, then an expression can be derived for F by the method of dimensions. Thus,

$$F = k\eta^x v^y r^z$$

where x , y and z are the indices to be found and k is a dimensionless constant. The dimensional equation is

$$[MLT^{-2}] = [ML^{-1}T^{-1}]^x [LT^{-1}]^y [L]^z$$

Equating indices of M , L and T on both sides, we have

$$1 = x$$

$$1 = -x + y + z$$

$$-2 = -x - y$$

Solving, we get $x = 1$, $y = 1$ and $z = 1$. Hence $F = k\eta v r$

A detailed treatment, first done by Stokes, gives $k = 6\pi$ and so

$$F = 6\pi\eta v r$$

The above relation is called the Stokes' law. According to this law: **Whenever a spherical body of radius r moves in a fluid of viscosity η with a velocity v , it experiences a viscous drag, F which is given by $F = 6\pi\eta v r$.**

The Stokes' law is valid under the following assumptions:

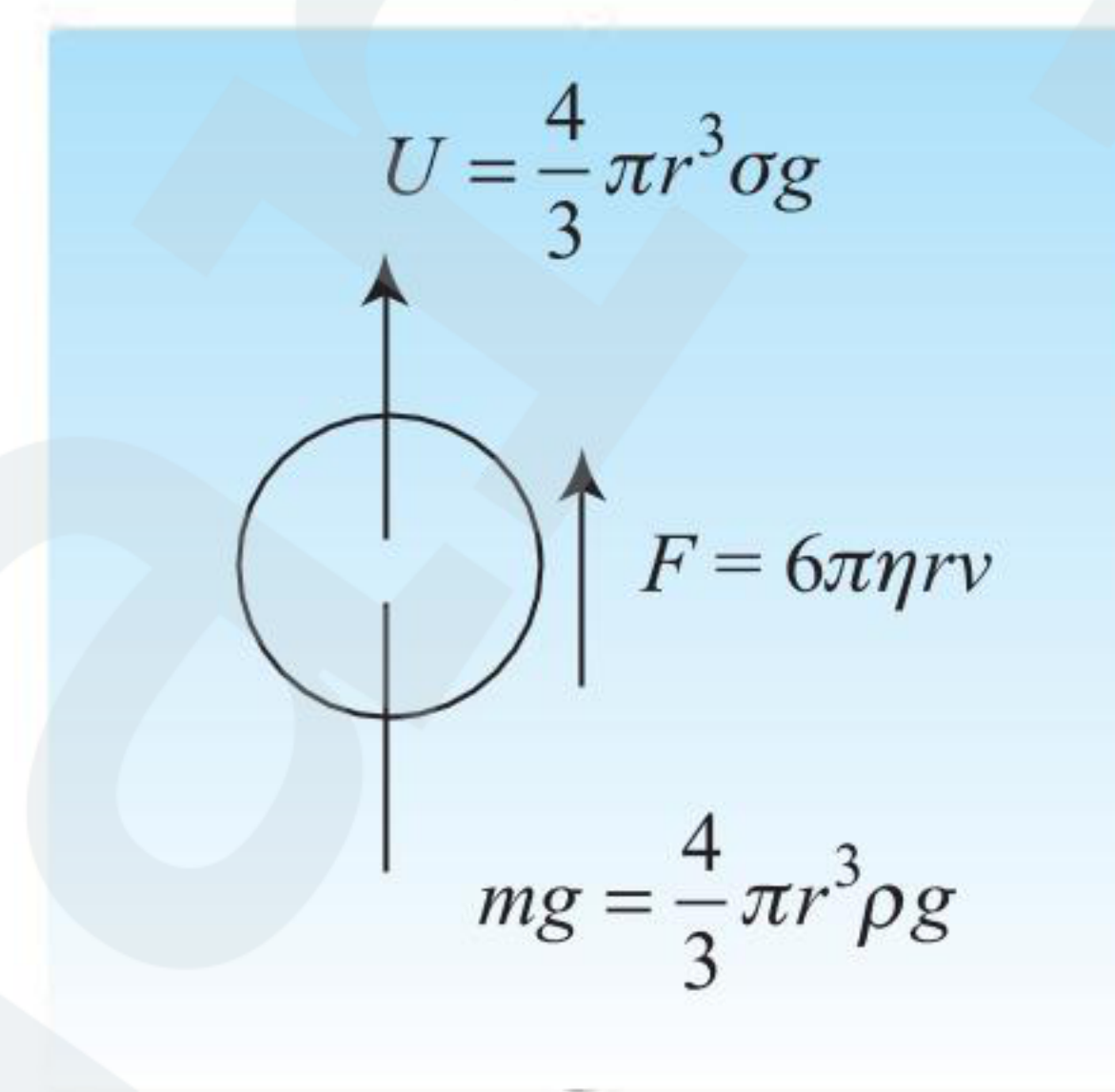
- The medium through which the body moves is of infinite extent.
- The moving body is perfectly smooth and rigid.
- Though the size of the moving body is small, it is much larger than the distance between the molecules of the medium and as such the medium may be considered as continuous.

- There is no slip between the moving body and the medium.
- The velocity of the moving body is less than the critical velocity of the medium and as such the motion is streamlined.

TERMINAL VELOCITY

When a body is dropped in a viscous fluid, it is first accelerated and then its acceleration becomes zero and it attains a constant velocity called terminal velocity. Consider a small ball of radius r , which is gently dropped into the liquid of infinite extent.

As the body falls, the following three forces act on the body [see figure].



- (i) Weight of the body acting vertically downwards. It is given by

$$mg = \frac{4}{3}\pi r^3 \rho g$$

- (ii) Upward thrust equal to the weight of the fluid displaced acting vertically upwards. It is given by

$$U = \frac{4}{3}\pi r^3 \sigma g$$

Therefore, effective or apparent weight of the body acting vertically downwards,

$$mg' = mg - U = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$

$$\text{Or } mg' = \frac{4}{3}\pi r^3 (\rho - \sigma)g$$

- (iii) The viscous force acting vertically upwards (in a direction opposite to the motion of the body). The viscous force goes on increasing as the velocity of the falling body increases. A stage comes, when the weight of the body becomes just equal to the sum of the upthrust and viscous force. At that stage, the body falls with constant maximum velocity, called the terminal velocity. If v is terminal velocity attained, then according to Stokes' law, the viscous force on the body is given by $F = 6\pi\eta v r$. In equilibrium, the effective weight mg' of the body is equal to the viscous force F acting on the body on attaining the terminal velocity [see figure] i.e.

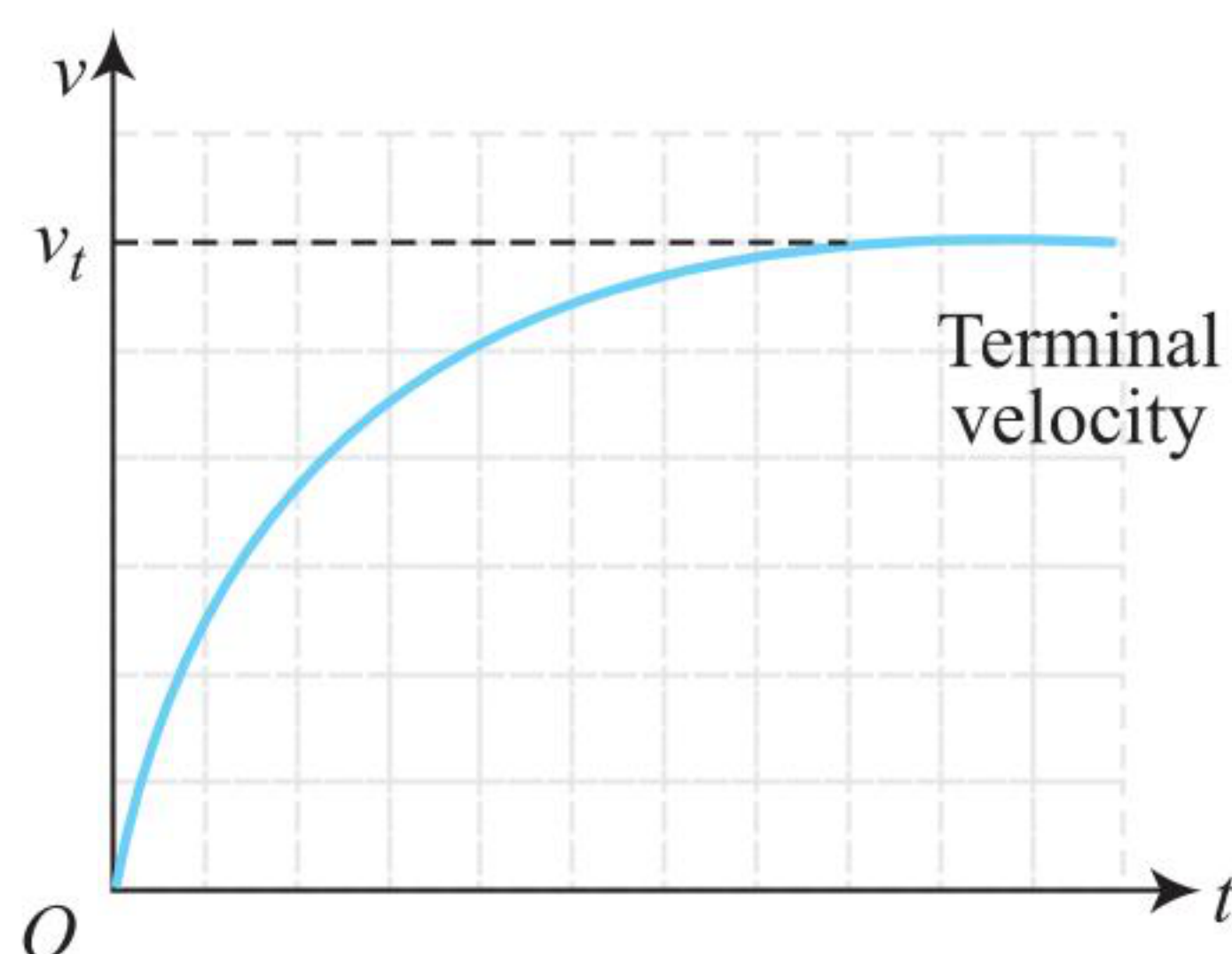
$$\frac{4}{3}\pi r^3 (\rho - \sigma)g = 6\pi\eta v r$$

$$\text{Or } v = \frac{2}{9} \times \frac{r^2 \times (\rho - \sigma)g}{\eta} \quad \dots(i)$$

The following results follow from Eq. (i):

- (i) The terminal velocity varies directly as the square of the radius of the body (i.e., $v_t \propto r^2$) and inversely as the coefficient of viscosity of the fluid, i.e., $v_t \propto 1/\eta$. It also depends upon the densities of the body and the fluid.

- (ii) If the density of the body (ρ) is less than the density of the fluid (σ), the value of v_t is negative. Therefore, the body will move up with uniform velocity. It is on account of this that gas bubbles rise up through soda water.
- (iii) By knowing the radius (r) of the body, densities (ρ, σ) of the body and the fluid and terminal velocity (v_t) [by noting the time taken by the body to cover a known distance in the fluid], the coefficient of viscosity of the fluid can be calculated.



If we plot the variation of velocity of the falling sphere with time, we obtain a graph as shown in the figure. The following points can be noted:

- Initially the velocity of the ball increases at a fast rate.
- The rate of increase of velocity decreases with time.
- Finally, the rate of increase of velocity becomes zero and the sphere acquires a terminal velocity.

Important Point:

Air bubble in water always goes up. It is because density of air (ρ) is less than the density of water (σ). So the terminal velocity for air bubble is negative, which implies that the air bubble will go up. Positive terminal velocity means the body will fall down.

EXAMPLES OF TERMINAL VELOCITY

Some examples of terminal velocity are discussed as follows:

- Determining the electronic charge by Millikan's experiment:** Stokes' formula is used in Millikan's method for determining the electronic charge. In this method, the formula is applied for finding out the radii of small oil drops by measuring their terminal velocities in air.
- Velocity of rain drops:** Rain drops are formed by the condensation of water vapour on dust particles. When they fall under gravity, their motion is opposed by the viscous drag in air. As the velocity of their fall increases, the viscous drag also increases and finally becomes equal to the effective force of gravity. The drops then attain a (constant) terminal velocity which is directly proportional to the square of the radius of the drops. In the beginning, the rain drops are very small in size and so they fall with such a small velocity that they appear floating in the sky as cloud. As they grow in size by further condensation, they reach the earth with appreciable velocity.
- Falling down with the help of a parachute:** When a person jumps down from a flying aeroplane, his parachute is closed. Hence, initially his velocity increases very rapidly while the viscosity of air tries to reduce his velocity. When the parachute opens, the viscosity of air exerts greater viscous force in upward direction (since viscous force is

directly proportional to surface area), due to which the velocity of the person begins to decrease and finally he attains the terminal velocity and reaches safely on ground.

- Formation of clouds:** When water vapours present in the atmosphere condense, small droplets are formed. The weight of these droplets in air is very small. Therefore, they attain the terminal velocity very soon due to viscosity of air. Because the value of their terminal velocity is very small, they appear to float in the sky as clouds.

ILLUSTRATION 8.28

A powder comprising particles of various sizes is stirred up in a vessel filled to a height of 10 cm with water. Assuming the particles to be spherical, find the size of the largest particle that will remain in suspension after 1 h (density of powder = 4 g/cm³, viscosity of water = 0.01 poise).

Sol. Terminal velocity of the largest particle which is just about to settle at the bottom of the vessel is

$$v_t = \frac{10 \times 10^{-2}}{3600} \text{ m/s}$$

Let r be the radius of that particle. Then

$$v_t = \frac{2}{9} r^2 \frac{(\sigma - \rho)}{\eta} g$$

where $\sigma = 4 \times 10^3 \text{ kg/m}^3$ and $\eta = \frac{0.01}{10} \text{ Ns/m}^3$

After solving, we get $r = 2.0 \times 10^{-6} \text{ m}$.

ILLUSTRATION 8.29

Spherical particles of pollen are shaken up in water and allowed to settle. The depth of water is $2 \times 10^{-2} \text{ m}$. What is the diameter of the largest particles remaining in suspension one hour later? Density of pollen = $1.8 \times 10^3 \text{ kg m}^{-3}$, viscosity of water = $1 \times 10^{-2} \text{ poise}$ and density of water = $1 \times 10^3 \text{ kg m}^{-3}$.

Sol. Terminal velocity, $v = \frac{2r^2 (\rho - \sigma) g}{9 \eta}$... (i)

But we know $v = \frac{s}{t}$

$$\therefore \frac{s}{t} = \frac{2r^2 (\rho - \sigma) g}{9 \eta} \Rightarrow r^2 = \frac{9s}{2t (\rho - \sigma) g} \eta$$

Given $s = 2 \times 10^{-2} \text{ m}$, $t = 1 \text{ h} = 3600 \text{ s}$

$$\therefore \eta = 1 \times 10^{-2} \text{ poise} = 1 \times 10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}$$

Substituting given values, we get

$$r^2 = \frac{9}{2} \times \frac{2 \times 10^{-2}}{3600} \times \frac{1 \times 10^{-3}}{(1.8 \times 10^3 - 1 \times 10^3) \times 10} = \frac{9}{36} \times \frac{1}{8} \times 10^{-10}$$

$$= \frac{1}{32} \times 10^{-10}$$

$$\therefore r = \sqrt{\frac{100}{32}} \times 10^{-6} \text{ m} = 1.77 \times 10^{-6} \text{ m}$$

$$\text{Diameter } D = 2r = 2 \times 1.77 \mu\text{m} = 3.54 \mu\text{m}$$

ILLUSTRATION 8.30

A small sphere falls from rest in a viscous liquid. Due to friction, heat is produced. Find the relation between the rate of production of heat and the radius of the sphere at terminal velocity.

Sol. Viscous force on a falling sphere in a liquid $F = \pi\eta r v_T$ where $v_T = (2/9)r^2(\rho - \sigma)g/\eta$ is terminal velocity, ρ = density of sphere, σ = density of liquid.

Rate of production of heat (= power) = $F v_T = 6\pi\eta r v_T^2$

$$= 6\pi\eta r \left[\frac{2}{9} \frac{r^2(\rho - \sigma)g}{\eta} \right]^2 = \frac{8}{27} \frac{\pi g^2(\rho - \sigma)^2}{\eta} r^5$$

Clearly, $\frac{dQ}{dt} \propto r^5$

ILLUSTRATION 8.31

With what terminal velocity will an air bubble 0.8 mm in diameter rise in a liquid of viscosity 0.15 N m⁻² s and specific gravity 0.9? Specific gravity of air = 0.001293. What is the terminal velocity of same bubble in water, if coefficient of viscosity of water is 10⁻³ N m⁻² s?

Sol. Here, radius of the air bubble, $r = 0.8/2 = 0.4$ mm = 4×10^{-4} m; coefficient of viscosity of the liquid, $\eta = 0.15$ N m⁻² s; and specific gravity of the liquid = 0.9.

Therefore, density of the liquid (medium) = $0.9 \text{ g cm}^{-3} = 0.9 \times 10^3 \text{ kg m}^{-3} = 900 \text{ kg m}^{-3}$. Specific gravity of air = 0.001293.

Therefore, density of air (spherical object),

$$\rho = 0.001293 \times 10^3 \text{ kg m}^{-3} = 1.293 \text{ kg m}^{-3}$$

Now, terminal velocity of the air bubble,

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta} = \frac{2 \times (4 \times 10^{-4})^2 \times (1.293 - 900) \times 9.8}{9 \times 0.15} \\ = -2.09 \times 10^{-3} \text{ m s}^{-1}$$

The negative sign indicates that the air bubble will rise up. Terminal velocity of air bubble in water:

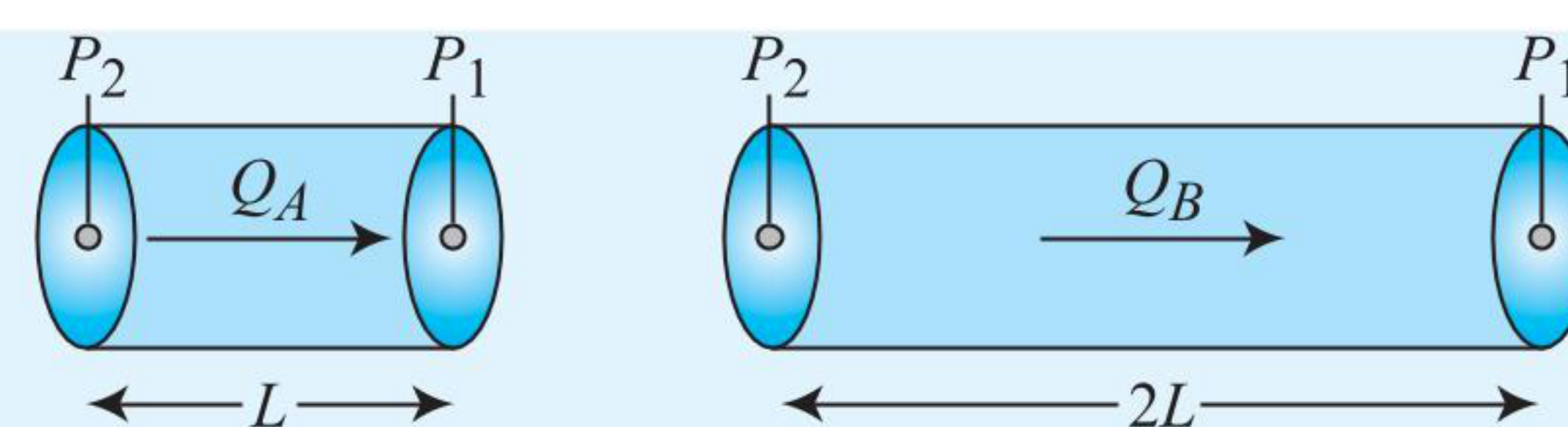
Density of water, $\sigma = 1000 \text{ kg m}^{-3}$;

Coefficient of viscosity of water, $\eta = 10^{-3} \text{ N m}^{-2} \text{ s}$

$$\text{Now, } v = \frac{2r^2(\rho - \sigma)g}{9\eta} = \frac{2 \times (4 \times 10^{-4})^2 (1.293 - 1000) \times 9.8}{9 \times 10^{-3}} \\ = -0.348 \text{ m s}^{-1}$$

CONCEPT APPLICATION EXERCISE 8.5

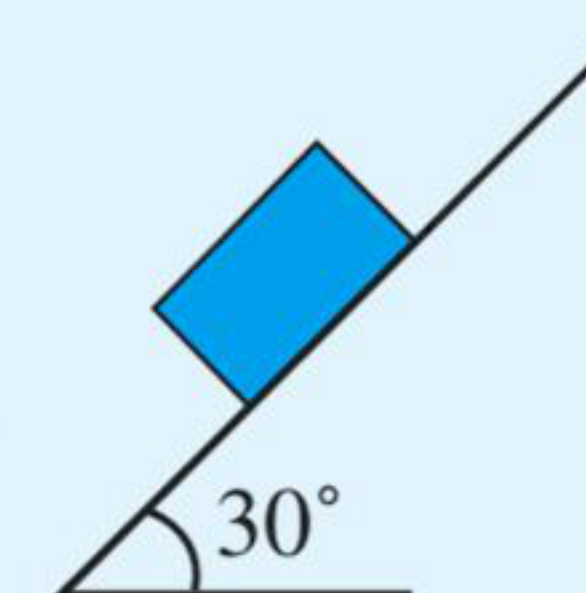
1. A viscous fluid is flowing through two horizontal pipes. The pressure difference $P_2 - P_1$ between the ends of each pipe is the same. The pipes have the same radius, although one is twice as long as the other. How does the volume flow rate Q_B in the longer pipe compare with the rate Q_A in the shorter pipe?



- (a) Q_B is the same as Q_A .
- (b) Q_B is twice as large as Q_A .
- (c) Q_B is four times as large as Q_A .
- (d) Q_B is one-half as large as Q_A .

2. A man is rowing a boat with a constant velocity ' v_0 ' in a river. The contact area of boat is ' A ' and coefficient of viscosity is η . The depth of river is ' D '. Find the force required to row the boat.

3. A cubical block (of side 2 m) of mass 20 kg slides on inclined plane lubricated with the oil of viscosity $\eta = 10^{-1}$ poise with constant velocity of 10 m s^{-1} . Find out the thickness of the layer of liquid (take $g = 10 \text{ m s}^{-2}$).



4. A drop of water of radius 0.0015 mm is falling in air. If the coefficient of viscosity of air is $1.8 \times 10^{-5} \text{ kg/(m s)}$, what will be the terminal velocity of the drop? Density of water = $1.0 \times 10^3 \text{ kg/m}^3$ and $g = 9.8 \text{ N/kg}$. Density of air can be neglected.
5. A metallic sphere of radius $1.0 \times 10^{-3} \text{ m}$ and density $1.0 \times 10^4 \text{ kg/m}^3$ enters a tank of water, after a free fall through a distance of h in the earth's gravitational field. If its velocity remains unchanged after entering water, determine the value of h . Given: coefficient of viscosity of water = $1.0 \times 10^{-3} \text{ N s/m}^2$, $g = 10 \text{ m s}^{-2}$ and density of water = $1.0 \times 10^3 \text{ kg/m}^3$.
6. Find the minimum force required to drag a hard polythene plate of area 2 m^2 on a thin film of oil of thickness 0.25 cm and $\eta = 15$ poise. Assume the speed of the plate is 10 cm s^{-1} .
7. A force of 3.14 N is required to drag a sphere of radius 4 cm with a speed of 5 m s^{-1} in a medium in gravity free space. Find the coefficient of the viscosity of the medium.
8. A square plate of 10 cm side moves parallel to another plate with a velocity of 10 cm s^{-1} , both plates immersed in water. If the viscous force is 200 dyne and viscosity of water is 0.01 poise, what is their distance apart?
9. In a hospital, a patient receives a 500 cm^3 blood transfusion through a needle with a length of 5 cm and inner radius of 0.03 cm. If the blood bag is kept 85 cm above the needle, how long the transfusion takes place? Given that the viscosity of blood is 0.017 poise and the density of blood is 1.02 g cm^{-3} .

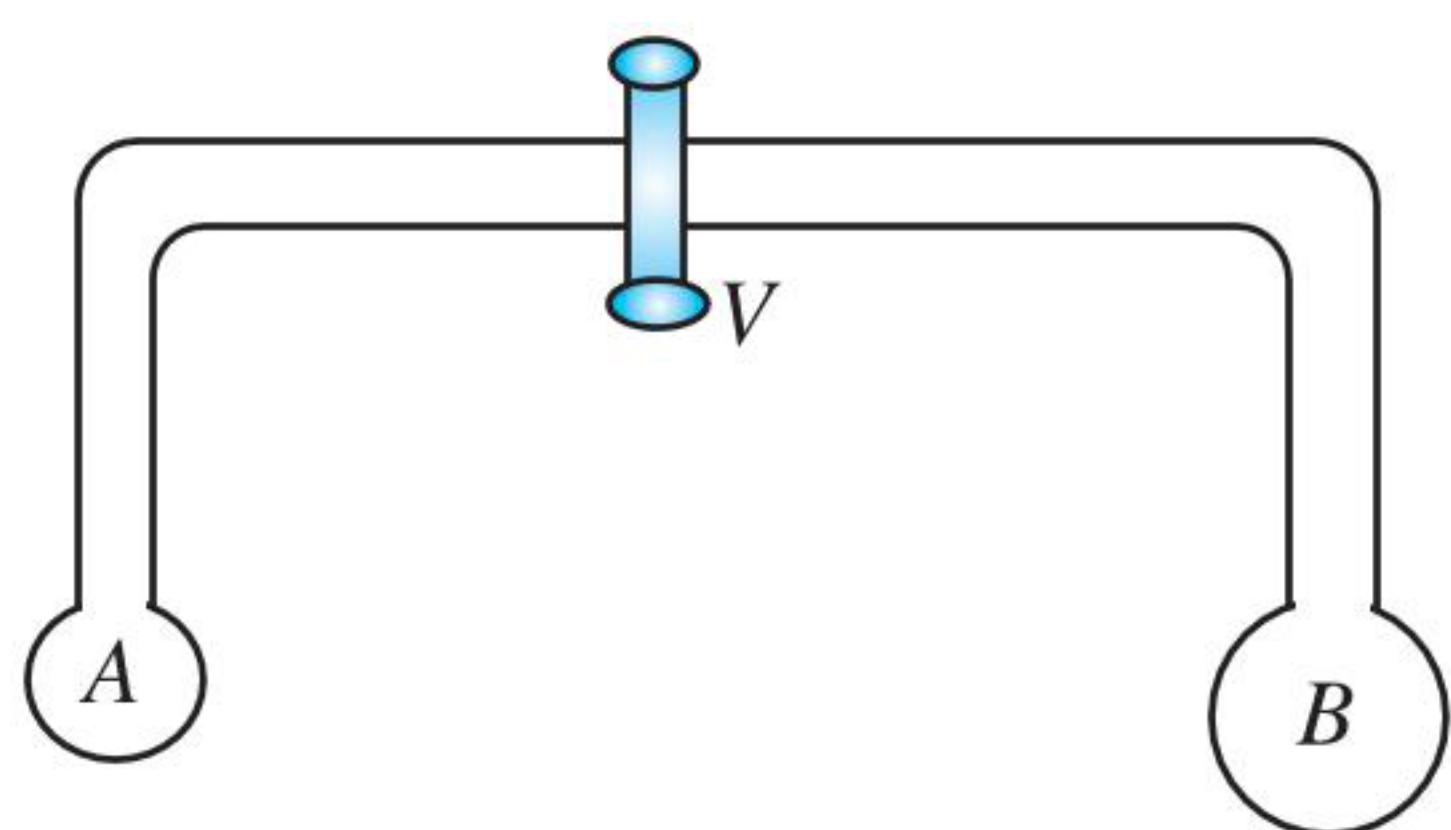
ANSWERS

- | | | | |
|-----------------------------------|---------------------------------------|--------------------------------------|---------|
| 1. (d) | 3. 4 mm | 4. $2.72 \times 10^{-4} \text{ m/s}$ | 5. 20 m |
| 6. $1.2 \times 10^7 \text{ dyne}$ | 7. $8.3 \times 10^{-5} \text{ poise}$ | 8. 0.05 cm | |
| 9. 1529 s | | | |

Exercises

Single Correct Answer Type

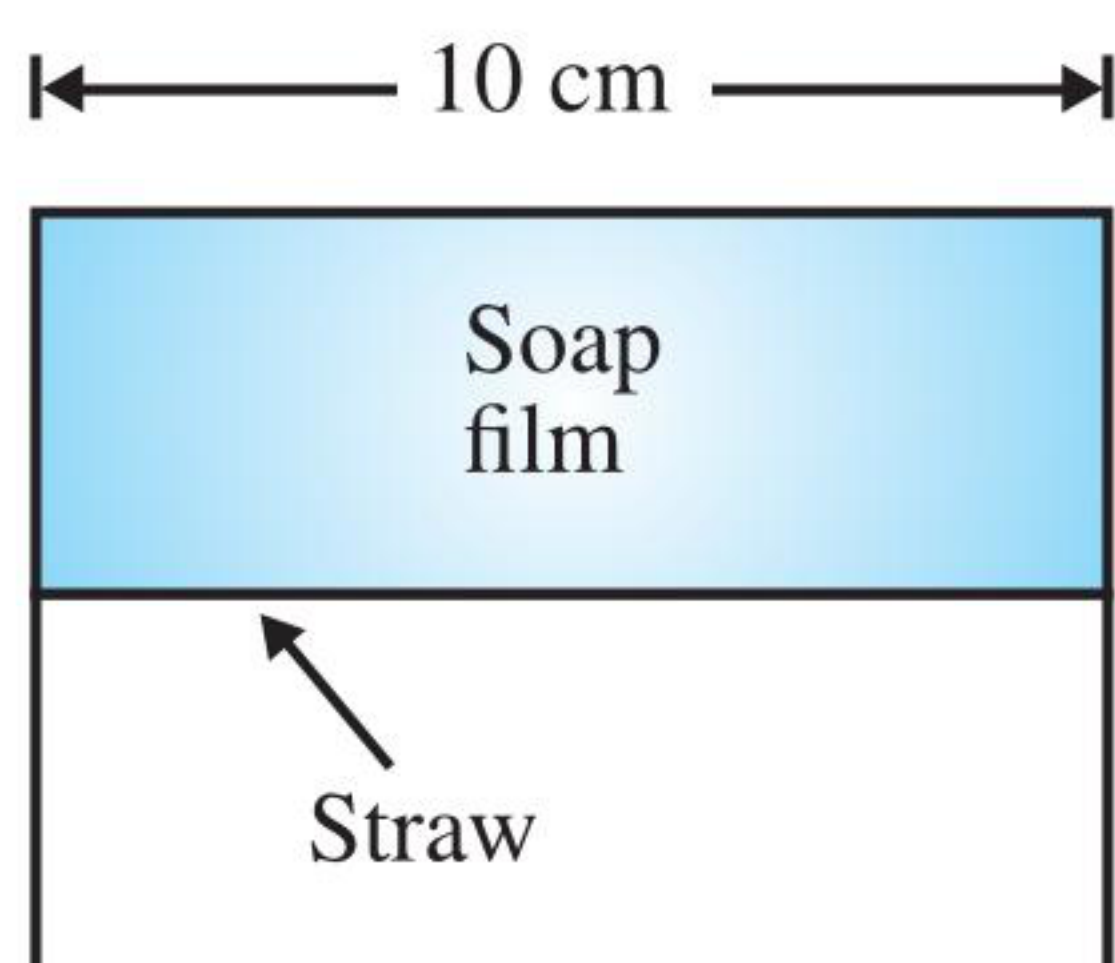
- A water drop is divided into eight equal droplets. The pressure difference between inner and outer sides of the big drop
 - will be the same as for smaller droplet
 - will be half of that for smaller droplet
 - will be one-fourth of that for smaller droplet
 - will be twice of that for smaller droplet
- The value V in the bent tube is initially kept closed. Two soap bubbles A (smaller) and B (larger) are formed at the two open ends of the tube. V is now opened and air can flow freely between the bubbles.



- There will be change in the size of the bubbles
 - The bubbles will become of equal size
 - A will become smaller and B will become larger
 - The sizes of A and B will be interchanged
- Two soap bubbles of different radii are in communication with each other. Then
 - air follows from the larger bubble into smaller bubble till both bubbles acquire same size
 - air follows from the smaller bubble into larger bubble and the larger bubble grows in size with decrease in size of the smaller bubble
 - air does not flow but the sizes of the bubbles changes
 - sizes of the bubbles remain unchanged
 - A paper disc of radius R from which a hole of radius r is cut out is floating in a liquid of the surface tension S . The force on the disc due to the surface tension is
 - $S \times 2\pi R$
 - $S \times 2\pi r$
 - $S \times 2\pi(R - r)$
 - $S \times 2\pi(R + r)$
 - If T is surface tension of soap solution, the amount of work done in blowing a soap bubble from a diameter D to a diameter $2D$ is
 - $2\pi D^2 T$
 - $4\pi D^2 T$
 - $6\pi D^2 T$
 - $8\pi D^2 T$
 - Work W is required to form a bubble of volume V from a given solution. What amount of work is required to be done to form a bubble of volume $2V$?
 - W
 - $2W$
 - $2^{1/3}W$
 - $4^{1/3}W$
 - The surface energy of a liquid drop is E . It is sprayed into 1000 equal droplets. Then its surface energy becomes
 - $1000E$
 - $100E$
 - $10E$
 - E

- In the previous question, the work done in spraying is
 - $999E$
 - $99E$
 - $9E$
 - E
- A small metal ball of diameter 4 mm and density 10.5 g/cm^3 is dropped in glycerine of density 1.5 g/cm^3 . The ball attains a terminal velocity of 8 cm s^{-1} . The coefficient of viscosity of glycerine is
 - 4.9 poise
 - 9.8 poise
 - 98 poise
 - 980 poise
- A sphere of brass released in a long liquid column attains a terminal speed v_0 . If the terminal speed is attained by a sphere of marble of the same radius and released in the same liquid is $n v_0$, then the value of n will be (Given: The specific gravities of brass, marble and liquid are 8.5, 2.5 and 0.8, respectively)
 - $\frac{5}{17}$
 - $\frac{17}{77}$
 - $\frac{11}{31}$
 - $\frac{17}{5}$
- Between a plate of area 100 cm^2 and another plate of area 100 m^2 there is a 1 mm, thick layer of water, if the coefficient of viscosity of water is 0.01 poise, then the force required to move the smaller plate with a velocity 10 cm s^{-1} with reference to large plate is
 - 100 dyn
 - 10^4 dyn
 - 10^6 dyn
 - 10^9 dyn
- A river 10 m deep is flowing at 5 m s^{-1} . The shearing stress between horizontal layers of the river is ($\eta = 10^{-3} \text{ SI units}$)
 - 10^{-3} N/m^2
 - $0.8 \times 10^{-3} \text{ N/m}^2$
 - $0.5 \times 10^{-3} \text{ N/m}^2$
 - 1 N/m^2
- A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator, the length of water column in the capillary tube will be
 - 20 cm
 - 4 cm
 - 10 cm
 - 8 cm
- A glass rod of diameter $d = 2 \text{ mm}$ is inserted symmetrically into a glass capillary tube of radius $r = 2 \text{ mm}$. Then the whole arrangement is vertically dipped into liquid having surface tension 0.072 N/m . The height to which liquid will rise on capillary is (Take $g = 10 \text{ m s}^{-2}$, $\rho_{\text{liq}} = 1000 \text{ kg/m}^3$. Assume contact angle to be zero of capillary tube to be long enough)
 - 1.44 cm
 - 6 cm
 - 4.86 cm
 - None of these
- Two soap bubbles, one of radius 50 mm and the other of radius 80 mm, are brought in contact so that they have a common interface. The radius of the curvature of the common interface is
 - 0.003 m
 - 0.133 m
 - 1.2 m
 - 8.9 m

16. A straw 6 cm long floats on water. The water film on one side has surface tension of 50 dyn/cm. On the other side, camphor reduces the surface tension to 40 dyn/cm. The resultant force acting on the straw is
 (1) $(50 \times 6 - 40 \times 6)$ dyn (2) 10 dyn
 (3) $\left(\frac{50}{6} - \frac{40}{6}\right)$ dyn (4) 90 dyn
17. Two glass plates are separated by water. If surface tension of water is 75 dyn/cm and the area of each plate wetted by water is 8 cm² and the distance between the plates is 0.12 mm, then the force applied to separate the two plates is
 (1) 10² dyn (2) 10⁴ dyn
 (3) 10⁵ dyn (4) 10⁶ dyn
18. A ring is cut from a platinum tube 8.5 cm internal and 8.7 cm external diameter. It is supported horizontally from the pan of a balance, so that it comes in contact with the water in a glass vessel. If an extra 3.103 gf is required to pull it away from water, the surface tension of water is
 (1) 72 dyn/cm (2) 70.80 dyn/cm
 (3) 63.35 dyn/cm (4) 60 dyn/cm
19. A soap film of surface tension 3×10^{-2} formed in a rectangular frame can support a straw as shown in figure. If $g = 10 \text{ ms}^{-2}$, the mass of the straw is



- (1) 0.006 g (2) 0.06 g
 (3) 0.6 g (4) 6 g
20. The lower end of a capillary tube is at a depth of 12 cm and water rises 3 cm in it. The mouth pressure required to blow an air bubble at the lower end will be x cm of water column, where x is
 (1) 12 (2) 15
 (3) 3 (4) 9
21. A film of soap solution is trapped between a vertical frame and a light wire ab of length 0.1 m. If $g = 10 \text{ ms}^{-2}$, then the load W that should be suspended from the wire to keep it in equilibrium is
 (1) 0.2 g (2) 0.3 g
 (3) 0.4 g (4) 0.5 g
22. The angle of contact between glass and water is 0° and water (surface tension 70 dyn/cm) rises in a glass capillary up to 6 cm. Another liquid of surface tension 140 dyn/cm, angle of contact 60° and relative density 2 will rise in the same capillary up to
 (1) 12 cm (2) 24 cm
 (3) 3 cm (4) 6 cm
23. A hollow sphere has a small hole in it. On lowering the sphere in a tank of water, it is observed that water enters into the hollow sphere at a depth of 40 cm below the surface. Surface tension of water is $7 \times 10^{-2} \text{ N/m}$. The diameter of the hole is

- (1) $\frac{1}{28} \text{ mm}$ (2) $\frac{1}{21} \text{ mm}$
 (3) $\frac{1}{14} \text{ mm}$ (4) $\frac{1}{7} \text{ mm}$

24. A capillary tube is attached horizontally to a constant pressure head arrangement. If the radius of the capillary tube is increased by 10%, then the rate of flow of the liquid shall change nearly by
 (1) +10% (2) 46%
 (3) -10% (4) -40%
25. A ball rises to the surface of a liquid with constant velocity. The density of the liquid is four times the density of the material of the ball. The frictional force of the liquid on the rising ball is greater than the weight of the ball by a factor of
 (1) 2 (2) 3
 (3) 4 (4) 6
26. A spherical ball falls through viscous medium with terminal velocity v . If this ball is replaced by another ball of the same mass but half the radius, then the terminal velocity will be (neglect the effect of buoyancy.)
 (1) v (2) $2v$
 (3) $4v$ (4) $8v$
27. Two parallel wires each of length 10 cm are 0.5 cm apart. A film of water is formed between them. If surface tension of water is 0.072 N/m, then the work done in increasing the distance between the wires by 1 mm is
 (1) $1.44 \times 10^{-5} \text{ J}$ (2) $1.72 \times 10^{-5} \text{ J}$
 (3) $1.44 \times 10^{-4} \text{ J}$ (4) $1.72 \times 10^{-4} \text{ J}$
28. The length of a needle floating on water is 2.5 cm. The minimum force in addition to its weight needed to lift the needle above the surface of water will be (surface tension of water is 0.072 N/m)
 (1) $3.6 \times 10^{-3} \text{ N}$ (2) 10^{-2} N
 (3) $9 \times 10^{-4} \text{ N}$ (4) $6 \times 10^{-4} \text{ N}$
29. A solid sphere falls with a terminal velocity of 20 ms^{-1} in air. If it is allowed to fall in vacuum,
 (1) terminal velocity will be 20 ms^{-1}
 (2) terminal velocity will be less than 20 ms^{-1}
 (3) terminal velocity will be greater than 20 ms^{-1}
 (4) no terminal velocity will be attained
30. A marble of mass x and diameter $2r$ is gently released in a tall cylinder containing honey. If the marble displaces mass y ($< x$) of the liquid, then the terminal velocity is proportional to
 (1) $x + y$ (2) $x - y$
 (3) $\frac{x + y}{r}$ (4) $\frac{x - y}{r}$
31. Water rises to a height h in a capillary tube of cross-sectional area A . The height to which water will rise in a capillary tube of cross-sectional area $4A$ will be
 (1) h (2) $h/2$
 (3) $h/4$ (4) $4h$
32. Neglecting the density of air, the terminal velocity obtained by a raindrop of radius 0.3 mm falling through the air of viscosity $1.8 \times 10^{-5} \text{ N/m}^2$ will be

- (1) 10.9 m/s (2) 8.3 m/s
(3) 9.2 m/s (4) 7.6 m/s
33. Water rises to a height of 2 cm in a capillary tube. If the tube is tilted 60° from the vertical, water will rise in the tube to a length of
(1) 4.0 cm (2) 2.0 cm
(3) 1.0 cm (4) water will not rise at all
34. A vessel, whose bottom has round holes with diameter 0.1 mm, is filled with water. The maximum height up to which water can be filled without leakage is
(1) 100 cm (2) 75 cm (3) 50 cm (4) 30 cm
35. Water rises to a height of 10 cm in a capillary tube and mercury falls to a depth of 3.42 cm in the same capillary tube. If the density of mercury is 13.6 g/c.c. and the angles of contact for mercury and water are 135° and 0° , respectively, the ratio of surface tension for water and mercury is
(1) 20 : 3 (2) 1 : 3
(3) 2 : 13 (4) 3 : 2
36. The velocity of small ball of mass M and density d_1 when dropped a container filled with glycerine becomes constant after some time. If the density glycerine is d_2 , the viscous force acting on ball is
(1) $\frac{Md_1 g}{d_2}$ (2) $Mg\left(1 - \frac{d_2}{d_1}\right)$
(3) $\frac{M(d_1 + d_2)}{g}$ (4) $Md_1 d_2$
37. A cube with a mass = 20 g wettable water floats on the surface of water. Each face of the cube is $\alpha = 3$ cm long. Surface tension of water is 70 dyn/cm. The distance of the lower face of the cube from the surface of water is ($g = 980 \text{ cm s}^{-2}$)
(1) 2.3 cm (2) 4.6 cm
(3) 9.7 cm (4) 12.7 cm
38. Consider vertical parallel of semi-circular cross section dipped in a liquid. Assume that the wetting of the tube is complete. The force of surface tension on the flat part and on curved part of the tube are in the ratio
(1) 2 : π (2) 1 : π
(3) 3 : π (4) 4 : π
39. Two vertical parallel glass plates are partially submerged in water. The distance between the plates is d and the length is l . Assume that the water between the plates does not reach the upper edges of the plates and that the wetting is complete. The water will rise to height (ρ = density of water and σ = surface tension of water)
(1) $\frac{2\sigma}{\rho g d}$ (2) $\frac{\sigma}{2\rho g d}$
(3) $\frac{4\sigma}{\rho g d}$ (4) $\frac{5\sigma}{\rho g d}$
40. In the previous question, the force of attraction between the plates is
(1) $\frac{l\rho g}{2} h^2$ (2) $\frac{l\rho g}{2} h$
(3) $\frac{l\rho g}{2}$ (4) $\frac{l\rho g}{2h}$
(Here h represents the height to which the liquid rises.)
41. In Q.39, if water is replaced by another liquid of surface tension 2σ , then the force F of attraction between two plates would become
(1) $2F$ (2) $4F$
(3) $8F$ (4) $16F$
42. A number of droplets, each of radius r , combine to form a drop of radius R . If T is the surface tension, the rise in temperature will be
(1) $\frac{2T}{r}$ (2) $\frac{3T}{R}$
(3) $2T\left[\frac{1}{r} - \frac{1}{R}\right]$ (4) $3T\left[\frac{1}{r} - \frac{1}{R}\right]$
43. A drop of liquid of density ρ is floating half-immersed in a liquid of density d . If ρ is the surface tension the diameter of the drop of the liquid is
(1) $\sqrt{\frac{\sigma}{g(2\rho - d)}}$ (2) $\sqrt{\frac{2\sigma}{g(2\rho - d)}}$
(3) $\sqrt{\frac{6\sigma}{g(2\rho - d)}}$ (4) $\sqrt{\frac{12\sigma}{g(2\rho - d)}}$
44. A drop of water of volume V is pressed between the two glass placed so as to spread to an area A . If T is the surface tension, the normal force required to separate the glass plates is
(1) $\frac{TA^2}{V}$ (2) $\frac{2TA^2}{V}$
(3) $\frac{4TA^2}{V}$ (4) $\frac{TA^2}{2V}$
45. A thin square plate of side 5 cm is suspended vertically from a balance so that lower side just dips into water with side to surface. When the plate is clean ($\theta = 0^\circ$), it appears to weigh 0.044 N. But when the plate is greasy ($\theta = 180^\circ$), it appears to weigh 0.03 N. The surface tension of water is
(1) $3.5 \times 10^{-2} \text{ N/m}$ (2) $7.0 \times 10^{-2} \text{ N/m}$
(3) $14.0 \times 10^{-2} \text{ N/m}$ (4) 1.08 N/m
46. A loop of 6.28 cm long thread is put gently on a soap film in a wire loop. The film is pricked with a needle inside the soap film enclosed by the thread. If the surface tension of soap solution is 0.030 N/m, then the tension in the thread is
(1) $1 \times 10^{-4} \text{ N}$ (2) $2 \times 10^{-4} \text{ N}$
(3) $3 \times 10^{-4} \text{ N}$ (4) $4 \times 10^{-4} \text{ N}$
47. A glass rod of radius r_1 is inserted symmetrically into a vertical capillary tube of radius r_2 such that their lower ends are at the same level. The arrangement is now dipped in water. The height to which water will rise into the tube will be (σ = surface tension of water, ρ = density of water)
(1) $\frac{2\sigma}{(r_2 - r_1)\rho g}$ (2) $\frac{\sigma}{(r_2 - r_1)\rho g}$
(3) $\frac{2\sigma}{(r_2 + r_1)\rho g}$ (4) $\frac{2\sigma}{(r_2^2 + r_1^2)\rho g}$
48. A large number of droplets, each of radius a , coalesce to form a bigger drop of radius b . Assume that the energy released in the process is converted into the kinetic energy

of the drop. The velocity of the drop is (σ = surface tension, ρ = density)

$$(1) \left[\frac{\sigma}{\rho} \left(\frac{1}{a} - \frac{1}{b} \right) \right]^{1/2} \quad (2) \left[\frac{2\sigma}{\rho} \left(\frac{1}{a} - \frac{1}{b} \right) \right]^{1/2}$$

$$(3) \left[\frac{3\sigma}{\rho} \left(\frac{1}{a} - \frac{1}{b} \right) \right]^{1/2} \quad (4) \left[\frac{6\sigma}{\rho} \left(\frac{1}{a} - \frac{1}{b} \right) \right]^{1/2}$$

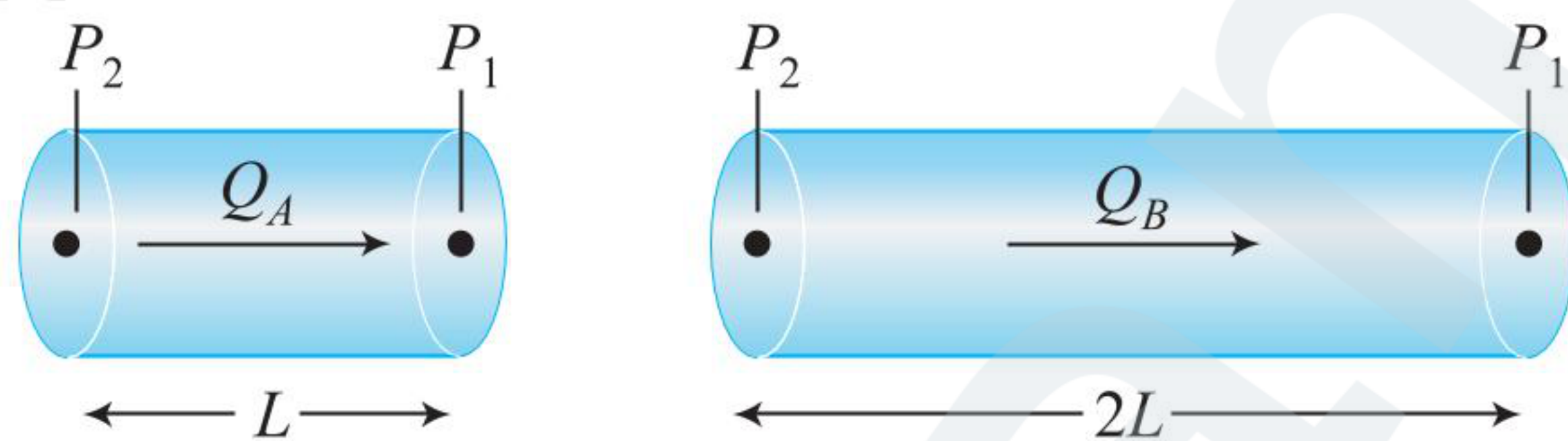
49. The space between two large horizontal metal plates, 6 cm apart, is filled with a liquid of viscosity 0.8 N/m^2 . A thin plate of surface area 0.01 m^2 is moved parallel to the length of the plate such that the plate is at a distance of 2 cm from one of the plates and 4 cm from the other. If the plate moves with a constant speed of 1 m/s , then

- (1) the layer of the fluid, which is having the maximum velocity, is lying mid-way between the plates
- (2) the layer of the fluid, which is in contact with the moving plate, is having the maximum velocity
- (3) the layer of the fluid, which is in contact with the moving plate and is on the side of farther plate, is moving with the maximum velocity
- (4) the layer of the fluid, which is in contact with the moving plate and is on the side of nearer plate, is moving with the maximum velocity

50. In the previous question, the net force exerted by fluid on moving plate is

- (1) 0.6 N backwards
- (2) 0.6 N forwards
- (3) 0.2 N backwards
- (4) 0.2 N forwards

51. A viscous fluid is flowing through two horizontal pipes. The pressure difference $P_2 - P_1$ between the ends of each pipe is the same. The pipes have the same radius, although one is twice as long as the other. How does the volume flow rate Q_B in the longer pipe compare with the rate Q_A in the shorter pipe?

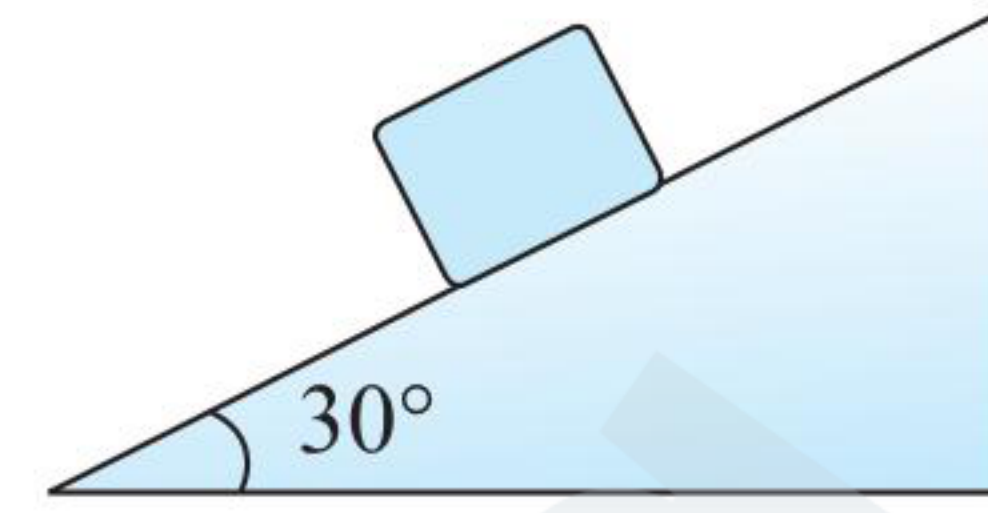


- (1) Q_B is the same as Q_A .
- (2) Q_B is twice as large as Q_A .
- (3) Q_B is four times as large as Q_A .
- (4) Q_B is one-half as large as Q_A .

52. A rain drop starts falling from a height of 2 km. It falls with a continuously decreasing acceleration and attains its terminal velocity at a height of 1 km. The ratio of the work done by the gravitational force in the first half and the second half of the drop's journey is

- (1) 1 : 1 and the times of fall of the drop in the two halves is $a : 1$ (where $a > 1$)
- (2) 1 : 1 and the times of fall of the drop in the two halves is $a : 1$ (where $a < 1$)
- (3) $a : 1$ (where $a > 1$) and the times of fall of the drop in the two halves is 1 : 1
- (4) $a : 1$ (where $a < 1$) and the times of fall of the drop in the two halves is 1 : 1

53. A cubical block (of side 2 m) of mass 20 kg slides on inclined plane lubricated with the oil of viscosity $\eta = 10^{-1}$ poise with constant velocity of 10 m/s ($g = 10 \text{ m/s}^2$). The thickness of layer of liquid is ($10^{-1} \text{ poise} = 10^{-2} \text{ N s/m}^2$)



- (1) 2.5 mm
- (2) 4 mm
- (3) 6 mm
- (4) 5.5 mm

54. A man is rowing a boat of mass m with a constant velocity ' v_0 ' in a river. The contact area of boat is ' A ' and coefficient of viscosity is η . The depth of river is ' D '. Assume that velocity gradient is constant. The force required to row the boat is

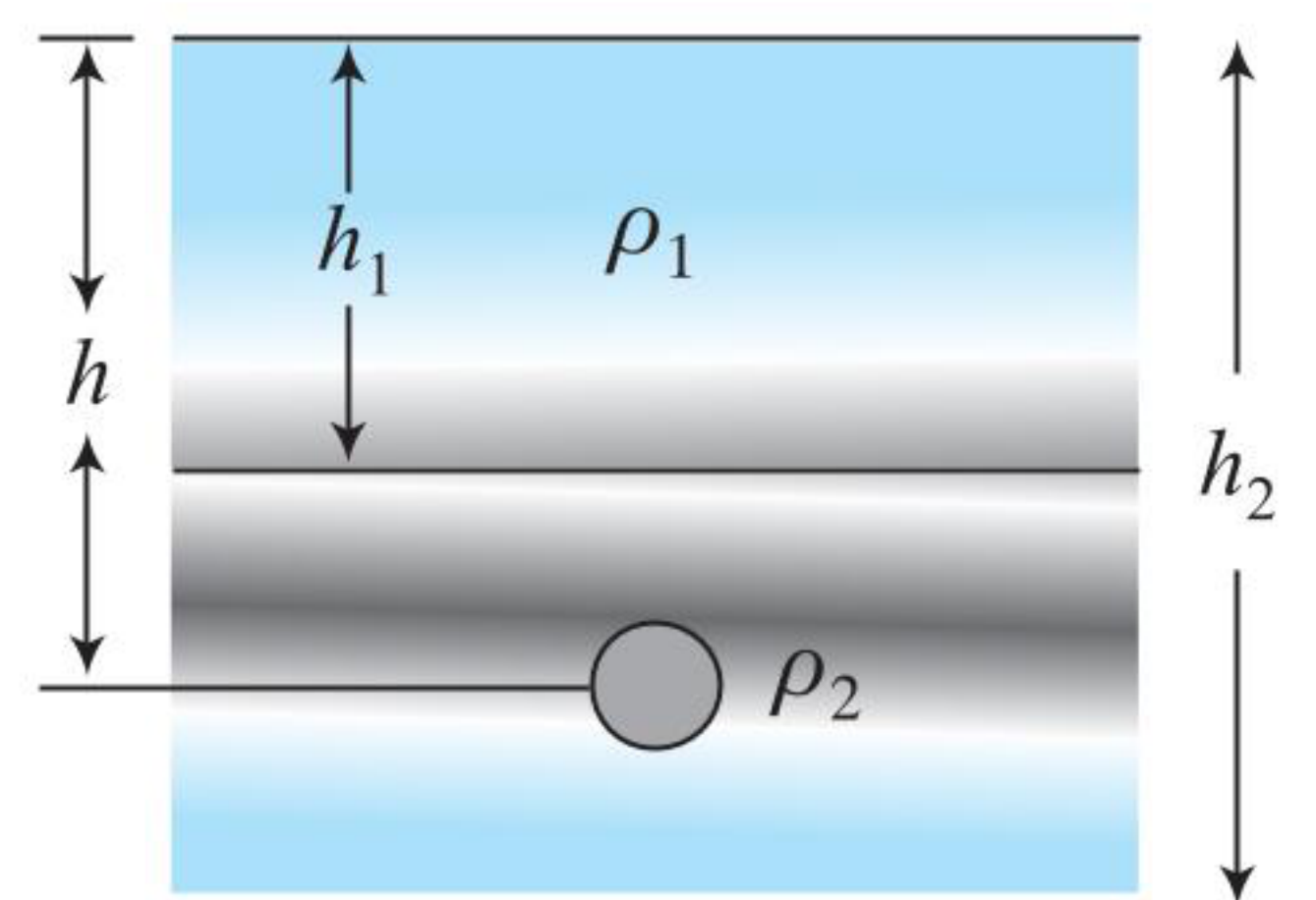
- (1) $\frac{\eta A v_0}{2D}$
- (2) $\frac{2\eta A v_0}{D}$
- (3) $\frac{\eta A v_0}{D}$
- (4) $\frac{3\eta A v_0}{2D}$

55. A soap bubble of radius R is surrounded by another soap bubble of radius $2R$, as shown. Take surface tension = S . Then, the pressure inside the smaller soap bubble, in excess of the atmospheric pressure, will be



- (1) $4S/R$
- (2) $3S/R$
- (3) $6S/R$
- (4) None of these

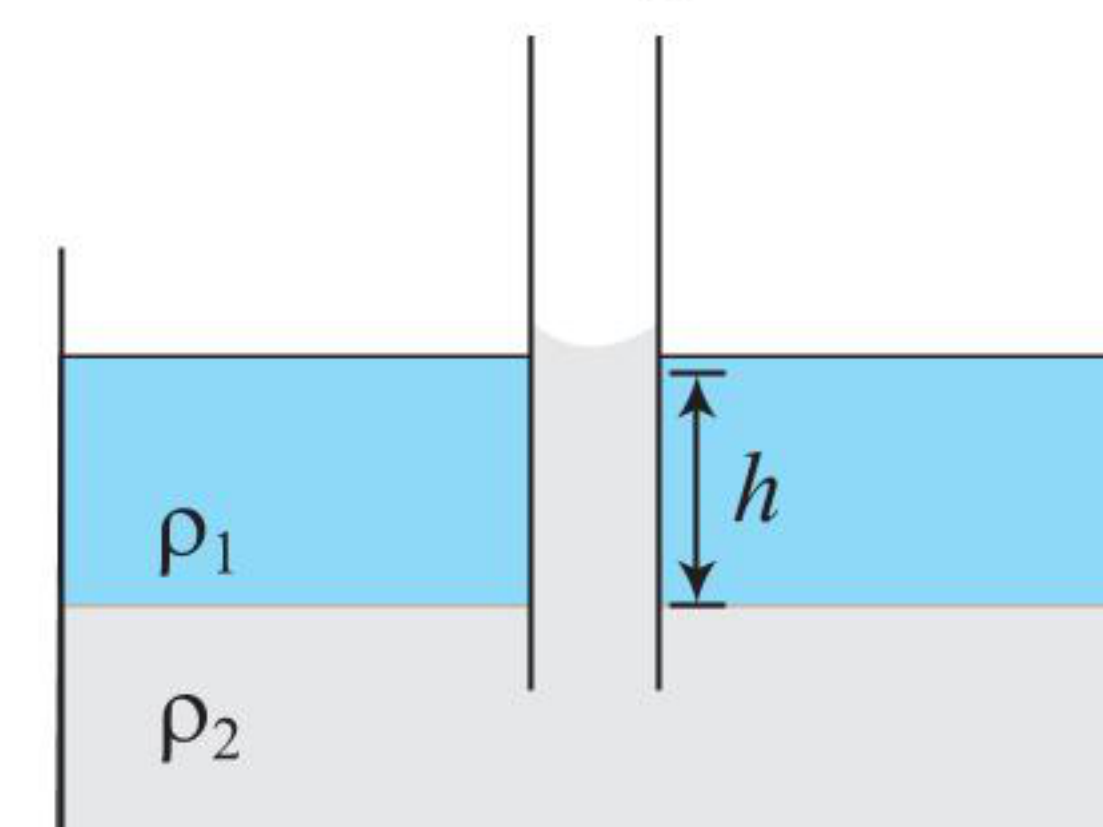
56. Calculate the pressure inside a small air bubble of radius r situated at a depth of h below the free surface of liquids of densities ρ_1 and ρ_2 and surface tensions T_1 and T_2 . The thickness of first and second liquids are h_1 and h_2 .



Take atmospheric pressure = P_0 .

- (1) $P_0 + \rho_1 g h_1 + \rho_2 g (h - h_1) - \frac{2T_2}{r}$
- (2) $P_0 + \rho_1 g h_1 + \rho_2 g (h - h_1) + \frac{2T_2}{r}$
- (3) $P_0 + \rho_1 g h_2 + \rho_2 g (h - h_1) + \frac{2T_2}{r}$
- (4) None of these

57. A container is partially filled with a liquid of density ρ_2 . A capillary tube of radius r is vertically inserted in this liquid. Now another liquid of density ρ_1 ($\rho_1 < \rho_2$) is slowly poured in the container to a height h as shown. There is only denser liquid in the capillary tube. The rise of denser liquid in the capillary tube is also h . Assuming zero contact angle, the surface tension of heavier liquid is

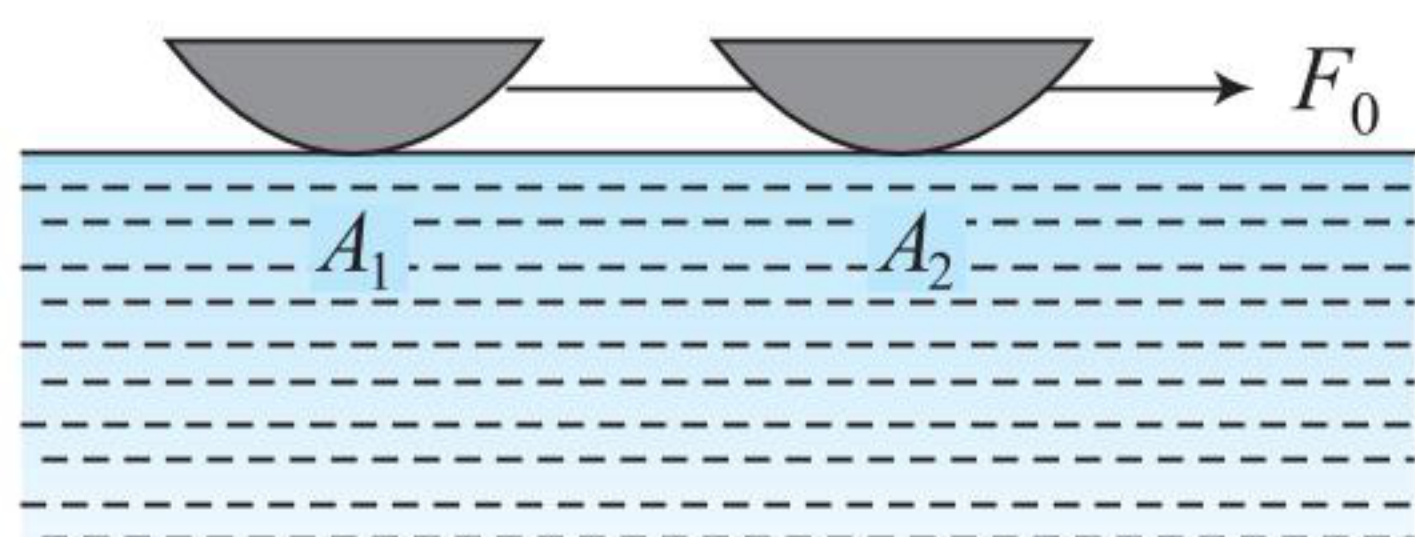


- (1) $r \rho_2 gh$ (2) $2\pi r \rho_2 gh$
 (3) $\frac{r}{2}(\rho_2 - \rho_1)gh$ (4) $2\pi r(\rho_2 - \rho_1)gh$

58. A glass plate of length 10 cm, breadth 1.54 cm and thickness 0.2 cm, weight 8.2 g in air. It is held vertically with the long side horizontal and the lower half under water. Surface tension of water is $7.3 \times 10^{-2} \text{ Nm}^{-1}$ and $g = 9.8 \text{ ms}^{-2}$. The apparent weight of the plate is

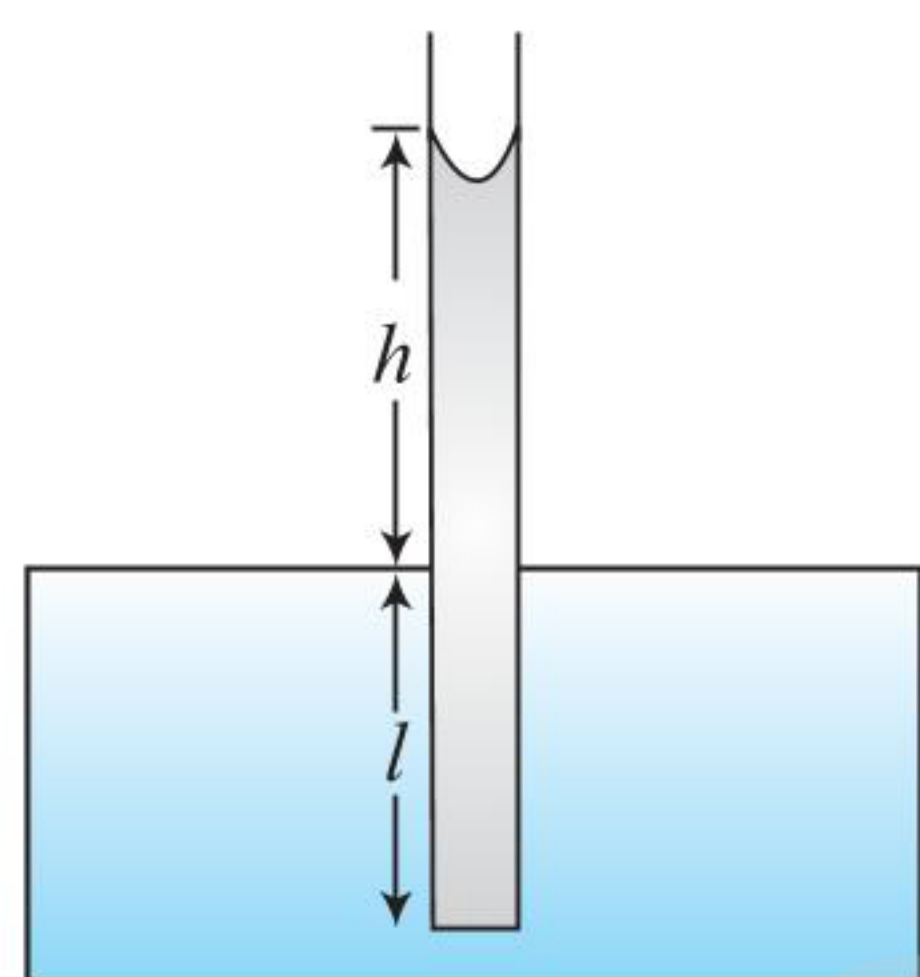
- (1) $40.16 \times 10^{-3} \text{ N}$ (2) $80.16 \times 10^{-3} \text{ N}$
 (3) $30.12 \times 10^{-3} \text{ N}$ (4) $25.15 \times 10^{-3} \text{ N}$

59. Two boats of base areas A_1 and A_2 , connected by a string are being pulled by an external force F_0 . The viscosity of water is η and depth of the water body is H . When the system attains a constant speed, the tension in the thread will be



- (1) $F_0 \left(\frac{A_1}{A_2} \right)$ (2) $F_0 \frac{A_2}{(A_1 + A_2)}$
 (3) $F_0 \frac{A_1}{(A_1 + A_2)}$ (4) $F_0 \left(\frac{A_2}{A_1} \right)$

60. Water rises to a height h in a capillary tube lowered vertically into water to a depth l as shown in the figure. The lower end of the tube is now closed, the tube is then taken out of the water and opened again. The length of the water column remaining in the tube will be

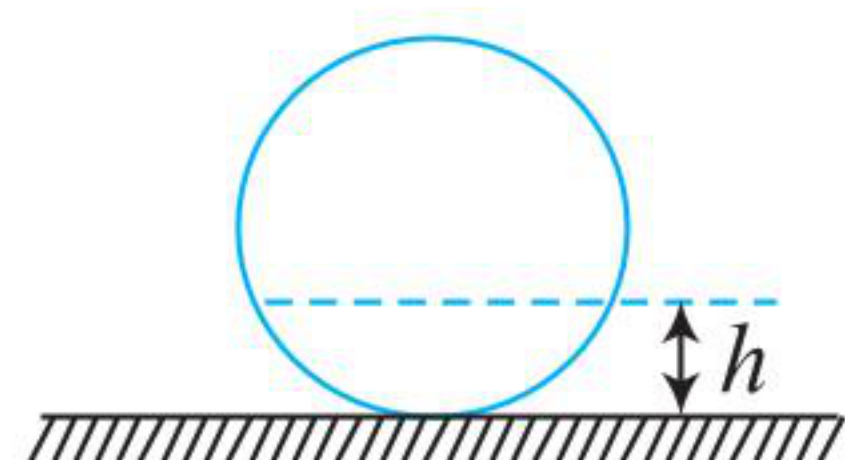


- (1) $2h$ if $l \geq h$ and $l + h$ if $l \leq h$
 (2) h if $l \geq h$ and $l + h$ if $l \leq h$
 (3) $4h$ if $l \geq h$ and $l - h$ if $l \leq h$
 (4) $\frac{h}{2}$ if $l \geq h$ and $l + h$ if $l \leq h$

61. Two parallel glass plates are dipped partly in the liquid of density d keeping them vertical. If the distance between the plates is x , surface tension for liquid is T and angle of contact is θ then rise of liquid between the plates due to capillary will be

- (1) $\frac{T \cos \theta}{xd}$ (2) $\frac{2T \cos \theta}{xdg}$
 (3) $\frac{2T}{x dg \cos \theta}$ (4) $\frac{T \cos \theta}{x dg}$

62. A liquid is filled in a spherical container of radius R till a height h . At this positions the liquid surface at the edges is also horizontal. The contact angle is



- (1) 0 (2) $\cos^{-1} \left(\frac{R-h}{R} \right)$
 (3) $\cos^{-1} \left(\frac{h-R}{R} \right)$ (4) $\sin^{-1} \left(\frac{R-h}{R} \right)$

Multiple Correct Answers Type

- Excess pressure can be $(2T/R)$ for
 - spherical drop
 - spherical meniscus
 - cylindrical bubble in air
 - spherical bubble in water
- If a liquid rises to the same height in two capillaries of the same material at the same temperature, then
 - the weight of liquid in both capillaries must be equal
 - the radius of meniscus must be equal
 - the capillaries must be cylindrical and vertical
 - the hydrostatic pressure at the base of capillaries must be same
- Viscous force is somewhat like friction as it opposes, the motion and is non-conservative but not exactly so, because
 - it is velocity dependent while friction is not
 - it is velocity independent while friction is
 - it is temperature dependent while friction is not
 - it is independent of area is like surface tension while friction is dependent
- When a capillary tube is dipped in a liquid, the liquid rises to a height h in the tube. The free liquid surface inside the tube is hemispherical in shape. The tube is now pushed down so that the height of the tube outside the liquid is less than h . Then
 - the liquid will come out of the tube like in a small fountain
 - the liquid will ooze out of the tube slowly
 - the liquid will fill the tube but not come out of its upper end
 - the free liquid surface inside the tube will not be hemispherical
- A vertical glass capillary tube, open at both ends, contains some water. Which of the following shapes may not be taken by the water in the tube?

(1)

(2)

(3)

(4)
- If n drops of a liquid, each with surface energy E , join to form a single drop, then
 - some energy will be released in the process
 - some energy will be absorbed in the process
 - the energy released or absorbed will be $E(n - n^{2/3})$
 - the energy released or absorbed will be $nE(2^{2/3} - 1)$

7. Eight identical droplets, each falling under gravity in the earth's atmosphere with terminal velocity v , combine to form a single drop. The terminal velocity of single drop and drag force on it are calculated. Find the correct statement from the following.
- Final terminal velocity is $4v$
 - Final terminal velocity is $8v$
 - Drag force is increased to four times
 - Drag force is increased to eight times
8. A sphere is dropped into a viscous liquid of viscosity η from some height. If the density of material and liquid are ρ and σ respectively ($\rho > \sigma$) then which of the following is/are incorrect.
- The acceleration of the sphere just after entering the liquid is $g\left(\frac{\rho - \sigma}{\rho}\right)$
 - Time taken to attain terminal speed $t \propto \rho^0$
 - At terminal speed, the viscous force is maximum
 - At terminal speed, the net force acting on the sphere is zero
9. Two separate bubbles (radii 0.002 m and 0.004 m) formed of the same soap solution (surface tension 0.07 N m^{-1}) comes together to form a double bubble. The internal film common to both the bubbles has radius R . Then,
- $R = 0.004 \text{ m}$
 - $R = 0.003 \text{ m}$
 - internal film is concave towards smaller bubble
 - internal film is concave towards bigger bubble
10. The velocity of liquid (v) in steady flow at a location through cylindrical pipe is given by $v = v_0 \left(1 - \frac{r^2}{R^2}\right)$, where r is the radial distance of that location from the axis of the pipe and R is the inner radius of pipe, if $R = 10 \text{ cm}$, volume rate of flow through the pipe is $\pi/2 \times 10^{-2} \text{ m}^3 \text{ s}^{-1}$ and the coefficient of viscosity of the liquid is 0.75 N s m^{-2}
- Magnitude of viscous force per unit area at $r = 4 \text{ cm}$ is 6 N/m^2
 - Maximum velocity of flow of liquid is 1 m/s
 - Viscous force per unit area increases with radial distance
 - Viscous force per unit area decreases with radial distance
11. A water drop and a mercury drop is taken inside identical horizontal conical glass pipes. Then
- the water drop tends to move towards the narrow end
 - the water drop tends to move towards the wide end
 - the mercury drop tends to move towards the wide end
 - the mercury drop tends to move towards the narrow end

Linked Comprehension Type

For Problems 1–2

A lead sphere of 1.0 mm diameter and relative density 11.20 attains a terminal velocity of 0.7 cm s^{-1} in a liquid of relative density 1.26.

1. Determine the coefficient of dynamic viscosity of the liquid.
- | | |
|--------------------------|--------------------------|
| (1) 0.45 N/m^2 | (2) 0.85 N/m^2 |
| (3) 0.56 N/m^2 | (4) 0.77 N/m^2 |

2. What is the value of the Reynolds number?

- | | |
|----------|----------|
| (1) 0.01 | (2) 0.03 |
| (3) 0.15 | (4) 0.26 |

For Problems 3–4

A long capillary tube of radius 0.2 mm is placed vertically inside a beaker of water.

3. If the surface tension of water is $7.2 \times 10^{-2} \text{ N/m}$ and the angle of contact between glass and water is zero, then determine the height of the water column in the tube.

- | | |
|----------|----------|
| (1) 3 cm | (2) 9 cm |
| (3) 7 cm | (4) 5 cm |

4. If the tube is now pushed into water so that only 5.0 cm of its length is above the surface, then determine the angle of contact between the liquid and glass surface.

- | | |
|---|---|
| (1) $\cos^{-1}\left(\frac{4}{5}\right)$ | (2) $\cos^{-1}\left(\frac{5}{7}\right)$ |
| (3) $\cos^{-1}\left(\frac{3}{5}\right)$ | (4) $\cos^{-1}\left(\frac{5}{4}\right)$ |

For Problems 5–7

An oil of relative density 0.9 and viscosity 0.12 kg/ms flows through a 2.5 cm diameter pipe with a pressure drop of 38.4 kN/m^2 in a length of 30 m. Determine

5. Determine the discharge

- | | |
|--|---|
| (1) $2.16 \times 10^{-4} \text{ m}^3/\text{s}$ | (2) $2.9 \times 10^{-3} \text{ m}^3/\text{s}$ |
| (3) $1 \times 10^{-4} \text{ m}^3/\text{s}$ | (4) $2 \times 10^{-4} \text{ m}^3/\text{s}$ |

6. Determine the shear stress at the pipe wall

- | | |
|--|---|
| (1) $8 \times 10^{-6} \text{ N/m}^2$ | (2) $3.9 \times 10^{-6} \text{ N/m}^2$ |
| (3) $2.3 \times 10^{-6} \text{ N/m}^2$ | (4) $10.6 \times 10^{-6} \text{ N/m}^2$ |

7. Determine the power required to maintain the flow

- | | |
|-----------|------------|
| (1) 2.2 W | (2) 3.84 W |
| (3) 5.6 W | (4) 9.3 W |

For Problems 8–10

Molecular forces exist between the molecules of a liquid in a container. The molecules on the surface have unequal force leading to a tension on the surface. If this is not compensated by a force, the equilibrium of the liquid will be a difficult task. This leads to an excess pressure on the surface. The nature of the meniscus can inform us of the direction of the excess pressure. The angle of contact of the liquid decided by the forces between the molecules, air and container can make the angle of contact.

8. The direction of the excess pressure in the meniscus of a liquid of angle of contact $2\pi/3$ is

- | | |
|----------------|--------------------------|
| (1) upward | (2) downward |
| (3) horizontal | (4) cannot be determined |

9. If the excess pressure in a soap bubble is p , the excess pressure in an air bubble is

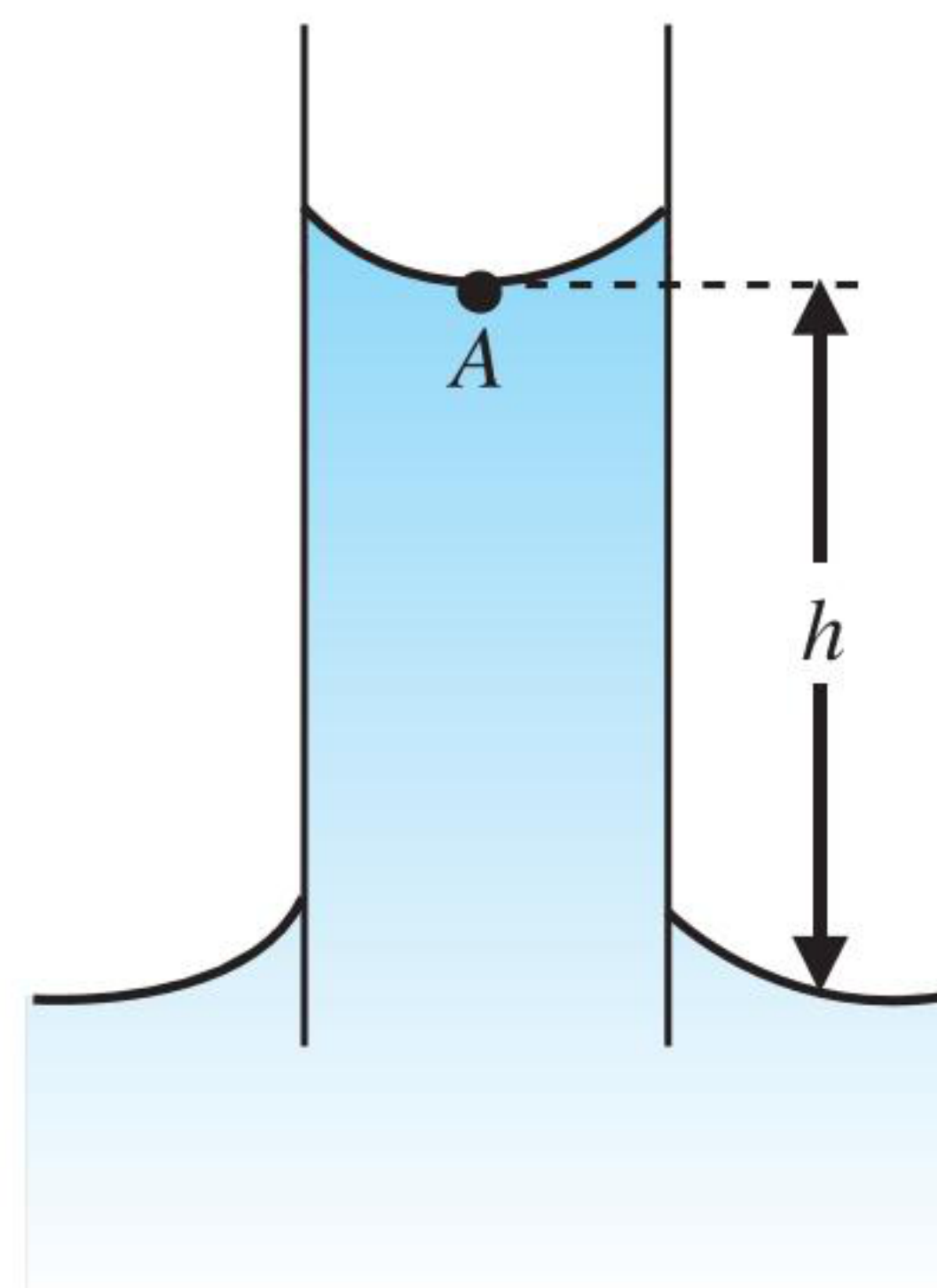
- | | |
|-------------------|----------|
| (1) $\frac{p}{2}$ | (2) p |
| (3) $2p$ | (4) $4p$ |

10. In a meniscus of radius r , with excess pressure p in atmospheric pressure p_0 , the force experienced is

- | | |
|------------------------|-----------------------|
| (1) $(p - p_0)\pi r^2$ | (2) $(p - p_0)2\pi r$ |
| (3) $p\pi r^2$ | (4) $p_0 2\pi r$ |

For Problems 11–13

Figure shows a capillary tube of radius r dipped into water. The atmospheric pressure is P_0 and the capillary rise of water is h . s is the surface tension for water–glass.



11. The pressure inside water at the point A (lowest point of the meniscus) is

(1) P_0 (2) $P_0 + \frac{2s}{r}$

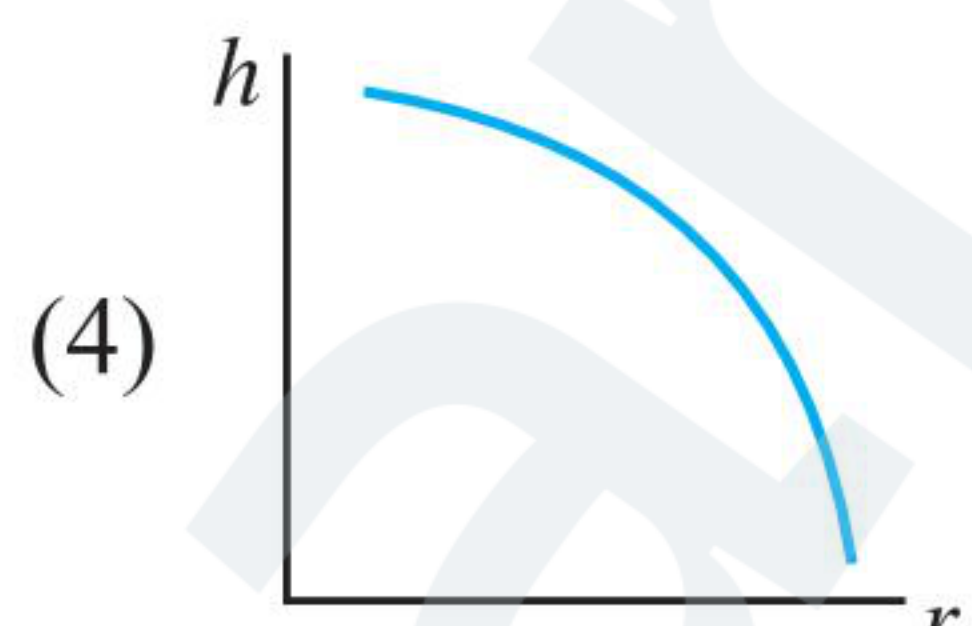
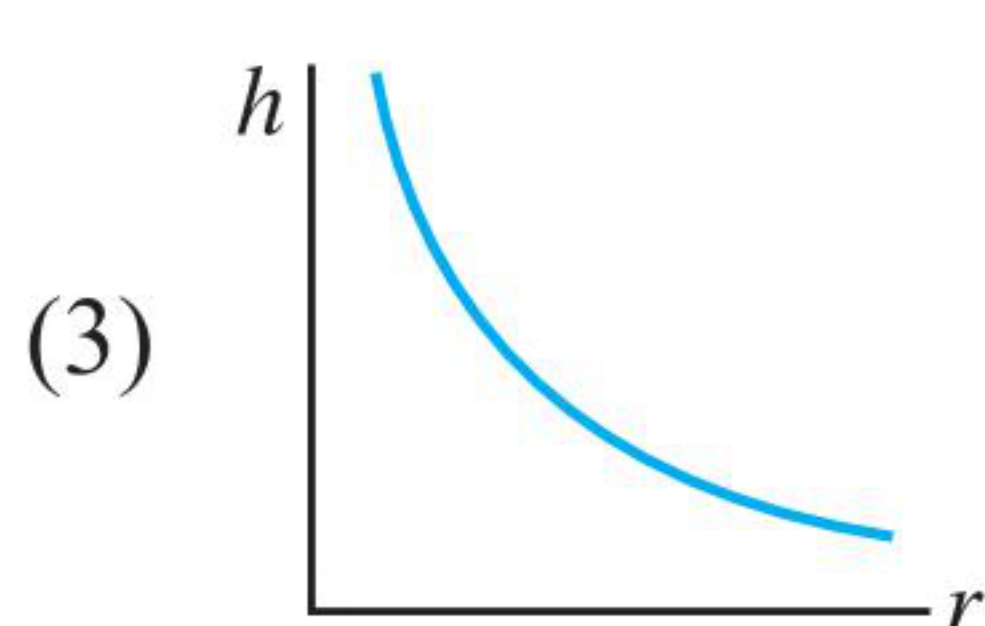
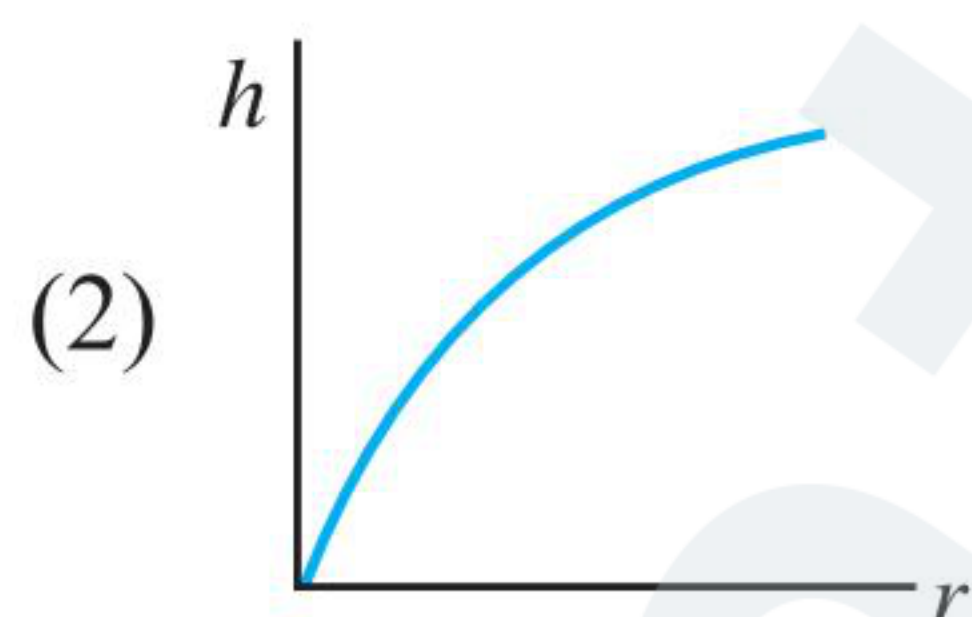
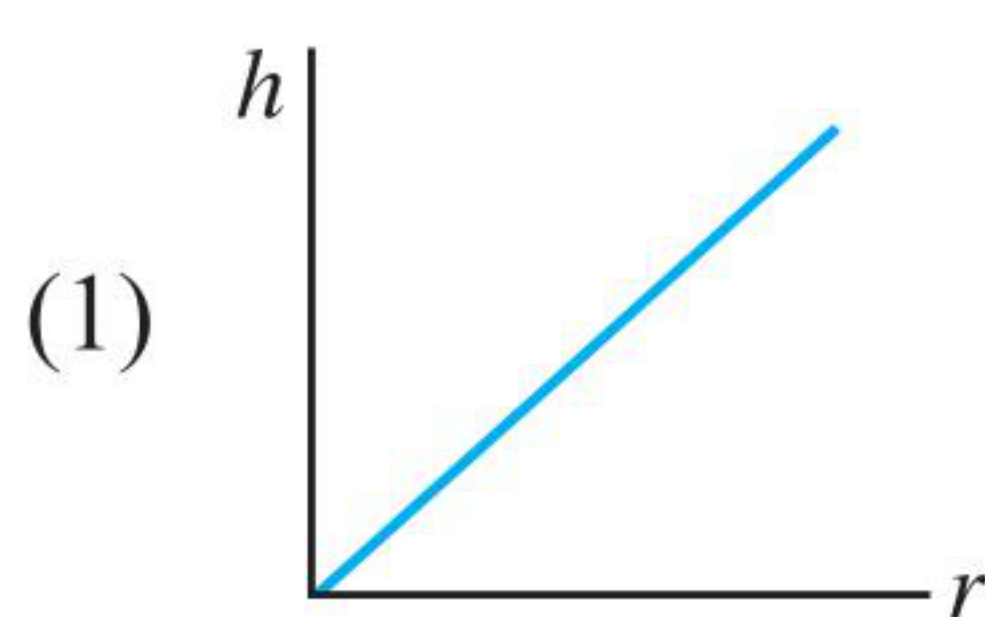
(3) $P_0 - \frac{2s}{r}$ (4) $P_0 - \frac{4s}{r}$

12. Initially, $h = 10$ cm. If the capillary tube is now inclined at 45° , the length of water rising in the tube will be

(1) 10 cm (2) $10\sqrt{2}$ cm

(3) $\frac{10}{\sqrt{2}}$ cm (4) none of these

13. Which of the following graphs may represent the relation between the capillary rise h and the radius r of the capillary?

**For Problems 14–15**

A man starts (from rest $u = 0$) rowing his stationary cuboidal boat of base area $A = 10 \text{ m}^2$. The driving force on the boat due to rowing is 100 N in the direction of motion. The depth of the lake is 10 m and the combined mass of man and the boat to be 150 kg and velocity gradient uniform. [Take coefficient of viscosity of water = 15 poise]

14. Find the maximum velocity that the boat can achieve.

(1) 50 m/s (2) 75 m/s

(3) $\frac{100}{3}$ m/s (4) $\frac{200}{3}$ m/s

15. The time (in seconds) in which he will attain half of this maximum velocity is

(1) $150 \ln 2$ s (2) $80 \ln 2$ s
(3) $100 \ln 2$ s (4) $200 \ln 2$ s

Matrix Match Type

1. Match the following:

Column I	Column II
i. Splitting of bigger drop into small droplets	a. Temperature increases
ii. Formation of bigger drop from small droplets	b. Temperature decreases
iii. Spraying of liquid	c. Surface energy increases
	d. Surface energy decrease

2. A glass plate of length 10 cm breadth 4 cm and thickness 0.4 cm weighs 20 g in air. It is held vertically with long side horizontal and half the plate immersed in water. What will be its apparent weight? Surface tension of water = 70 J dyn/cm.

Column I	Column II
i. Buoyant force due to liquid	a. 20 gf
ii. Force due to surface tension	b. 8 gf
ii. Apparent weight of the plate	c. 1.5 gf
	d. 13.5 gf

3. Column II depends on physical quantity/physical law given in Column I. Match the following columns and select the correct option from the codes given below.

Column I	Column II
i. stroke's law	a. radius
ii. terminal velocity	b. density of material of the body
iii. excess pressure inside mercury drop	c. coefficient of viscosity
iv. viscous force on a plate moving horizontally on a liquid surface	d. surface tension
	e. velocity gradient

Codes

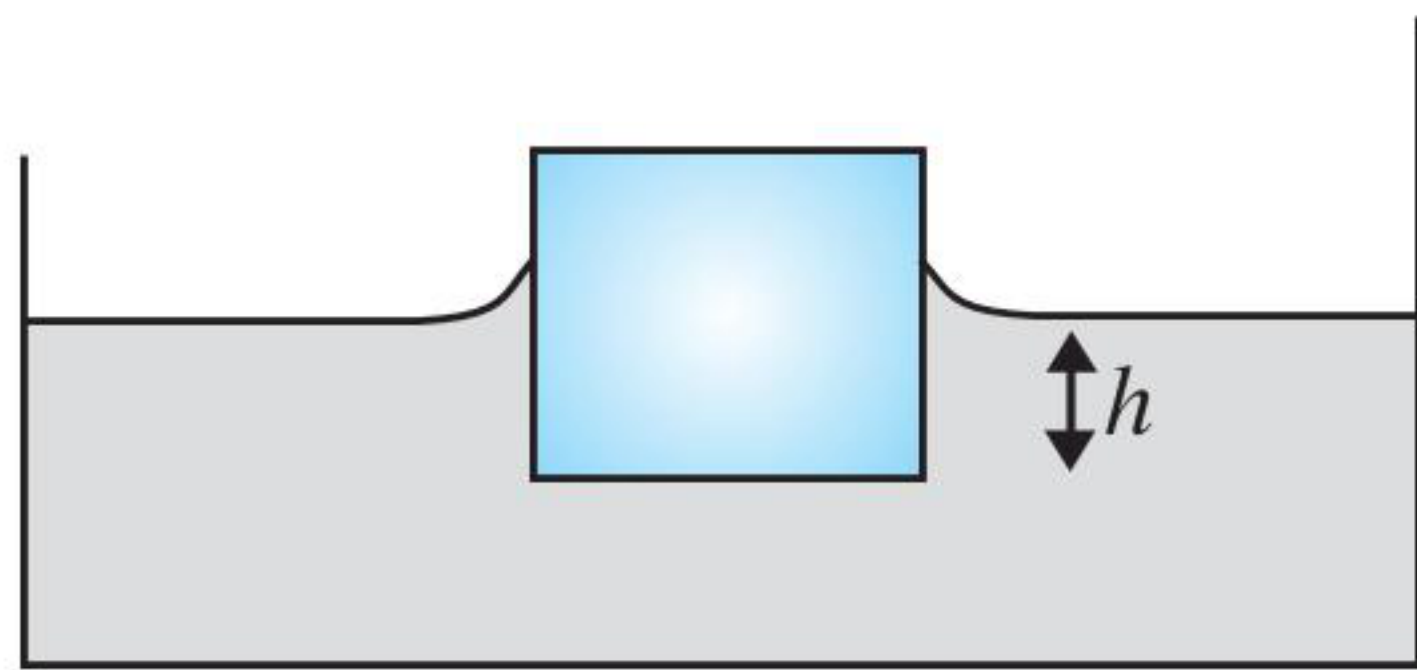
	i	ii	iii	iv
(1)	a, c	a, b, c	a, d	c, e
(2)	a, b, c	c, e	a, c	a, d
(3)	a, d	a, c	c, e	a, b, c, d
(4)	c	a, c	a	a, d

Numerical Value Type

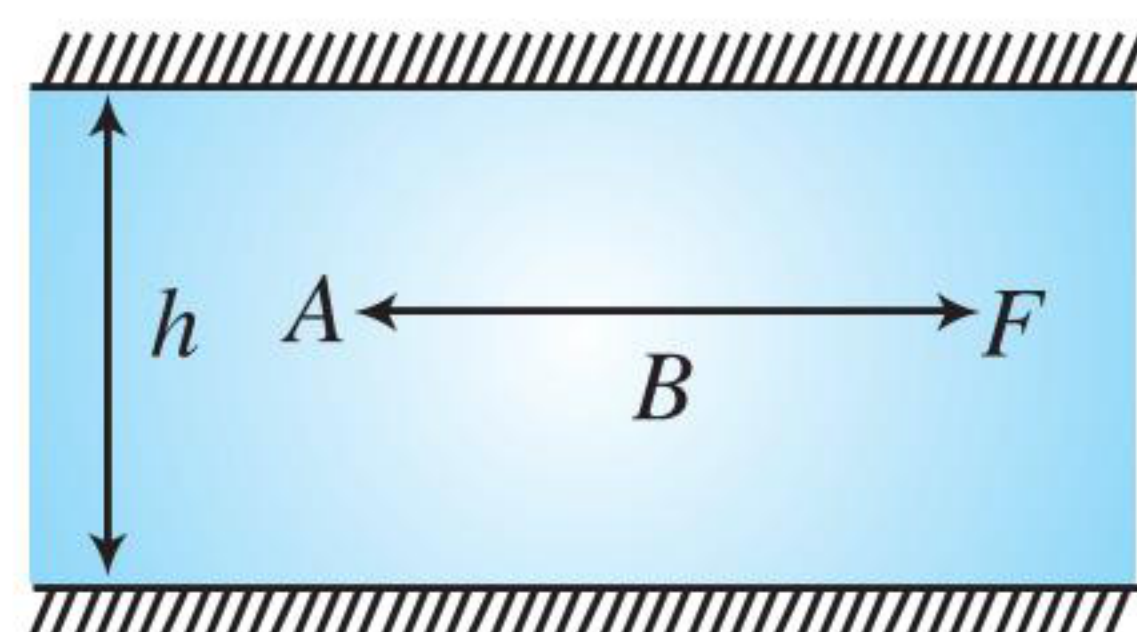
1. A cube of side a and mass m just floats on the surface of water as shown in the figure. The surface tension and density of water are T and ρ_w , respectively. If angle of contact between cube and water surface is zero, find the

distance h (in metres) between the lower face of cube and surface of the water.

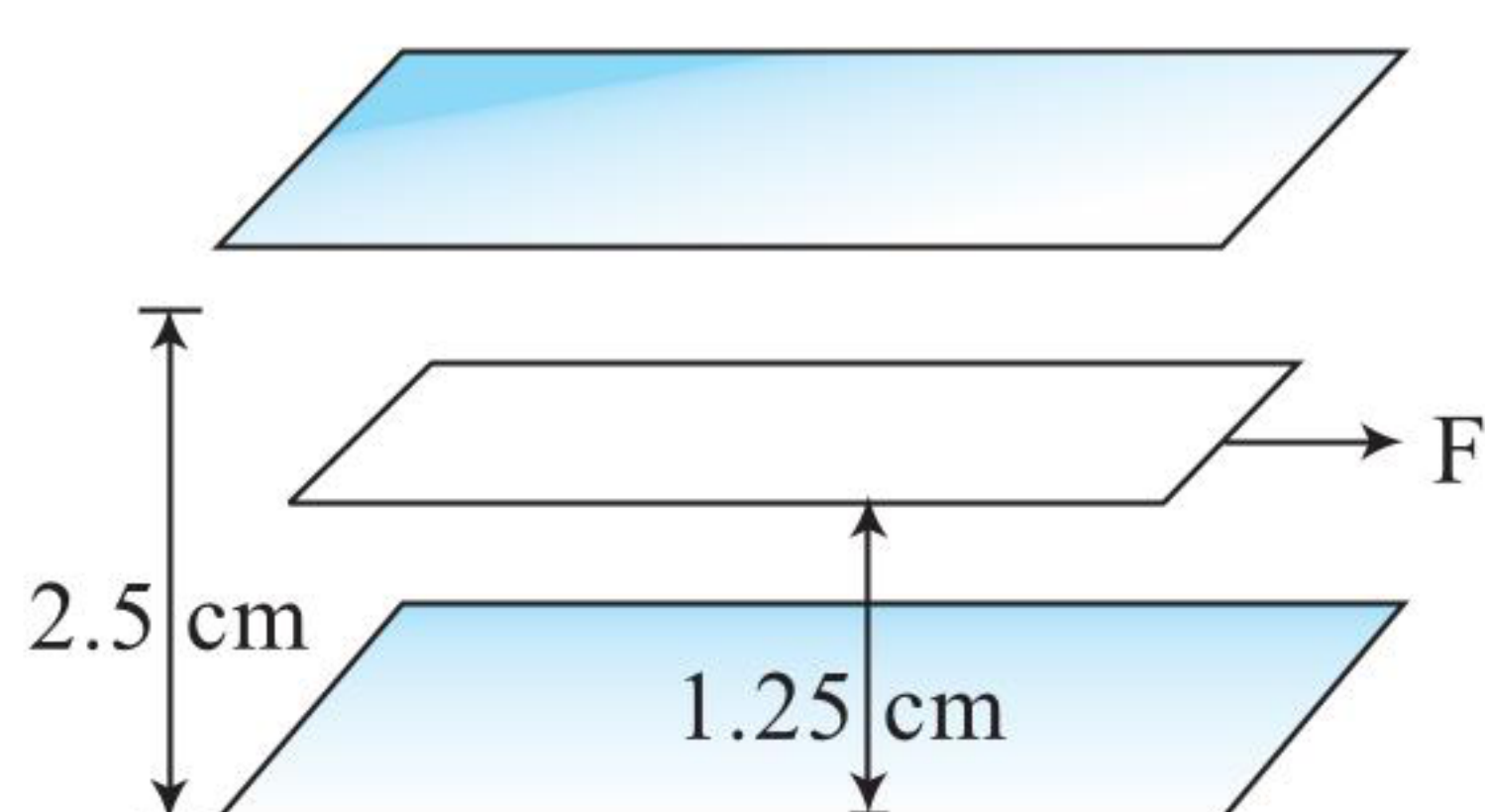
(Take $m = 1 \text{ kg}$, $g = 10 \text{ m s}^{-2}$, $aT = \frac{10}{4}$ unit and $\rho_w a^2 g = 10$ unit)



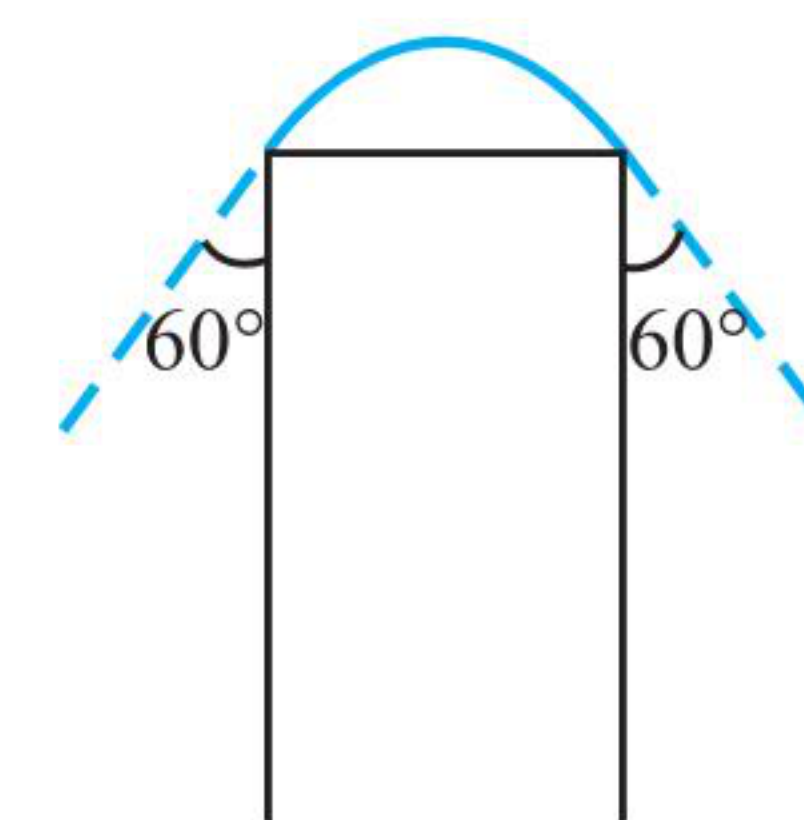
2. A thin plate AB of large area A is placed symmetrically in a small gap of height h filled with water of viscosity η_0 and the plate has a constant velocity v by applying a force F as shown in the figure. If the gap is filled with some other liquid of viscosity $0.75 \eta_0$, at what minimum distance (in cm) from top wall should the plate be placed in the gap, so that the plate can again be pulled at the same constant velocity V , by applying the same force F ? (Take $h = 20 \text{ cm}$)



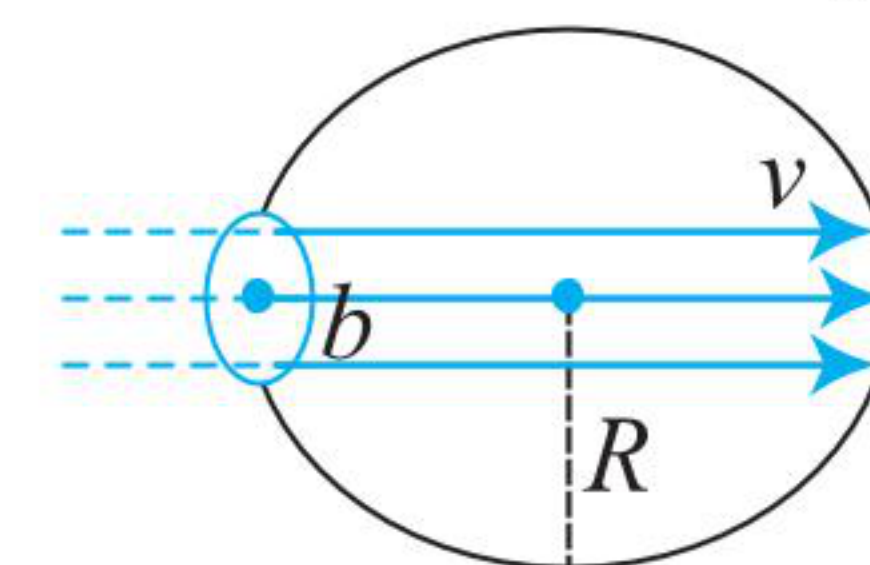
3. The diameter of a gas bubble formed at the bottom of a pond is $d = 4 \text{ cm}$. When the bubble rises to the surface, its height is (tension of water $= T = 0.07 \text{ N m}^{-1}$.)
4. n drops of water, each of radius 2 mm , fall through air at a terminal velocity of 8 cm s^{-1} . If they coalesce to form a single drop, then the terminal velocity of the combined drop is 32 cm s^{-1} . Find n .
5. A substance breaks down under a stress of 10^5 Pa . If the density of the wire is $2 \times 10^3 \text{ kg m}^{-3}$, find the minimum length of the wire which will break under its own weight ($g = 10 \text{ m s}^{-2}$).
6. The capillaries of radius r and $2r$ and lengths l and $2l$ respectively are joined in series. The streamline flow is maintained in capillaries. The pressure difference across first and second capillary is P_1 and P_2 respectively. Find P_1/P_2 .
7. The coefficient of viscosity of blood is $2.12 \times 10^{-3} \text{ Pa s}$ and its density is $2.12 \times 10^{-3} \text{ Pa s}$. Find the blood flow rate (in $\pi \times 10^{-6} \text{ m}^3 \text{ s}^{-1}$), corresponding to the largest average velocity in an artery of radius $2 \times 10^{-3} \text{ m}$ for the flow to be laminar.
8. A plate of area 2 m^2 is made to move horizontally with a speed of 2 m s^{-1} by applying a horizontal tangential force over the free surface of a liquid. The depth of the liquid is 1 m and the liquid in contact with the bed is stationary. Coefficient of viscosity of liquid $= 0.01$ poise. Find the tangential force needed to move the plate (in N).
9. A space 2.5 cm wide between two large plane surfaces is filled with oil. Force required to drag a very thin plate of area 0.5 m^2 just midway the surfaces at a speed of 0.5 m s^{-1} is 1 N . Find the coefficient of viscosity of oil (in $\times 10^{-3} \text{ kg-sm}^{-2}$).



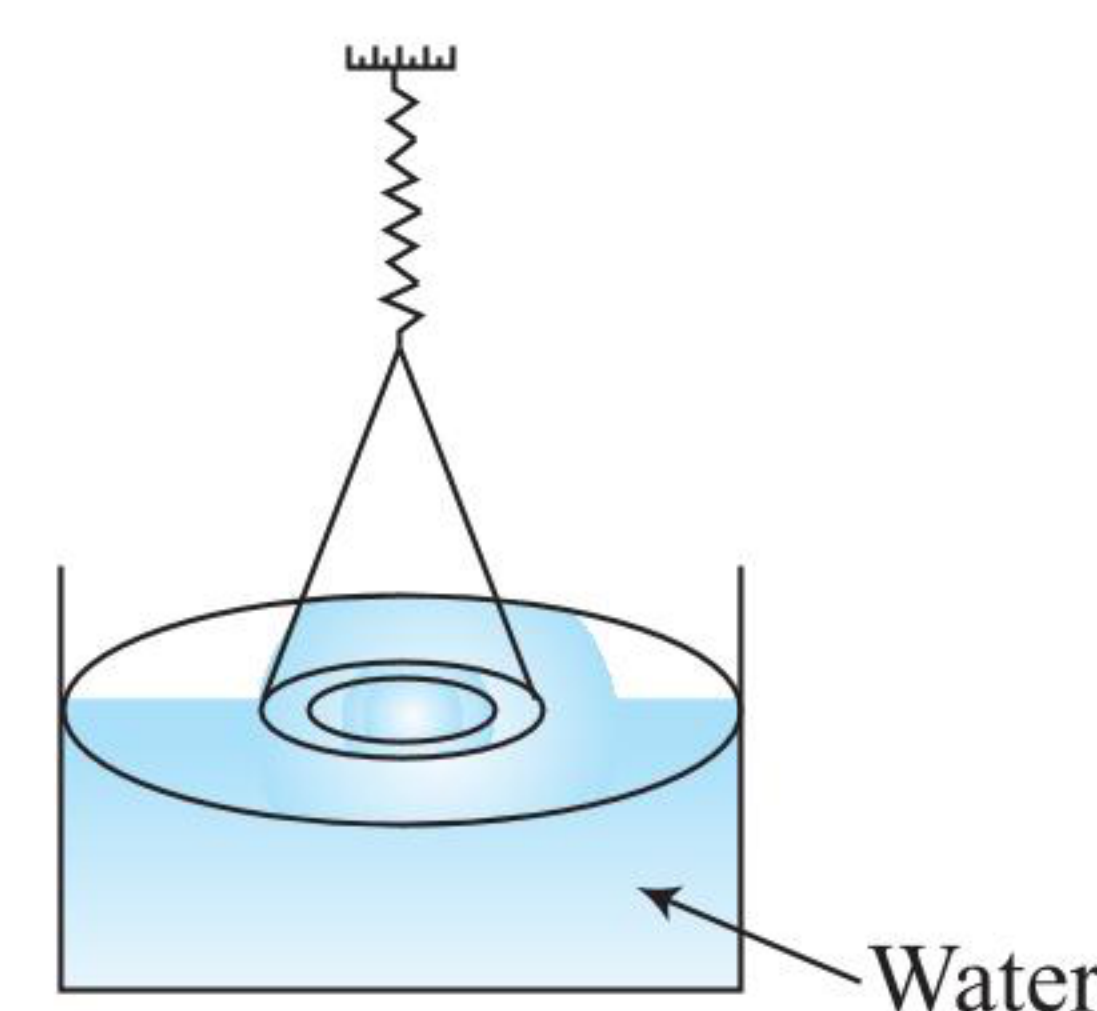
10. A soap bubble is being blown on a tube of radius 1 cm . The surface tension of the soap solution is 0.05 Nm^{-1} and the bubble makes 60° with the tube as shown in the figure. Find the excess of pressure over the atmosphere pressure in the tube (in Pa)



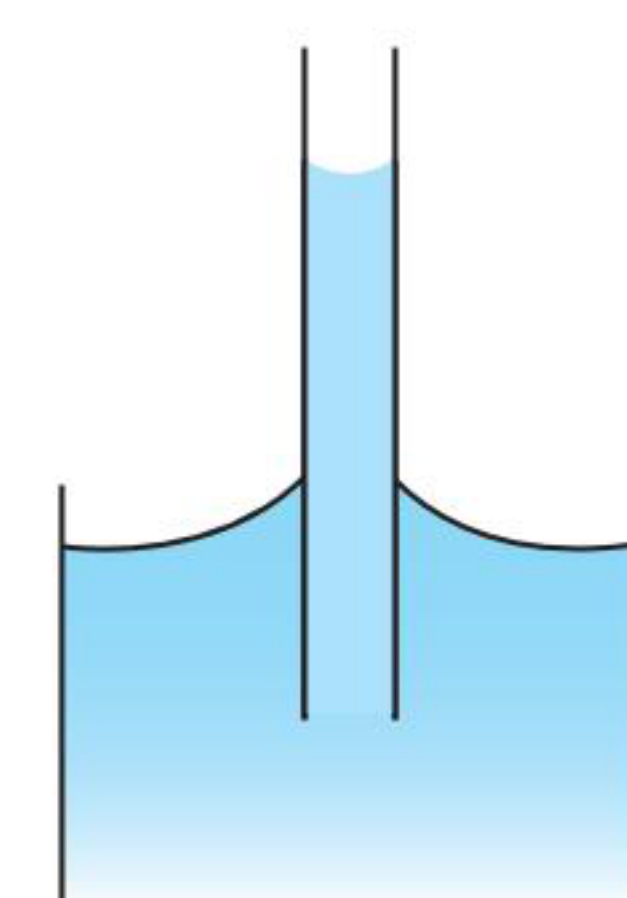
11. A bowl whose bottom has round holes of diameter 1 mm is filled with water. Assuming that surface tension acts only at holes, find the maximum height (in cm) up to which water can be filled in the vessel without leakage. (Given, surface tension of water $= 75 \times 10^{-3} \text{ Nm}^{-1}$, $g = 10 \text{ m s}^{-2}$ and density of water $= 1000 \text{ kg m}^{-3}$).
12. A soap film is made by dropping a circular frame of radius b in soap solution. A bubble is formed by blowing air with speed v in the form of a cylinder. Air stops after striking surface of soap bubble. Density of air is ρ . The radius of the bubble is $R > b$. The radius R of the bubble when the soap bubble separates from the ring is $\frac{\beta T}{\rho v^2}$. Find the value of β . (where, surface tension of liquid is T).



13. A metallic sphere of radius $1.0 \times 10^{-3} \text{ m}$ and density $10 \times 10^4 \text{ kg m}^{-3}$ enters a tank of water, after a free fall through a distance of h in the earth's gravitational field. If its velocity remains unchanged after entering water, determine the value of h (in m). Given: coefficient of viscosity of water $= 1.0 \times 10^{-3} \text{ N-s m}^{-2}$, $g = 10 \text{ m s}^{-2}$ and density of water $= 1.0 \times 10^3 \text{ kg m}^{-3}$.
14. A small steel ball falls through a syrup at a constant speed of 10 cm s^{-1} . If the steel ball is pulled upwards with a force equal to twice its effective weight, find the terminal velocity of the ball (in cm/s) in this case.
15. A circular ring has inner and outer radii equal to 20 mm and 50 mm respectively. Mass of the ring is $m = 0.8 \text{ g}$. It gently pulled out vertically from a water surface by a sensitive spring. When the spring is stretched 4 cm from its equilibrium position the ring is on verge of being pulled out from the water surface. If spring constant is $k = 0.9 \text{ Nm}^{-1}$ find the surface tension of water.



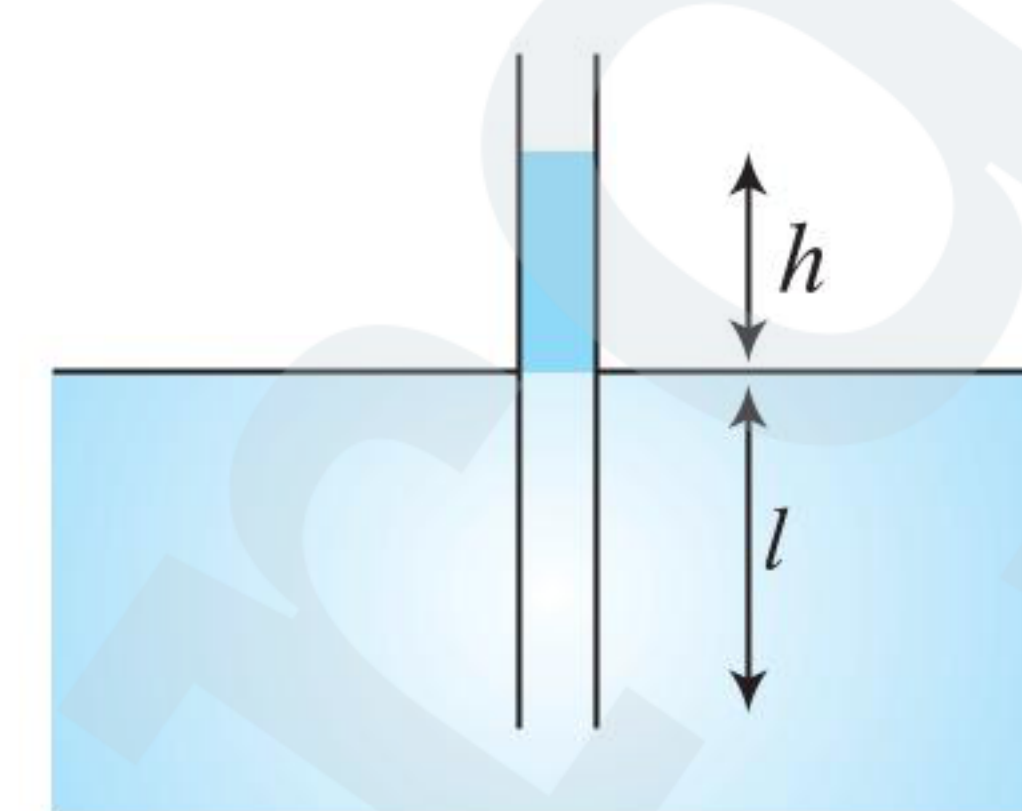
16. A long thin walled capillary tube of mass $M = 10 \text{ g}$ and radius $r = 7 \text{ mm}$ is partially immersed in a liquid of surface tension $T = 0.75 \text{ Nm}^{-1}$. The angle of contact for the liquid and the tube wall is 60° . How much force is needed to hold the tube vertically? Neglect buoyancy force on the tube.



17. A tapering glass capillary tube A of length 0.1 m has diameters 10^{-3} m and 5×10^{-4} m at the ends. When it is just immersed in a liquid at 0°C with larger radius in contact with liquid surface, the liquid rises 8×10^{-2} m in the tube. In another experiment, in a cylindrical glass capillary tube B , when immersed in the same liquid at 0°C , the liquid rises to 6×10^{-2} m height. The rise of liquid in tube B is only 5.5×10^{-2} m when the liquid is at 50°C . Find the rate at which the surface tension changes (in $\times 10^{-4}$ N/m $^\circ\text{C}$) with temperature considering the change to be linear. The density of liquid is $(1/14) \times 10^4$ kg/m³ and the angle of contact is zero. Effect of temperature on the density of liquid and glass is negligible.
18. There is an air bubble of radius 1.0 mm in a liquid of surface tension 0.075 N/m and density 1000 kg/m³. The bubble is at a depth of 10 cm below the free surface. By what amount

is the pressure (in Pa) inside the bubble greater than the atmospheric pressure? (Take $g = 9.8$ m/s²)

19. The capillary tube is dipped in water vertically. It is sufficiently long so that water rises to maximum height $h = 10$ cm in the tube. The length of the portion immersed in water is $l > h$. The lower end of the tube is closed, the tube is taken out and opened again. Then, find the length of the water column (in cm) remaining in the tube.

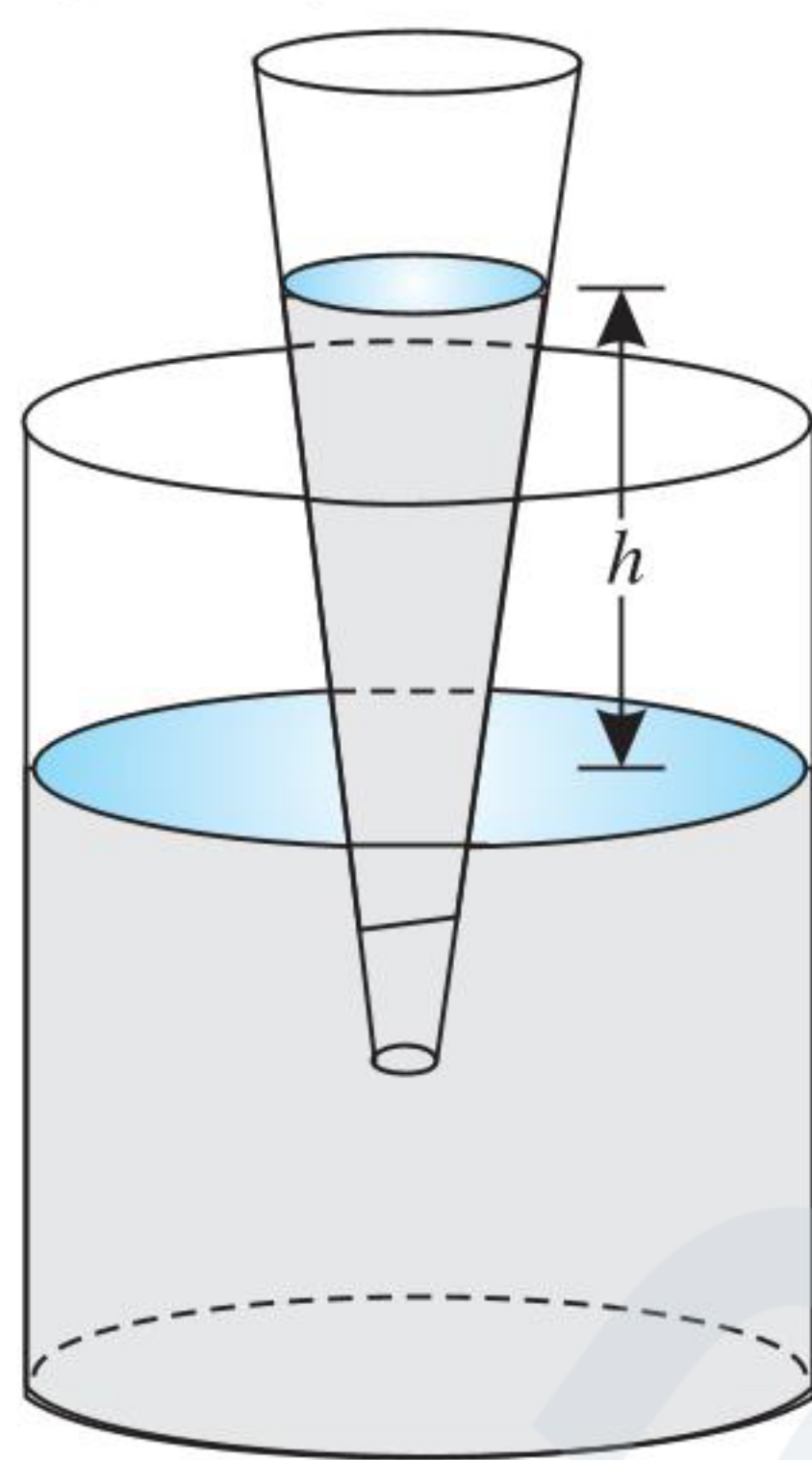


Archives

JEE ADVANCED

Single Correct Answer Type

1. A glass capillary tube is of the shape of truncated cone with an apex angle α so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to a height h , where the radius of its cross section is b . If the surface tension of water is S , its density is ρ , and its contact angle with glass is θ , the value of h will be (g is the acceleration due to gravity)



- (1) $\frac{2S}{b\rho g} \cos(\theta - \alpha)$ (2) $\frac{2S}{b\rho g} \cos(\theta + \alpha)$
 (3) $\frac{2S}{b\rho g} \cos(\theta - \alpha/2)$ (4) $\frac{2S}{b\rho g} \cos(\theta + \alpha/2)$

(JEE Advanced 2014)

2. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho} \frac{d\rho}{dt}\right)$ is constant. The velocity v of any point

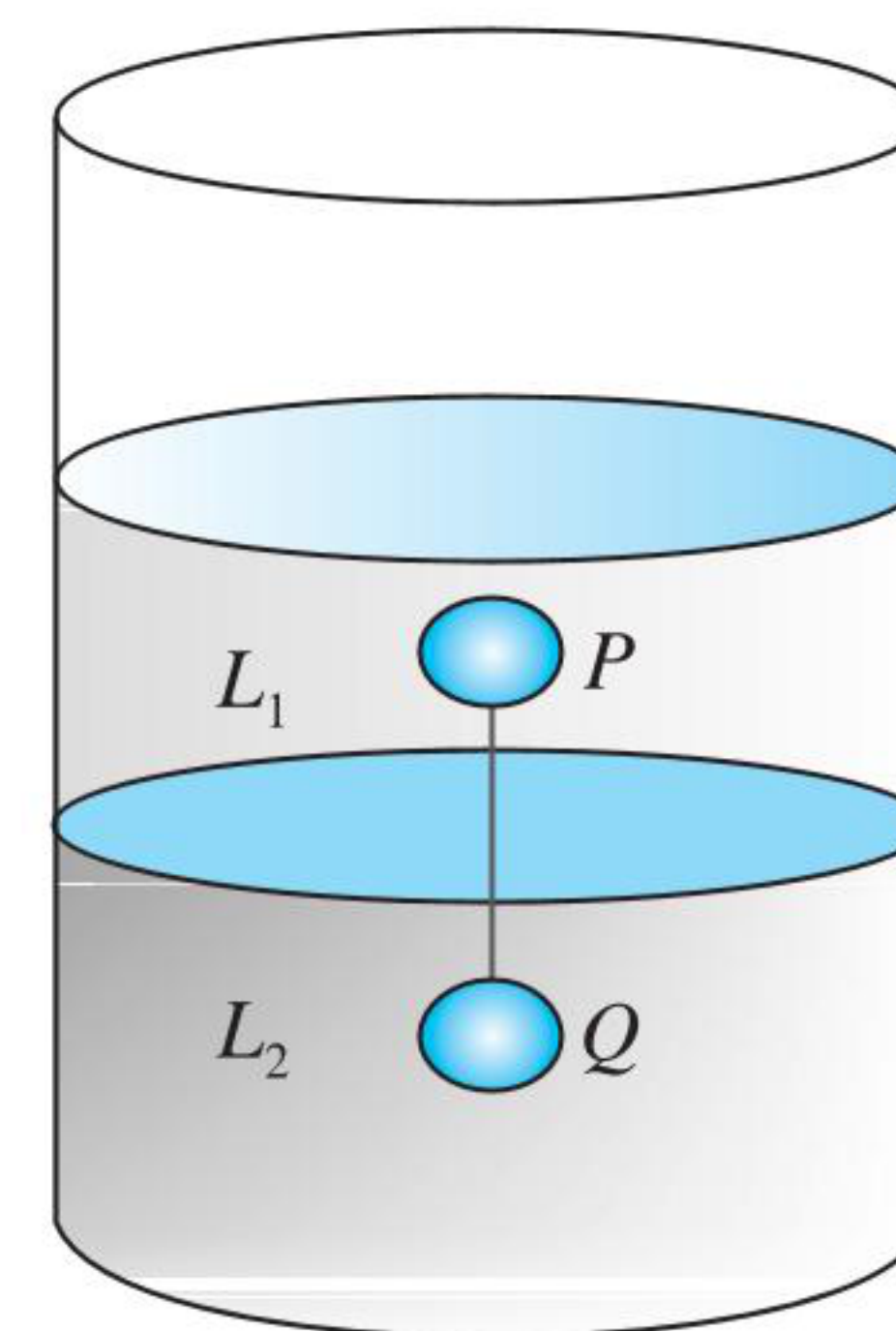
on the surface of the expanding sphere is proportional to

- (1) R^3 (2) $\frac{1}{R}$
 (3) R (4) $R^{2/3}$

(JEE Advanced 2017)

Multiple Correct Answers Type

1. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2 , respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2 , respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then



- (1) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2}$ (2) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_2}{\eta_1}$
 (3) $\vec{V}_P \cdot \vec{V}_Q > 0$ (4) $\vec{V}_P \cdot \vec{V}_Q < 0$

(JEE Advanced 2015)

2. A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ . The angle of contact between water and the wall of the capillary tube is θ . Ignore the mass of water in the meniscus. Which of the following statements is (are) true?

- (1) For a given material of the capillary tube, h decreases with increase in r .
 (2) For a given material of the capillary tube, h is independent of σ .
 (3) If this experiment is performed in a lift going up with a constant acceleration, then h decreases.
 (4) h is proportional to contact angle θ .

(JEE Advanced 2018)

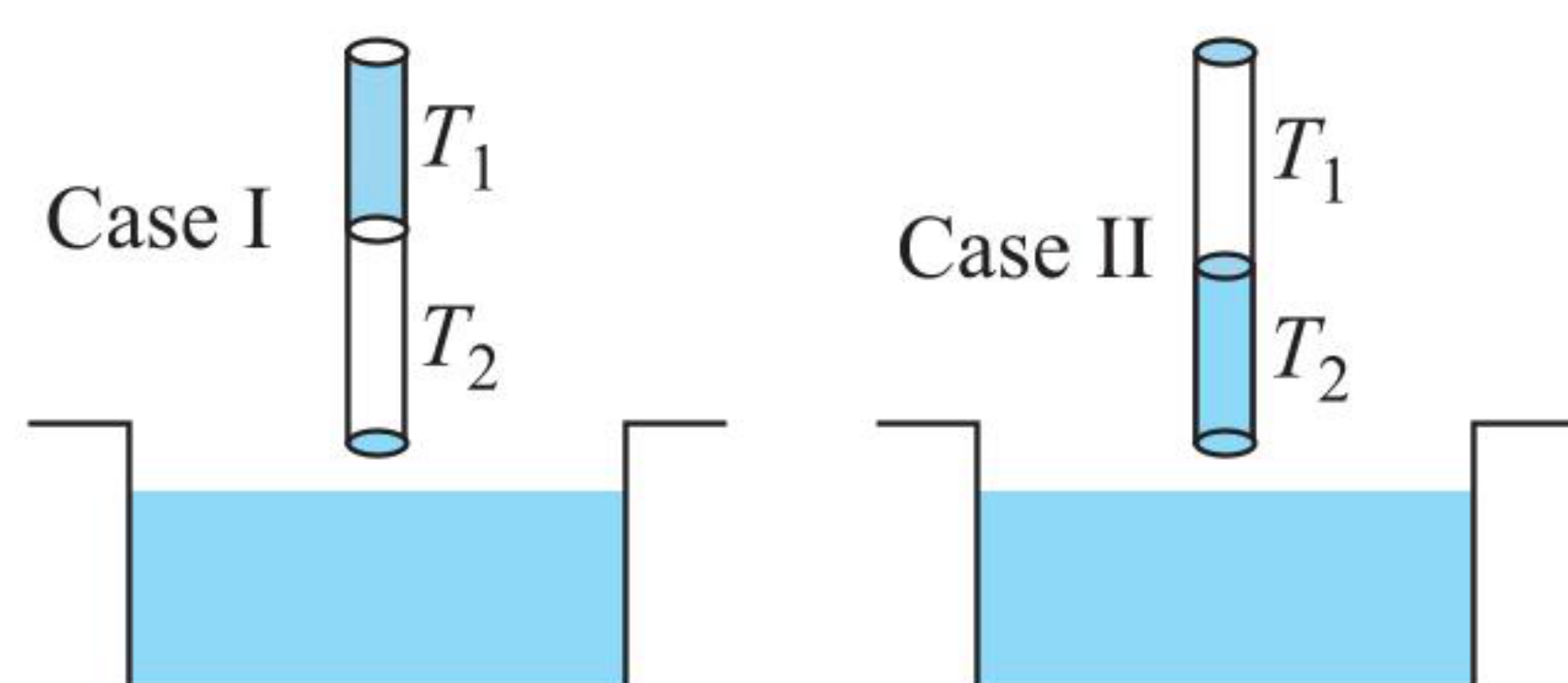
3. Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity u_0 . Which of the following statements is (are) true?

- (1) The resistive force of liquid on the plate is inversely proportional to h
- (2) The resistive force of liquid on the plate is independent of the area of the plate
- (3) The tangential (shear) stress on the floor of the tank increases with u_0
- (4) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid

(JEE Advanced 2018)

4. A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T_1 and T_2 of different materials having water contact angles of 0° and 60° , respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is (are) correct?

(Surface tension of water = 0.075 N/m , density of water = 1000 kg/m^3 , take $g = 10 \text{ m/s}^2$)



- (1) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.
- (2) For case I, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)
- (3) For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)
- (4) For case II, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)

(JEE Advanced 2019)

Linked Comprehension Type

For Problems 1–3

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R . When the force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

(IIT-JEE 2010)

1. If the radius of the opening of the dropper is r , the vertical force due to the surface tension on the drop of radius R (assuming $r \ll R$) is

- (1) $2\pi rT$
- (2) $2\pi RT$
- (3) $2\pi r^2T/R$
- (4) $2\pi R^2T/r$

2. If $r = 5 \times 10^{-4} \text{ m}$, $\rho = 10^3 \text{ kg m}^{-3}$, $g = 10 \text{ m s}^{-2}$, $T = 0.11 \text{ Nm}^{-1}$, the radius of the drop when it detaches from the dropper is approximately

- (1) $1.4 \times 10^{-3} \text{ m}$
- (2) $3.3 \times 10^{-3} \text{ m}$
- (3) $2.0 \times 10^{-3} \text{ m}$
- (4) $4.1 \times 10^{-3} \text{ m}$

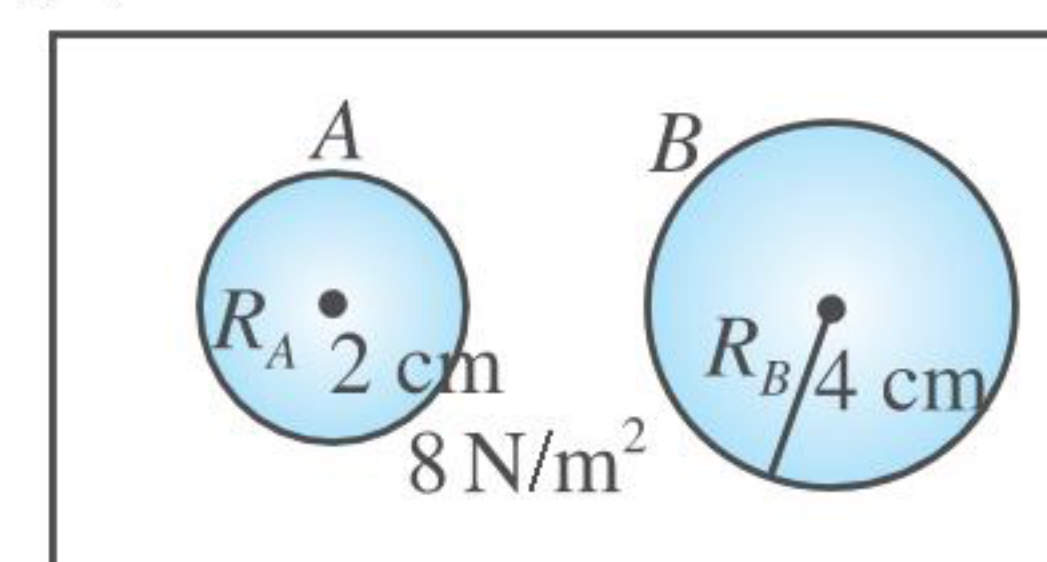
3. After the drop detaches, its surface energy is

- (1) $1.4 \times 10^{-6} \text{ J}$
- (2) $2.7 \times 10^{-6} \text{ J}$
- (3) $5.4 \times 10^{-6} \text{ J}$
- (4) $8.1 \times 10^{-6} \text{ J}$

Numerical Value Type

1. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2 . The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap water used to make bubbles is 0.04 N/m . Find the ratio n_B/n_A , where n_A and n_B are the number of moles of air in bubbles A and B , respectively. (Neglect the effect of gravity.)

(IIT-JEE 2009)



2. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is

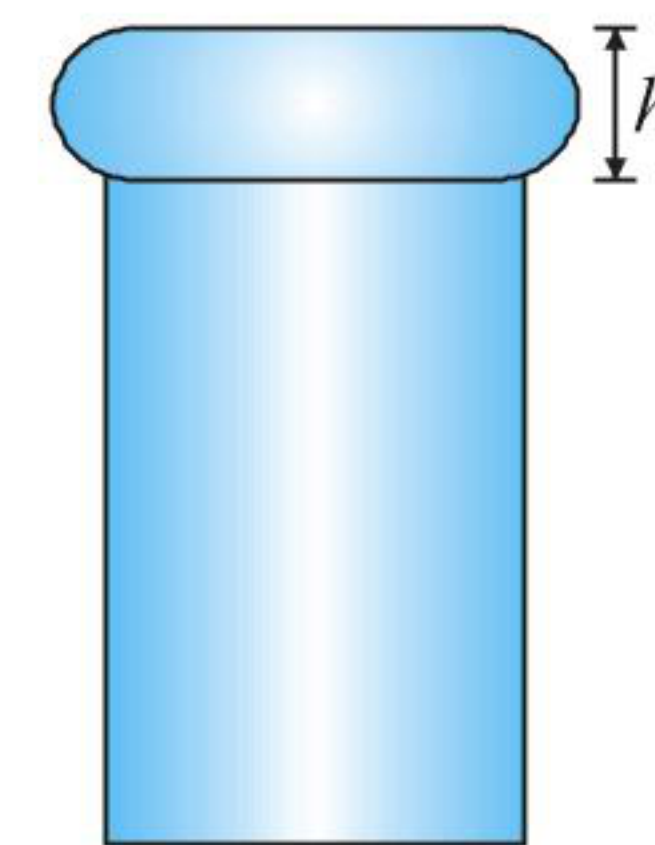
(JEE Advanced 2016)

3. A drop of liquid of radius $R = 10^{-2} \text{ m}$ having surface tension $S = \frac{0.1}{4\pi} \text{ Nm}^{-1}$ divides itself into K identical drops. In this

process the total change in the surface energy $\Delta U = 10^{-3} \text{ J}$. If $K = 10^\alpha$ then the value of α is _____.

(JEE Advanced 2017)

4. When water is filled carefully in a glass, one can fill it to a height h above the rim of the glass due to the surface tension of water. To calculate h just before water starts flowing, model the shape of the water above the rim as a disc of thickness h having semicircular edges, as shown schematically in the figure.



When the pressure of water at the bottom of this disc exceeds what can be withstood due to the surface tension, the water surface breaks near the rim and water starts flowing from there. If the density of water, its surface tension and the acceleration due to gravity are 10^3 kg m^{-3} , 0.07 Nm^{-1} and 10 ms^{-2} , respectively, the value of h (in mm) is _____.

(JEE Advanced 2020)

Answers Key

EXERCISES

Single Correct Answer Type

1. (2)

6. (4)

11. (1)

16. (1)

21. (4)

26. (2)

31. (2)

36. (2)

41. (2)

46. (3)

51. (4)

56. (2)

61. (2)
2. (3)

7. (3)

12. (3)

17. (3)

22. (3)

27. (1)

32. (1)

37. (1)

42. (4)

47. (1)

52. (1)

57. (3)

62. (2)
3. (2)

8. (3)

13. (1)

18. (1)

23. (3)

28. (1)

33. (1)

38. (1)

43. (4)

48. (4)

53. (2)

58. (2)
4. (4)

9. (2)

14. (1)

19. (3)

24. (2)

29. (4)

34. (4)

39. (1)

44. (2)

49. (2)

54. (3)

59. (3)
5. (3)

10. (2)

15. (2)

20. (2)

25. (2)

30. (4)

35. (3)

40. (1)

45. (2)

50. (1)

55. (3)

60. (1)

Multiple Correct Answers Type

1. (1),(2),(3),(4)

4. (3),(4)

7. (1),(4)

10. (1),(2),(3)
2. (1),(2)

5. (1),(2),(3)

8. (1),(2),(3)

11. (1),(3)
3. (1),(3)

6. (1),(3)

9. (1),(3)

Linked Comprehension Type

1. (4)

6. (1)

11. (3)
2. (1)

7. (2)

12. (2)
3. (3)

8. (1)

13. (3)
4. (2)

9. (1)

14. (4)
5. (3)

10. (3)

15. (3)

Matrix Match Type

1. i. → b., c.; ii. → a., d.; iii. → b., c.

2. i. → b.; ii. → c.; iii. → d.

3. (1)

Numerical Value Type

1. (2)

6. (8.00)

11. (3.00)

16. (0.133)
2. (5)

7. (4.00)

12. (4.00)

17. (1.4)
3. (5)

8. (0.004)

13. (20)

18. (1130)
4. (8)

9. (25)

14. (10)

19. (20.00)
5. (5)

10. (10)

15. (0.08)

ARCHIVES

JEE Advanced

Single Correct Answer Type

1. (4)

2. (3)

Multiple Correct Answers Type

1. (1),(3)

2. (1),(3)

3. (1),(3),(4)

4. (1),(3),(4)

Linked Comprehension Type

1. (3)

2. (1)

3. (2)

Numerical Value Type

1. (6)

2. (3)

3. (6)

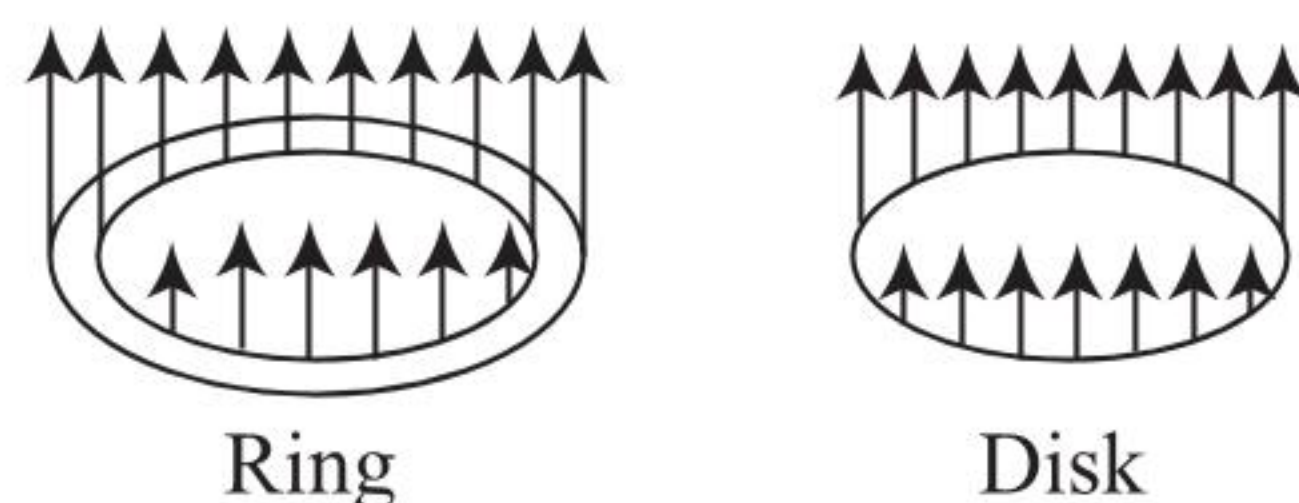
4. (3.74)

Chapter 8

Concept Application Exercise

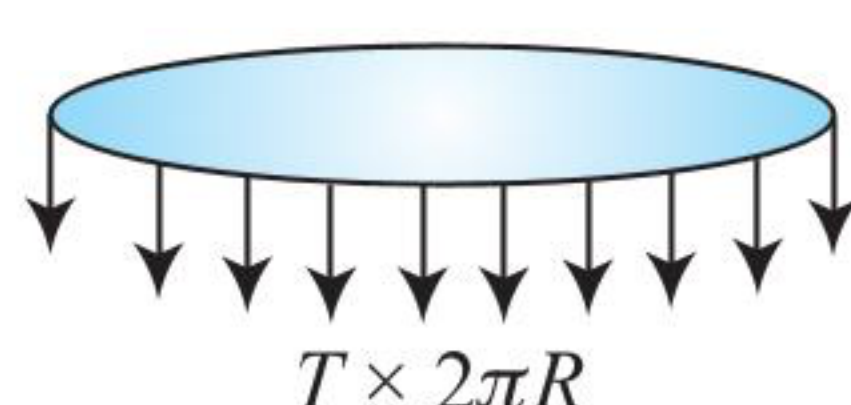
Exercise 8.1

1. (c)



Ring have double surface than that of disk.

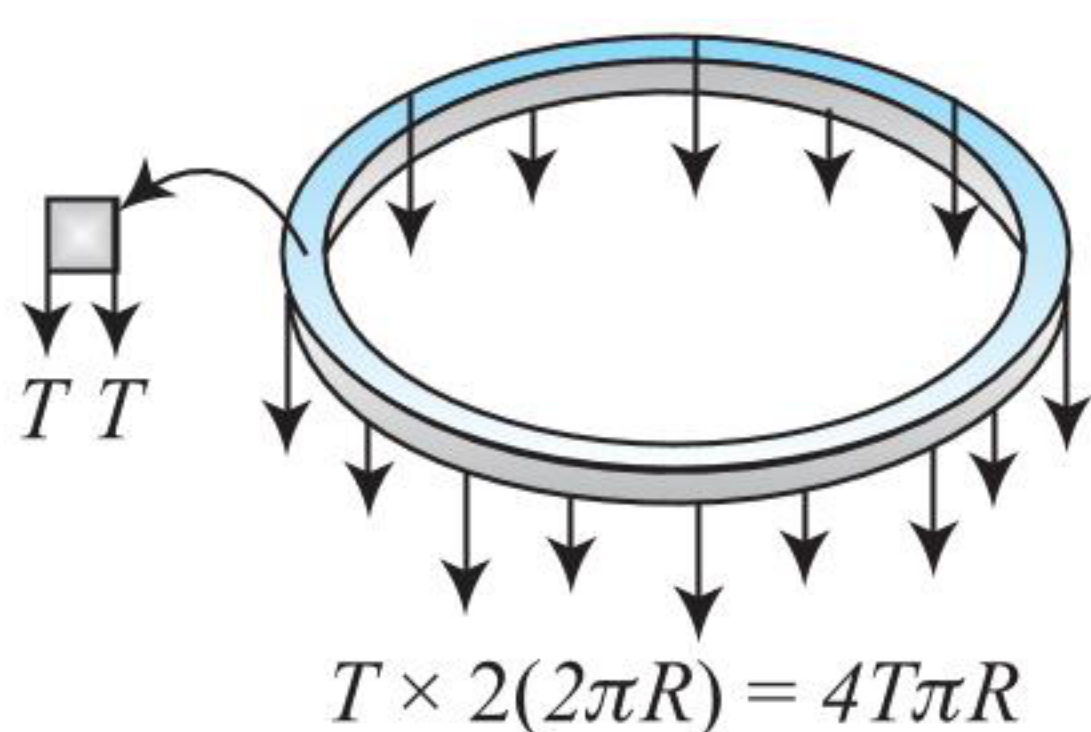
2. Force due to surface tension acts all along the circumference of the circular plate. Therefore, force required to take away the plate.



$$\text{Force due to surface tension } F = T \cdot \Delta L = T(2\pi R) \\ = (0.70 \text{ N/m})(2 \times \pi \times 0.10 \text{ m}) = 0.44 \text{ N}$$

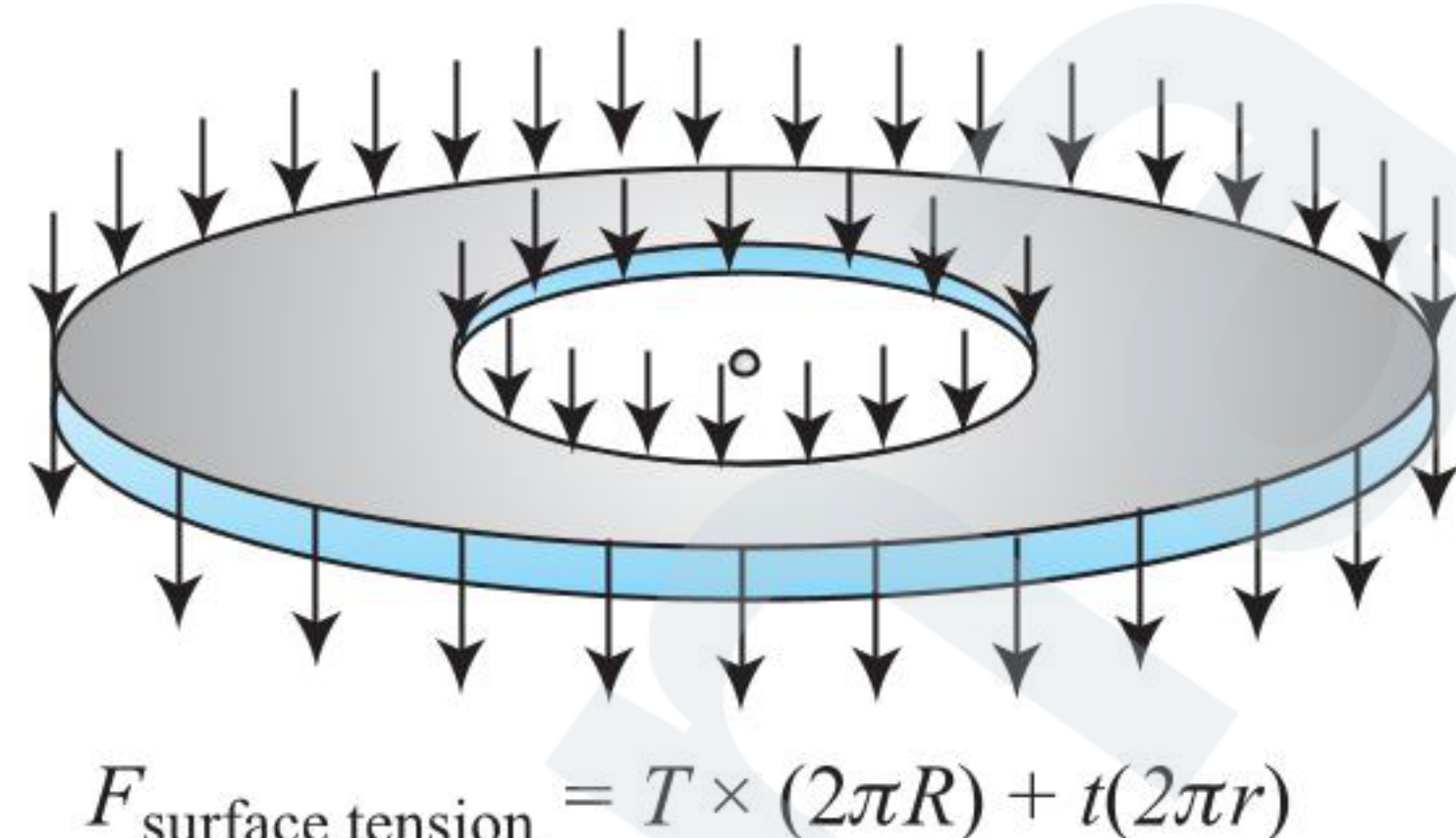
Hence force required to lift the plate = Force due to surface tension = 0.44 N

3. Force due to surface tension $F = T \cdot \Delta L = T \times 2(2\pi R)$
 $= (0.70 \text{ N/m})2 \times (2 \times \pi \times 0.10 \text{ m}) = 0.88 \text{ N}$



Hence force required to lift the plate = Force due to surface tension = 0.88 N

4. Force due to surface tension, $F = T \times 2\pi(r + R)$
 $= (0.70 \text{ N/m})(2 \times \pi \times (0.10 + 0.20) \text{ m}) = 1.32 \text{ N}$



Hence force required to lift the plate = Force due to surface tension = 1.32 N

5. The weight supported in cases (b) and (c) is also $5.0 \times 10^{-2} \text{ N}$. This is due to the reason that the weight is supported by the force (F) of surface tension (T) where $F \propto l$, l being the length of the film. Since T and l are the same, F is also the same in all the cases.

6. When the ring is about to leave the water surface, surface tension force on it is

$$F_{ST} = 2\pi RT + 2\pi rT = 2\pi(R + r)T$$

Spring force $F_s = kx$

$$\therefore kx = 2\pi(R + r)T + mg$$

$$\therefore T = \frac{kx - mg}{2\pi(R + r)}$$

7. The glass plate is held inside water as shown in figure. Here, $l = 50 \text{ cm}$; $b = 40 \text{ cm}$ and $t = 5 \text{ cm}$; $T = 50 \times 10^{-3} \text{ N/m}$.

The glass plate is under the action of the following forces

- (i) weight mg acting vertically downwards,
 - (ii) force due to surface tension acting vertically downwards and
 - (iii) upthrust experienced due to the half immersed part of the plate.
- Thus, apparent weight of the glass plate,

$$mg' = mg + 2(l + t) \times T - \left(l \times t \times \frac{b}{2} \right) \rho g \\ = 3 \times 10 + 2(50 + 5) \times 10^{-2} \times 0.5 - \left(50 \times 5 \times \frac{40}{2} \right) \times 10^{-6} \times 10^3 \times 10 \\ = 30 + 0.55 - 25 = 5.55 \text{ N}$$

Exercise 8.2

1. (a) When two droplets merge with each other, their surface energy decreases.

$$W = T(\Delta A) = (\text{negative}) \text{ i.e. energy is released.}$$

2. (c) Work done to form a bubble of radius R

$$W_1 = 8\pi R^2 T_1$$

Work done to form a bubble of radius $2R$

$$W_2 = 8\pi (2R)^2 T_2 = 32\pi R^2 T_2 \quad \therefore \frac{W_1}{W_2} = \frac{T_1}{4T_2}$$

If surface tension of soap solution is same then

$$W_2 = 4W_1$$

But in the problem temperature of solution is increased so its surface tension decreases.

$$\therefore W_2 < 4W_1$$

3. Work done to form a soap bubble

$$W = 8\pi R^2 T \quad (\text{As } V \propto R^3 \therefore R \propto V^{1/3})$$

$$\therefore W \propto V^{2/3}$$

$$\frac{W_2}{W_1} = \left(\frac{V_2}{V_1} \right)^{2/3} = (2)^{2/3} \Rightarrow W_2 = (4)^{1/3} W$$

4. Here, $T = 30 \text{ dyne cm}^{-1}$; initial surface area of the soap bubble = 0.50 cm^2 ; final surface area of the soap bubble = 1.10 cm^2 . Since the soap bubble has two free surfaces, effective increase in the surface area of the soap bubble,

$$A = 2 \times (1.10 - 0.50) = 1.20 \text{ cm}^2$$

The work done in blowing up the soap bubble to the new size,

$$W = T \times A = 30 \times 1.20 = 36 \text{ erg}$$

5. Let r be the radius of each of the tiny drops. Clearly, volume of 27 tiny drops = volume of the liquid drop of diameter D

$$\text{or } 27 \times \frac{4\pi}{3} r^3 = \frac{4\pi}{3} \left(\frac{D}{2} \right)^3 = \frac{4\pi}{3} \times \frac{D^3}{8}$$

$$\text{or } r^3 = \frac{D^3}{8 \times 27} \Rightarrow r = \frac{D}{2 \times 3} = \frac{D}{6}$$

$$\text{Original surface area of the drop} = 4\pi \left(\frac{D}{2} \right)^2 = \pi D^2$$

Final surface area of 27 droplets = $27 \times 4\pi r^2$

$$= 27 \times 4\pi \left(\frac{D}{6} \right)^2 = 3\pi D^2$$

Increase in surface area, $\Delta A = 3\pi D^2 - \pi D^2 = 2\pi D^2$

Increase in potential energy, $W = T\Delta A = 2\pi D^2 T$

Exercise 8.3

1. Pressure difference on two sides of a curved surface is inversely proportional to the radius of curvature.

$$2. \quad p + \rho gh = \frac{2T}{R} + \rho gh = \frac{2 \times 0.075}{1.0 \times 10^{-3}} + (1000 \times 10 \times 0.1)$$

$$\text{Or } P = 150 + 1000 = 1150 \text{ Pa}$$

$$3. \text{ Here, } \frac{r}{r'} = \frac{1}{2}$$

Now, excess of pressure inside a bubble,

$$p_i - p_0 = \frac{4T}{r}$$

$$\frac{p_i - p_0}{p_i' - p_0} = \frac{4T}{r} \times \frac{r'}{4T} = \frac{r'}{r} = \frac{1}{1/2} = \frac{2}{1} \text{ i.e., } 2:1$$

Also, work done in blowing a bubble of radius r ,

$$W = T \times 4\pi r^2$$

$$\therefore \frac{W}{W'} = \frac{T \times 4\pi r^2}{T \times 4\pi r'^2} = \left(\frac{r}{r'}\right)^2 = \left(\frac{1}{1/2}\right)^2 = \frac{1}{4} \text{ i.e., } 1:4$$

4. Since the drag force balances the gravity, the necessary condition must be that the variation in hydrostatic pressure inside the drop should be negligible compared to the excess pressure due to surface tension

$$\rho g 2R \ll \frac{2T}{R} \Rightarrow R \ll \sqrt{\frac{T}{\rho g}}$$

5. Let P_1 and P_2 be pressures in the tubes 1 and 2 just above the meniscus. Then, at the points just below the meniscus of the liquid in the two tubes, the pressures will be

$$P_1' = P_1 - \frac{2T}{R_1} \text{ and } P_2' = P_2 - \frac{2T}{R_2}$$

Here, R_1 and R_2 are radii of the meniscus of the liquid in the two tubes.

As shown in Fig. (b), the radius (R) of the meniscus of liquid, the radius (r) of the tube and the angle of contact (θ) are related to each other as

$$\frac{r}{R} = \cos \theta \text{ or } R = \frac{r}{\cos \theta}$$

$$R_1 = \frac{r_1}{\cos \theta} \text{ and } R_2 = \frac{r_2}{\cos \theta}$$

$$P_1' = P_1 - \frac{2T \cos \theta}{r_1} \text{ and } P_2' = P_2 - \frac{2T \cos \theta}{r_2}$$

Hence, $P_2' - P_1' =$ pressure exerted by height h of the water column

$$\text{Or } \left(P_2 - \frac{2T \cos \theta}{r_2}\right) - \left(P_1 - \frac{2T \cos \theta}{r_1}\right) = h\rho g$$

or true pressure difference,

$$P_2 - P_1 = h\rho g - 2T \cos \theta \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

On substituting the corresponding values we get

$$P_2 - P_1 = 1850 \text{ N/m}^2$$

Exercise 8.4

1. (d) If radius of capillary is reduced to half, the rise of liquid column will be two times as $h \propto 1/r$.

$$2. \text{ (a) } h = \frac{2T}{Rdg}$$

$$hR = \frac{2T}{dg} = \text{constant}$$

When h decreases, R increases.

3. Here, $T = 72 \text{ dyne cm}^{-1}$;

length of the tube above the surface of water,

$$H = 80 \text{ mm} = 8 \text{ cm}$$

$$\text{Now, } h = \frac{2T \cos \theta}{r\rho g}$$

For water: $\theta = 0^\circ$ and $\rho = 1 \text{ g cm}^{-3}$

$$h = \frac{2 \times 75 \times \cos 0^\circ}{0.03 \times 1 \times 1000} = 5.0 \text{ cm}$$

When the tube is lowered:

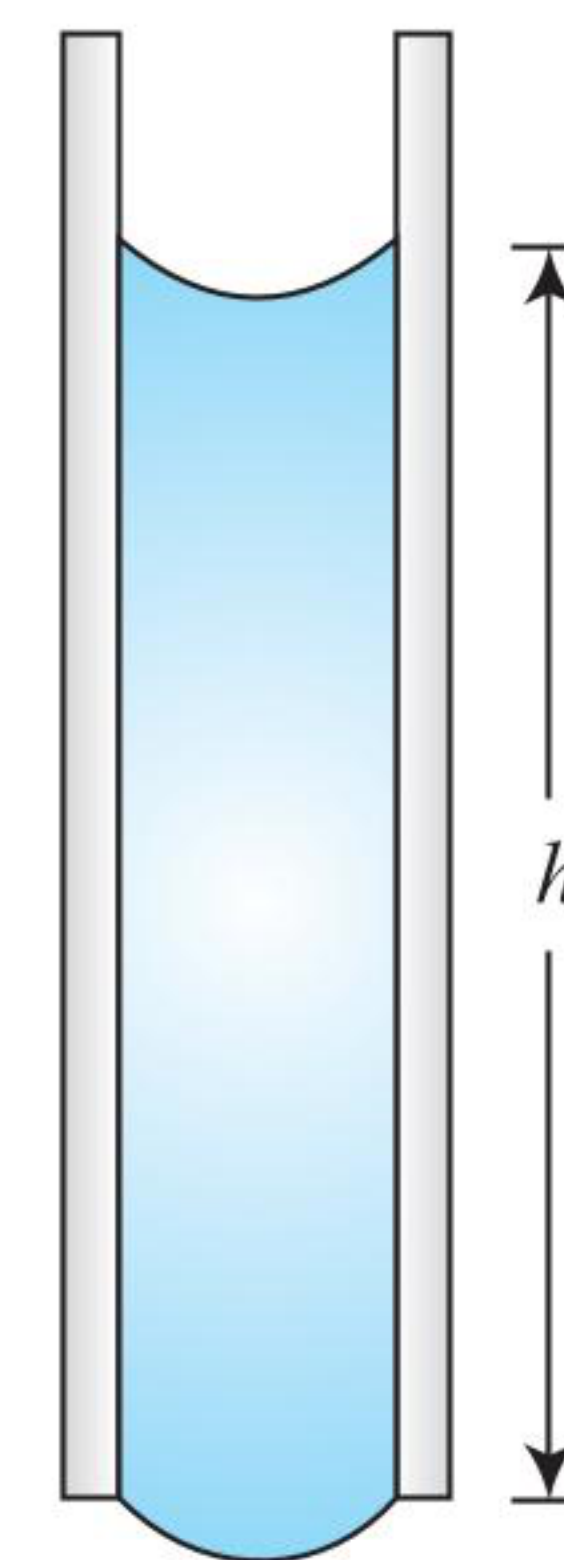
Length of the tube above the surface of water,

$$H' = 30 \text{ mm} = 3 \text{ cm}$$

Since $h > H'$, it follows that tube is of insufficient length. The water will rise upto the top end of the tube. However, it will not overflow. It will change the radius of curvature of the meniscus from r to R , where $RH' = rh$

$$\text{or } R = \frac{rh}{H'} = \frac{0.03 \times 5.0}{3} = 0.05 \text{ cm}$$

4. When a capillary tube filled with water is placed vertically, a column of water of length h will be held by the upper and the lower meniscus of water as shown in figure.



If r is radius of the tube, then weight of the water in the tube,

$$mg = \pi r^2 h \rho g$$

The weight of the liquid is supported by the force due to surface tension in the two menisci i.e.

$$F = 2\pi r T + 2\pi r T = 4\pi r T,$$

where T is surface tension of the water.

In equilibrium, $mg = F$

$$\text{i.e. } \pi r^2 h \rho g = 4\pi r T$$

$$\text{or } h = \frac{4T}{r\rho g} = \frac{4 \times 72}{0.1 \times 1 \times 980} = 2.94 \text{ cm}$$

5. Let $r_1 = r$. Then, $r_2 = r/3$

Here, $h_1 = 2 \text{ cm}$

Let h_2 be the height, to which liquid rises in the second tube

$$\text{Now, } h = \frac{2T \cos \theta}{r\rho g} \text{ or } h \propto \frac{1}{r}$$

$$\frac{h_1}{h_2} = \frac{r_2}{r_1} \text{ or } h_2 = \frac{r_1 h_1}{r_2}$$

$$\Rightarrow h_2 = \frac{r \times 2}{(r/3)} = 6 \text{ cm}$$

When the capillary tube is inclined: In the ascent formula, h refers to the vertical height of the liquid in the capillary tube. If the tube is inclined at an angle θ with the vertical, then length of liquid column in the tube,

$$L = \frac{h}{\cos \theta} = \frac{6}{\cos 60^\circ} = \frac{6}{0.5} = 12 \text{ cm}$$

Exercise 8.5

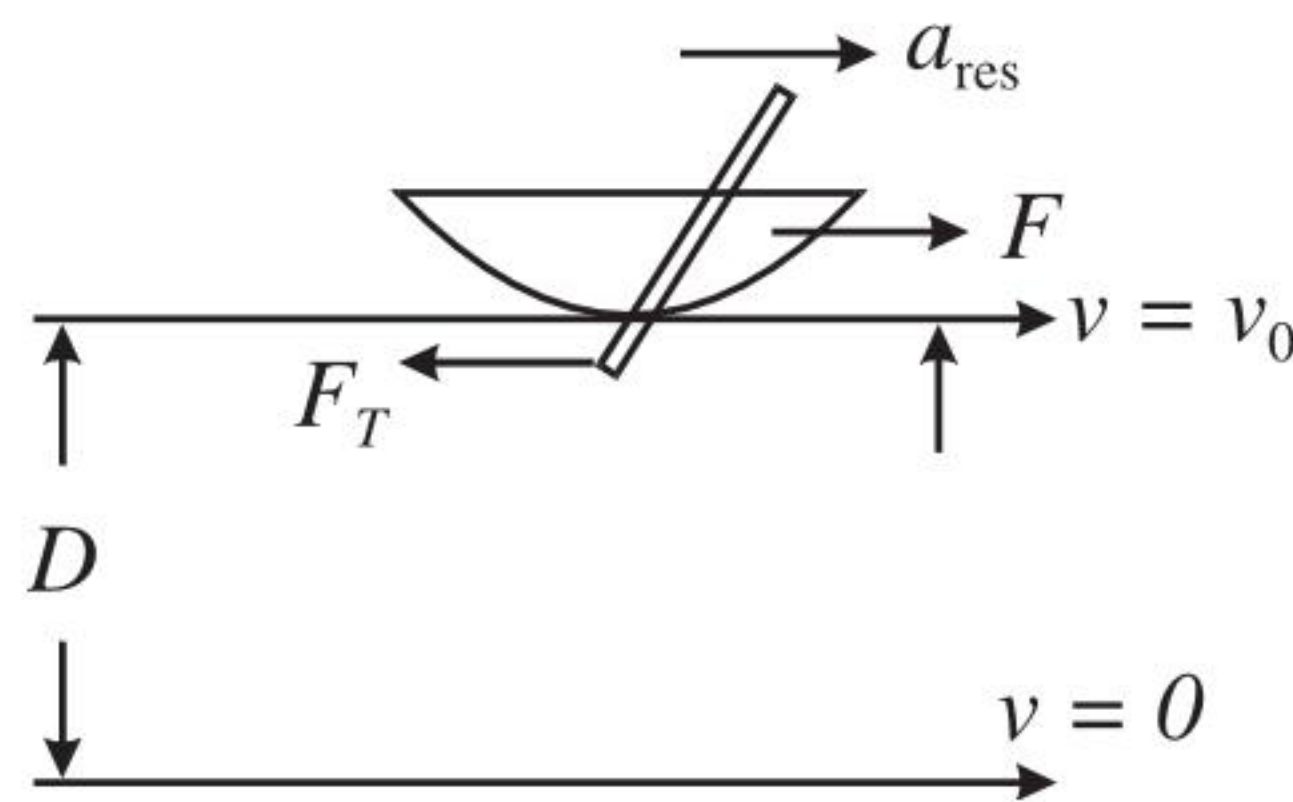
- (d) A longer pipe offers a greater resistance to the flow of a viscous fluid than a shorter pipe does. The volume flow rate depends inversely on the length of the pipe. For a given pipe radius and pressure difference between the ends of the pipe, the volume flow rate is less in longer pipes. In this case the longer pipe is twice as long, so its volume flow rate Q_B is one-half that of Q_A .

$$2. F - F_T = ma_{\text{res}}$$

As boat moves with constant velocity $a_{\text{res}} = 0$

$$F = F_T$$

We know $F_T = \eta A \frac{dv}{dy}$, but $\frac{dv}{dy} = \frac{v_0 - 0}{D} = \frac{v_0}{D}$



$$\text{then } F = F_T = \frac{\eta A v_0}{D}$$

$$3. F = F' = \eta A \frac{dv}{dy} = mg \sin \theta \quad \left[\frac{dv}{dy} = \frac{v}{h} \right]$$

$$20 \times 10 \times \sin 30^\circ = \eta \times 4 \times \frac{10}{h}$$

$$h = \frac{40 \times 10^{-2}}{100} \eta \quad [\eta = 10^{-1} \text{ poise} = 10^{-2} \text{ Ns/m}^2]$$

$$= 4 \times 10^{-3} \text{ m} = 4 \text{ mm}$$

- By Stoke's law, the terminal velocity of a water drop of radius r is given by

$$v = \frac{2}{9} \frac{r^2 (\rho - \sigma) g}{\eta}$$

where ρ is the density of water, σ is the density of air and η the coefficient of viscosity of air. Here σ is negligible and $r = 0.0015 \text{ mm} = 1.5 \times 10^{-3} \text{ mm} = 1.5 \times 10^{-6} \text{ m}$. Substituting the values:

$$v = \frac{2}{9} \times \frac{(1.5 \times 10^{-6})^2 \times (1.0 \times 10^3) \times 9.8}{1.8 \times 10^{-5}} = 2.72 \times 10^{-4} \text{ m/s}$$

- The velocity attained by the sphere in falling freely from a height h is

$$v = \sqrt{2gh} \quad \dots(i)$$

This is the terminal velocity of the sphere in water. Hence by Stoke's law, we have

$$v = \frac{2}{9} \frac{r^2 (\rho - \sigma) g}{\eta}$$

where r is the radius of the sphere, ρ is the density of the material of the sphere, σ ($= 1.0 \times 10^3 \text{ kg/m}^3$) is the density of water and η is coefficient of viscosity of water.

$$\therefore v = \frac{2 \times (1.0 \times 10^{-3})^2 (1.0 \times 10^4 - 1.0 \times 10^3) \times 10}{9 \times 1.0 \times 10^{-3}} = 20 \text{ m/s}$$

$$\text{From Eq. (i), we have } h = \frac{v^2}{2g} = \frac{20 \times 20}{2 \times 10} = 20 \text{ m}$$

$$6. F = \eta A \frac{dy}{dt} = \eta A \frac{v}{h} = 15 \times 2 \times 10^4 \times \frac{10}{0.25} = 1.2 \times 10^7 \text{ dyne}$$

$$7. F = 6\pi\eta r v$$

$$\eta = \frac{F}{6\pi r v} = \frac{3.14}{(6)(3.14)(4)(5 \times 10^2)} = 8.3 \times 10^{-5} \text{ poise}$$

- Here, coefficient of viscosity, $\eta = 0.01$ poise; viscous force, $F = 200$ dyne; velocity of one plate w.r.t. the other plate, $dv = 10 \text{ cm}^{-1}$; and side of the square plate $= 10 \text{ cm}$.

Therefore, area of the plate $= 10 \times 10 = 100 \text{ cm}^2$

$$\text{Now, } F = \eta A \frac{dv}{dx}$$

$$dx = \frac{\eta A dv}{F} = \frac{0.01 \times 100 \times 10}{200} = 0.05 \text{ cm}$$

- Here, length of the needle, $l = 5 \text{ cm}$; radius of the needle, $r = 0.03 \text{ cm}$; viscosity of blood, $\eta = 0.017$ poise; density of blood, $p = 1.02 \text{ g cm}^{-3}$; and height of the blood bag, $h = 85 \text{ cm}$.

Pressure head, $p = h \rho g = 85 \times 1.02 \times 980 \text{ dyne cm}^{-2}$

Therefore, volume of the blood given to the patient per second,

$$V = \frac{\pi p r^4}{8 \eta l} = \frac{\pi \times 85 \times 1.02 \times 980 \times (0.03)^4}{8 \times 0.017 \times 5} = 0.327 \text{ cm}^3 \text{ s}^{-1}$$

Therefore, time for which the transfusion takes place,

$$t = \frac{\text{total volume of the blood}}{\text{rate of flow of the blood}} = \frac{500}{0.327} = 1529 \text{ s}$$

Exercises

Single Correct Answer Type

- (2) Suppose, R = radius of water drop and r = radius of droplets

$$\therefore \frac{4}{3} \pi R^3 = 8 \times \frac{4}{3} \pi r^3$$

(Since volume remains constant)

$$\therefore r = \frac{R}{2}$$

$$\text{Since excess pressure inside drop} = \frac{2T}{R}$$

(T —surface tension, R —radius)

Therefore, pressure difference between inner and outer surface of big drop will be half of that for smaller droplet.

- (3) The air pressure is greater inside the smaller bubble ($4T/r$). Hence, air flows from the smaller to the larger bubble.
- (2) When two bubbles come into contact, they try to minimize the surface area for which air flows from bubble having higher pressure (smaller radius) to bubble having lower pressure.
- (4) Surface tension force, $F = S \times \text{length}$ or $F = S \times [2\pi R + 2\pi r]$ as liquid surface is on the both sides.

5. (3) Work done = (increase in surface area) $\times$ surface tension

$$= 2 \times \left[4\pi \left(\frac{2D}{2} \right)^2 - 4\pi \left(\frac{D}{2} \right)^2 \right] \times T$$

(Since for soap bubble, there are two free surfaces)

$$= 2 \times (4\pi D^2 - \pi D^2) T = 6\pi D^2 T$$

6. (4) $V \times \frac{4}{3}\pi R^3$; $2V = \frac{4}{3}\pi R'^3$; $2 \times \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R'^3$;

$$R' = 2^{1/3} R$$

$$W' = 2 \times 4\pi [2^{1/3} R]^2 \sigma = 2^{2/3} \times 2 \times 4\pi R^2 \sigma = 4^{1/3} W$$

7. (3) Final surface energy = $1000 \times 4\pi r^2 \sigma$

Initial surface energy, $E = 4\pi R^2 \sigma$

$$\text{Again, } \frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3 \quad \text{or} \quad R = 10r$$

Now, final energy

$$1000 \times 4\pi r^2 \sigma = 10 \times 4\pi R^2 \sigma = 10E$$

8. (3) Work done = $10E - E = 9E$.

9. (2) $v_0 = \frac{2r^2(\rho - \rho')g}{9\eta}$ or $\eta = \frac{2r^2(\rho - \rho')g}{9v_0}$

$$= \frac{2 \times 0.2 \times 0.2 \times 9 \times 980}{9 \times 8} \text{ poise} = 9.8 \text{ poise}$$

10. (2) For the same radius, terminal velocity is proportional to the density difference.

11. (1) $F = \frac{0.01 \times 100 \times 10}{0.1} \text{ dyn} = 100 \text{ dyn}$

12. (3) Shearing stress = $\frac{\text{viscous force}}{\text{area}}$

$$= \frac{\eta A \frac{dv}{dx}}{A} = \eta \frac{dv}{dx}$$

$$= 10^{-3} \times \frac{5}{10} \text{ N/m}^2 = 0.5 \times 10^{-3} \text{ N/m}^2$$

13. (1) In the condition of weightlessness, water rises to the whole of the available length.

14. (1) Here, surface tension force support the weight of liquid.

$$S \times 2\pi \left[r + \frac{d}{2} \right] = \left(r^2 - \frac{d^2}{4} \right) h \rho g$$

$$\Rightarrow h = 0.0144 \text{ m} = 1.44 \text{ cm}$$

15. (2) $\left[P_0 + \frac{4\sigma}{R_2} \right] - \left[P_0 + \frac{4\sigma}{R_1} \right] = \frac{4\sigma}{R}$

$$\text{or } \frac{1}{R} = \frac{1}{R_2} - \frac{1}{R_1}$$

$$\text{or } R = \frac{R_1 R_2}{R_1 - R_2} = \frac{50 \times 80}{30} \text{ mm} = \frac{400}{3} \text{ mm}$$

$$= \frac{400}{3 \times 1000} \text{ m} = \frac{4}{30} \text{ m} = 0.133 \text{ m}$$

16. (1) $F = (\sigma_1 - \sigma_2) l$

17. (3) $F = \frac{2A\sigma}{t} = \frac{2 \times 8 \times 75 \times 10}{0.12} \text{ dyn} = 10^5 \text{ dyn}$

18. (1) $\left[2\pi \times \frac{8.7}{2} + 2\pi \times \frac{8.5}{2} \right] \sigma = 3.97 \times 980$

$$\text{or } \sigma = \frac{3.97 \times 980 \times 7}{22 \times 17.2} \text{ dyn/cm} = 72 \text{ dyn/cm}$$

19. (3) $m \times 10 = 2 \times 3 \times 10^{-2} \times \frac{10}{100}$

$$\text{or } m = 6 \times 10^{-4} \text{ kg} = 6 \times 10^{-4} \times 10^3 \text{ g} = 0.6 \text{ g}$$

20. (2) Pressure due to 15 cm long liquid column needs to be balanced.

21. (4) $25 \times 10^{-3} \times 2 \times 0.1 = m \times 10$

$$\text{or } m = \frac{5 \times 10^{-3}}{10} \text{ kg} = 0.5 \text{ g}$$

22. (3) $\frac{h_2}{h_1} = \frac{\sigma_2 \cos \theta_2}{\rho_2} \times \frac{\rho_1}{\sigma_1 \cos \theta_1}$

$$\text{or } \frac{h_2}{h_1} = \frac{140 \times \frac{1}{2}}{2} \times \frac{1}{70 \times 1} \quad \text{or } h_2 = \frac{h_1}{2} = \frac{6}{2} \text{ cm} = 3 \text{ cm}$$

23. (3) $\frac{40}{100} \times 1000 \times 9.8 = \frac{2 \times 7 \times 10^{-2}}{R}$

$$\text{or } R = \frac{14 \times 10^{-2} \times 100}{40 \times 1000 \times 9.8} \text{ m} = \frac{14 \times 1000}{40 \times 1000 \times 9.8} \text{ mm} = \frac{1}{28} \text{ mm}$$

$$\text{Diameter} = 2R = \frac{1}{14} \text{ mm}$$

24. (2) $V \propto r^4, V' \propto \left(r + \frac{1}{10} r \right)^4$

$$\frac{V'}{V} = \frac{11 \times 11 \times 11 \times 11}{10 \times 10 \times 10 \times 10} = 1.4641$$

$$\text{or } \left(\frac{V'}{V} - 1 \right) \times 100 = 46.4\%$$

25. (2) $V\rho g + F = V(4\rho)g$

$$\text{or } F = 3V\rho g = 3mg \quad \text{or } \frac{F}{mg} = 3$$

26. (2) $m = \frac{4}{3}\pi r^3 \rho m$

Keeping m constant, if r is halved, ρ will increased by a factor of 8.

$$\text{Now, } v_0 \propto r^2 \rho$$

$$v'_0 \propto \frac{r^2}{4} (8\rho) \quad \text{or } v'_0 \propto 2r^2 \rho$$

$$\text{Dividing, } \frac{v'_0}{v_0} = 2 \quad \text{or } v'_0 = 2v_0$$

$$\text{or } v'_0 = 2v$$

27. (1) As we increase the separation between the wires by 1 mm, the increase in surface area of the film is

$$\Delta A = 2 \times 0.1 \times 10^{-3} = 2 \times 10^{-4} \text{ m}^2$$

(as there are two surfaces of film)

$$\text{So, increase in surface energy, } \Delta U = S\Delta A = 1.44 \times 10^{-5} \text{ J}$$

$$\text{Work done} = \Delta U = 1.44 \times 10^{-5} \text{ J}$$

28. (1) Surface tension force,

$$F = S \times 2l = 0.072 \times 2 \times \frac{2.5}{100} = 3.6 \times 10^{-3} \text{ N}$$

29. (4) As in vacuum, there is no viscosity, no resistive force and hence no terminal velocity can be acquired.

$$30. (4) v_0 = \frac{2}{9} \frac{r^2 (\rho - \rho') g}{\eta}$$

$$\text{Now, } x = \frac{4}{3} \pi r^3 \rho \text{ or } \rho \propto \frac{x}{r^3}$$

$$\text{Similarly, } \rho' \propto \frac{y}{r^3}$$

$$\therefore v_0 \propto \frac{x-y}{r}$$

$$31. (2) h = \frac{2S \cos \theta}{r \rho g}$$

Cross-sectional area increases four times, which means radius gets doubled.

$$\text{So, } h' = \frac{2S}{2(r \rho g)} = \frac{h}{2}$$

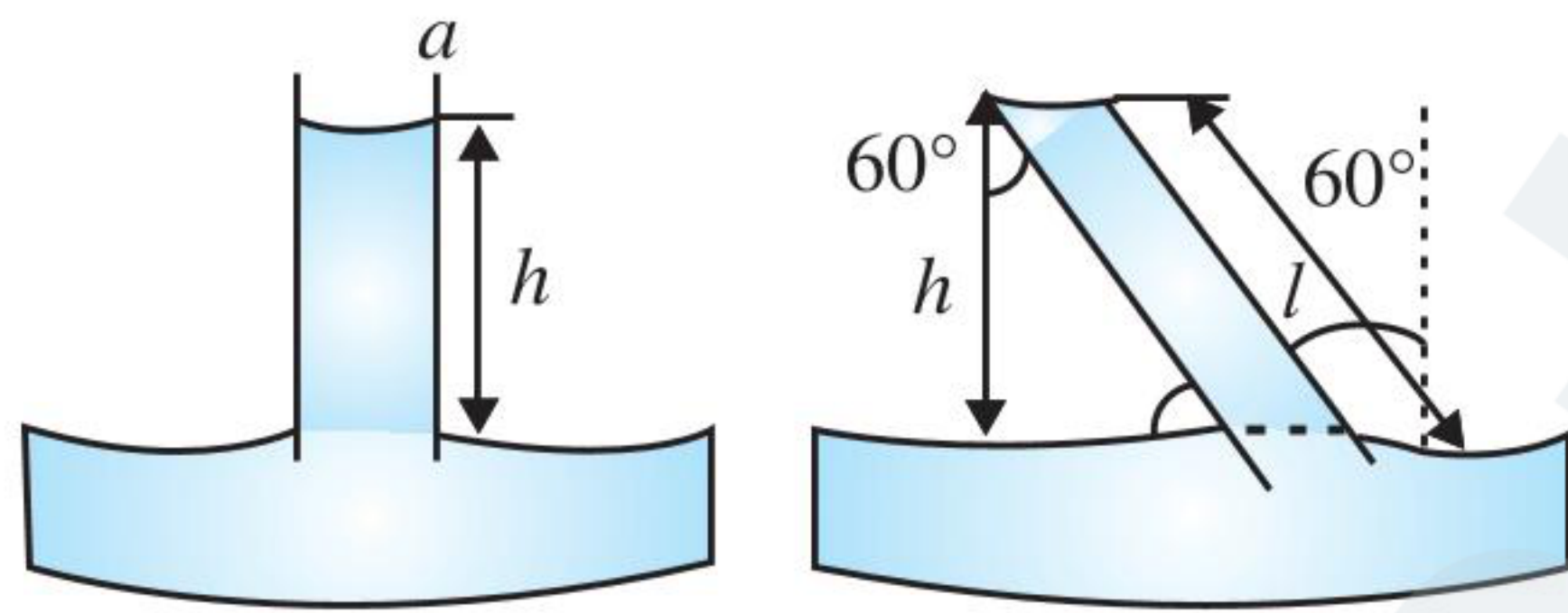
32. (1) The terminal velocity of the spherical raindrop of radius r is given by

$$v_t = \frac{2r^2 \rho g}{9\eta}$$

where ρ is the density of water and η the viscosity of air. Substituting $r = 0.3 \text{ mm} = 0.3 \times 10^{-3} \text{ m}$, $\rho = 10^3 \text{ kg/m}^3$; $g = 9.8 \text{ ms}^{-2}$ and $\eta = 1.8 \times 10^{-5} \text{ Ns/m}^2$.

$$\text{We get } v_t = \frac{2 \times (0.3)^2 \times 10^{-3} \times 9.8}{9 \times 1.8 \times 10^{-5}} = 10.9 \text{ m/s}$$

33. (1) Since water rises to height of 2 cm in a capillary



If tube is at 60° , in this case height must be equal to

$$h = 2 \text{ cm} \Rightarrow \cos 60^\circ = \frac{h}{l}$$

$$\therefore l = \frac{h}{\cos 60^\circ} = \frac{2}{1/2} = 4 \text{ cm}$$

34. (4) For equilibrium,

Total upward force by surface tension

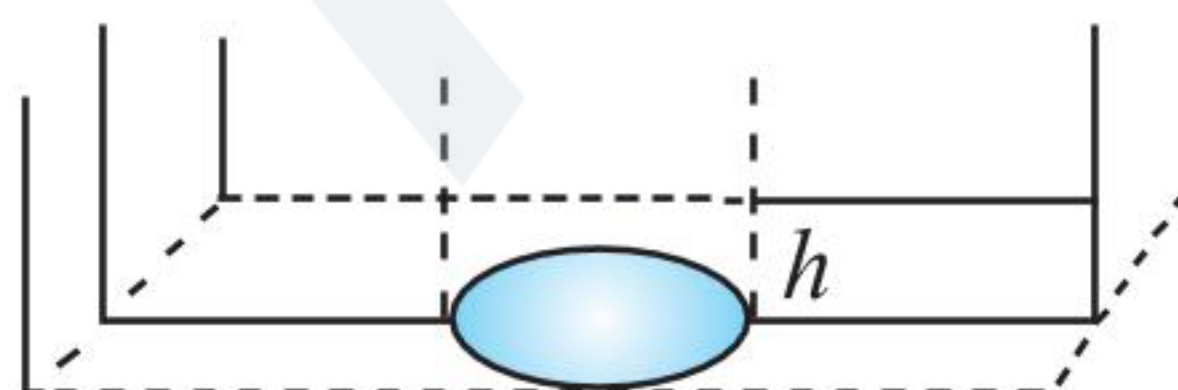
= Weight of the water in tube

$$\Rightarrow \pi \times D \times \text{surface tension (circumference)}$$

$$= \pi (D/2)^2 \times h \times \text{density} \times g \text{ (cross section)}$$

where D (diameter) = $0.1 \text{ mm} = 0.01 \text{ cm}$

Density of water = $1 \times 10^{-3} \text{ g/cm}^3$



$$\Rightarrow \pi \times (0.01) \times 75 \times 10^{-3} = \pi \times \left(\frac{0.01}{2}\right)^2 \times h \times 1 \times 10^{-3} \times 1000$$

$$\therefore h = \frac{0.75 \times 0.01 \times 4}{0.01 \times 0.01} = 0.3 \text{ m} = 30 \text{ cm}$$

$$35. (3) \text{ Height, } h = \frac{2T \cos \theta}{r \rho g}$$

$$\therefore \text{ For water, } h_w = \frac{2 \times T_w \times \cos 0^\circ}{r \times 1 \times g}$$

And, for mercury,

$$h_m = -\frac{2 \times T_m \times \cos 135^\circ}{r \times 13.6 \times g}$$

$$\therefore \frac{h_w}{h_m} = \frac{2 \times T_w \times 1}{r \times 1 \times g} \times \frac{r \times 13.6 \times g \times \sqrt{2}}{2 \times T_m \times 1} \left[\because \cos 135^\circ = -\frac{1}{\sqrt{2}} \right]$$

$$\Rightarrow \frac{10}{3.42} = \frac{T_w}{T_m} \times 13.6 \times \sqrt{2}$$

$$\therefore \frac{T_w}{T_m} = \frac{10}{3.42 \times 13.6 \times 1.414} \approx \frac{1}{6.5} = \frac{2}{13}$$

$$\therefore T_w : T_m = 2/13$$

36. (2) Since, ball gains the constant velocity. Therefore, net force on the ball = zero.

$$\Rightarrow \text{Weight of the ball} = \text{Buoyancy force} + \text{Viscous force } (F_v)$$

$$\Rightarrow Mg = \frac{M}{d_1} \times d_2 \times g + F_v$$

$$\therefore F_v = Mg \left(1 - \frac{d_2}{d_1} \right)$$

37. (1) If y is the required distance, then

$$a^2 y \rho g = mg + 4 a \sigma$$

$$\text{or } y = \frac{mg + 4 a \sigma}{a^2 \rho g} = \frac{20 \times 980 + 4 \times 3 \times 70}{3 \times 3 \times 1 \times 980} \text{ cm} = 2.3 \text{ cm}$$

38. (1) Effective length of flat part = $2r$. Effective length of curved part is πr .

$$F = \sigma l$$

So, required ratio is $2:\pi$.

39. (1) Total upward force due to surface tension = $2\sigma l$

Weight of lifted liquid = $(hld) \rho g$

$$\text{Equating, we get } h = \frac{2\sigma}{\rho g d}$$

40. (1) Increase in PE = $hld \rho g \left(\frac{h}{2} \right) = \frac{ld \rho g}{2} h^2$

If F is the force of attraction between the plates, then

$$Fd = \frac{ld \rho g}{2} h^2 \quad \text{or} \quad F = \frac{l \rho g}{2} h^2$$

41. (2) $F \propto h^2$

But $h^2 \propto \sigma^2$

$$\therefore F \propto \sigma^2$$

If σ is doubled, then F will become $4F$.

42. (4) $\frac{4}{3} \pi R^3 = n \times \frac{4}{3} \pi r^3 \quad \text{or} \quad R^3 = nr^3$

Energy evolved = $(n \times 4\pi r^2 - 4\pi R^2) T$

$$\text{Now, } \theta = \frac{Q}{V \rho S} = \frac{Q}{V} \quad (\text{In CGS system, } \rho = 1 \text{ and } S = 1)$$

$$\therefore \theta = \frac{[n \times 4\pi r^2 - 4\pi R^2] T}{\frac{4}{3} \pi R^3}$$

$$= \frac{3T[nr^2 - R^2]}{R^3} = 3T \left[\frac{nr^2}{nr^3} - \frac{R^2}{R^3} \right] = 3T \left[\frac{1}{r} - \frac{1}{R} \right]$$

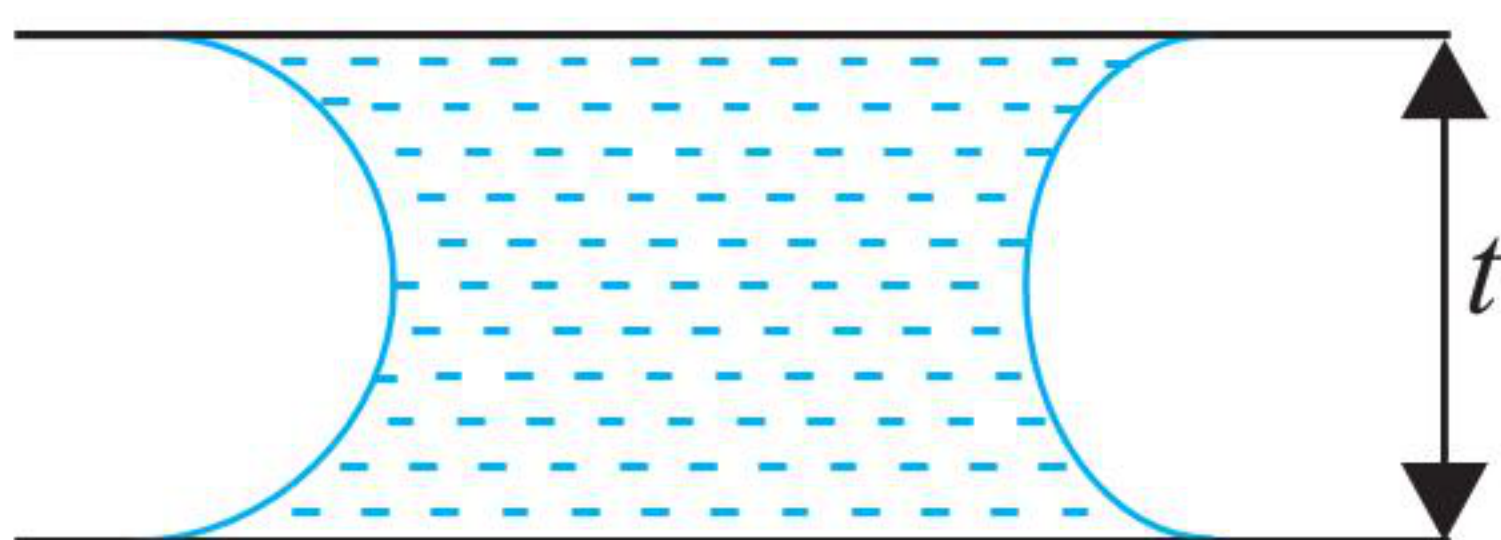
$$43. (4) \quad 2\pi r\sigma + \frac{1}{2} \times \frac{4}{3} \pi r^3 dg = \frac{4}{3} \pi r^3 \rho g$$

$$\text{or } 2\pi r\sigma = \frac{\pi r^3 g}{3} [4\rho - 2d] \quad \text{or } r^2 = \frac{3 \times 2\pi\sigma}{\pi g(4\rho - 2d)}$$

$$\text{or } r^2 = \frac{3\sigma}{g(2\rho - d)} \quad \text{or } r = \sqrt{\frac{3\sigma}{g(2\rho - d)}}$$

$$\text{Diameter} = 2r = \sqrt{\frac{12\sigma}{g(2\rho - d)}}$$

44. (2)



$$\text{Pressure difference} = \frac{T}{R} = \frac{T}{t/2} = \frac{2T}{t}$$

$$\text{Force} = \frac{2T}{t} A$$

$$\text{But } V = At \text{ or } t = \frac{V}{A}$$

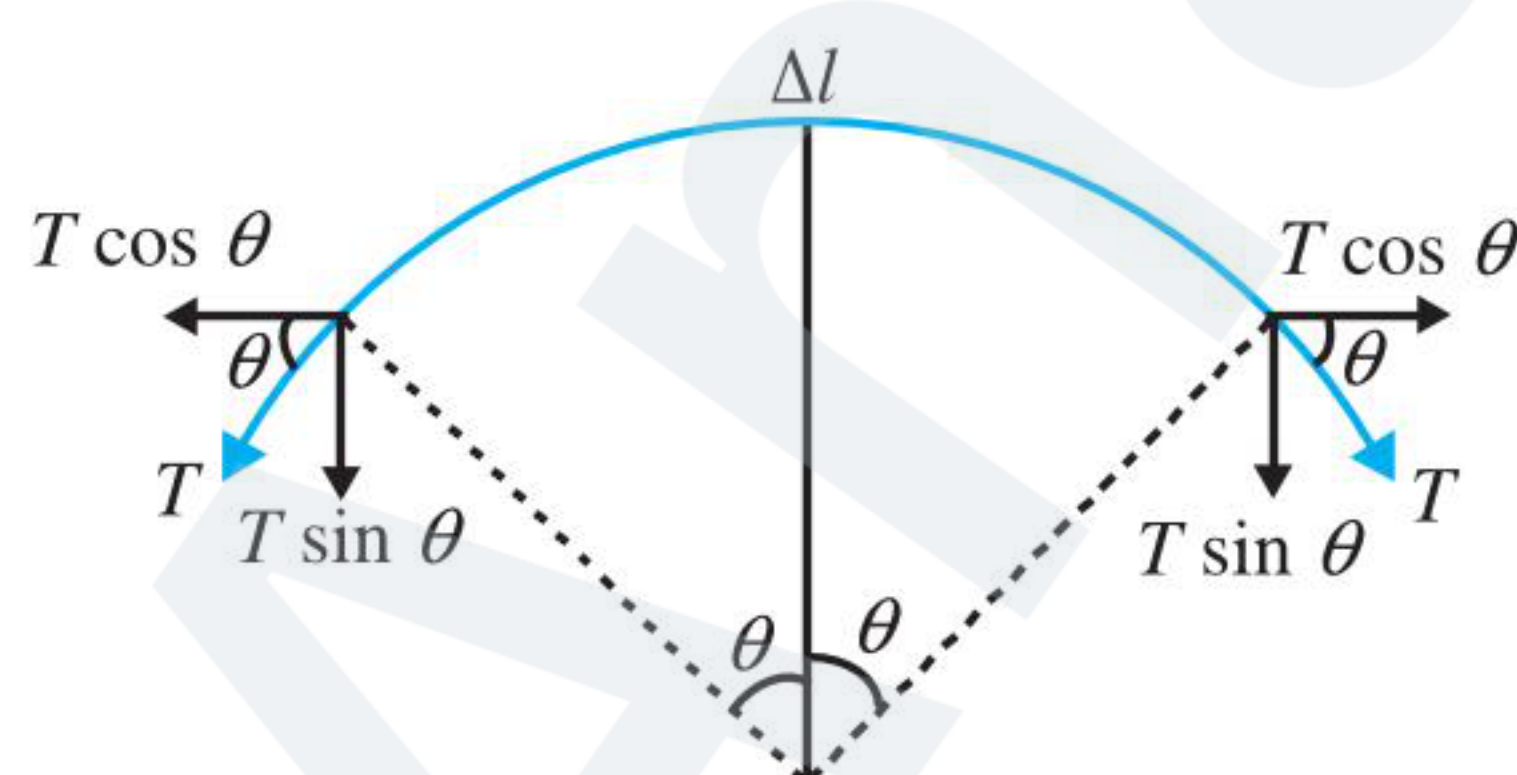
$$\therefore \text{Force} = \frac{2TA^2}{V}$$

45. (2) Difference in apparent weights is due to differences in forces of surface tension. Due to 180° , the force surface tension in one case is opposite to the force of surface tension in the other case.

$$\therefore 2 \times \sigma_w \times \frac{10}{100} = 0.044 - 0.03$$

$$\text{or } \sigma_w \times \frac{0.014}{1} \times 5 \text{ N/m} = 0.07 \text{ N/m}$$

46. (3) When the soap film enclosed by the thread is pricked, the thread would take up a circular shape. Consider a small element of the thread. While horizontal components of tension get balanced, the vertical components get added up.



$$\text{or } 2T \sin \theta = \sigma \Delta l$$

$$\text{or } 2T\theta = \sigma (2\theta \times r)$$

$$\text{or } T = \sigma r = \sigma \times \frac{2\pi r}{2\pi} = 0.030 \times \frac{6.28 \times 10^{-2}}{2\pi} \text{ N}$$

$$= 3 \times 10^{-4} \text{ N}$$

47. (1) Total upward force due to surface tension $= \sigma(2r_1 + 2r_2)$. This supports the weight of the liquid column of height h . Weight of liquid column $= h [\pi r_2^2 - \pi r_1^2] \rho g$

$$\text{Equating, we get } h\pi (r_2 + r_1) (r_2 - r_1) \rho g = 2\pi\sigma (r_1 + r_2)$$

$$\text{or } h (r_2 - r_1) \rho g = 2\sigma \quad \text{or } h = \frac{2\sigma}{(r_2 - r_1) \rho g}$$

48. (4) Energy released $= [n \times 4\pi a^2 - 4\pi b^2] \sigma$

$$\text{Now, } n \times \frac{4}{3} \pi a^3 = \frac{4}{3} \pi b^3 \quad \text{or } n = \frac{b^3}{a^3}$$

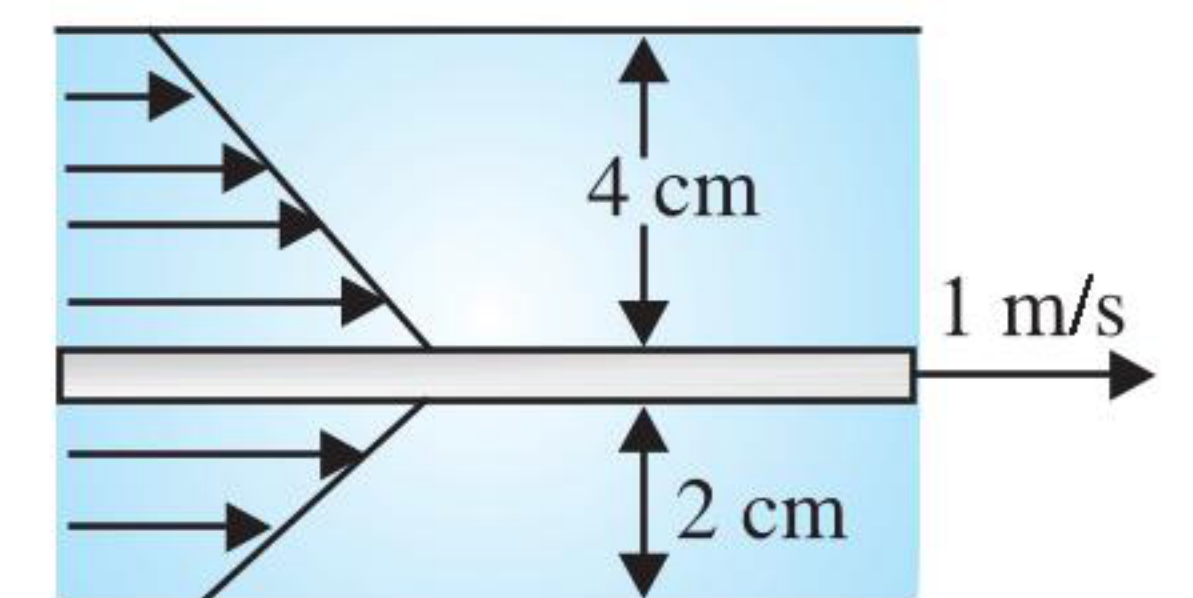
Therefore, energy released

$$= \left[\frac{b^3}{a^3} \times 4\pi a^2 - 4\pi b^2 \right] \sigma = 4\pi b^2 \left[\frac{b}{a} - 1 \right] \sigma$$

$$\text{Now, } \frac{1}{2} \left(\frac{4}{3} \pi b^3 \right) \rho v^2 = 4\pi b^2 \left[\frac{b}{a} - 1 \right] \sigma$$

$$\text{or } v = \left[\frac{6\sigma}{\rho} \left(\frac{1}{a} - \frac{1}{b} \right) \right]^{1/2}$$

49. (2) The profile of the velocity of fluid is as shown in the figure. The velocity of the layer of fluid, which is in contact with metal plates (fixed), is zero. As we move towards the centre from either plate, the velocity of the layer of fluid increases and it becomes maximum at the location of moving plate. This maximum value is same as that of the velocity of plate.



50. (1) The force experienced by the plate due to fluid is shown in the figure.



F_1 is the force exerted by the layer in contact with plate on upper side and F_2 is the force exerted by the layer in contact with plate on lower side.

$$F_1 = \eta A \times \frac{v}{4 \text{ cm}} = 0.8 \times 0.01 \times \frac{1}{0.04} = 0.2 \text{ N}$$

where v is the velocity of layer in contact with plate.

$$F_2 = \eta A \times \frac{v}{2 \text{ cm}} = 0.8 \times 0.01 \times \frac{1}{0.02} = 0.4 \text{ N}$$

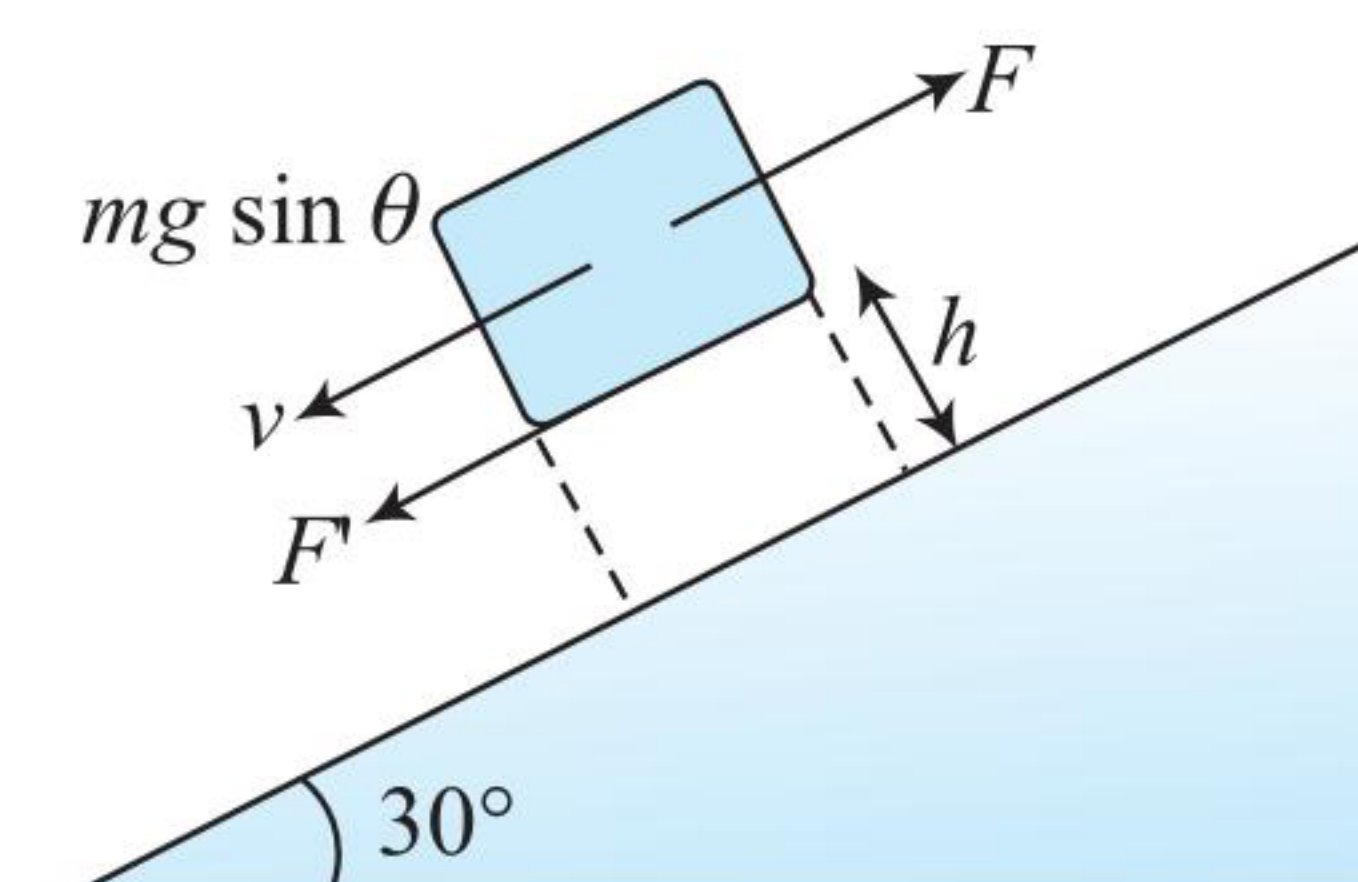
Total force, $F = F_1 + F_2 = 0.6 \text{ N}$ in backward direction.

51. (4) A longer pipe offers a greater resistance to the flow of a viscous fluid than a shorter pipe does. The volume flow rate depends inversely on the length of the pipe. For a given pipe radius and pressure difference between the ends of the pipe, the volume flow rate is less in longer pipes. In this case the longer pipe is twice as long, so its volume flow rate Q_B is one-half that of Q_A .

52. (1) Taking g constant, work done in both the cases is $-mg$ (1000) since the max velocity of the journey is attained at a height of 1 km, and the second half journey is travelled by max velocity, the time taken for second half journey will be less.

$$53. (2) \quad F = F' = \eta A \frac{dv}{dz} = mg \sin \theta$$

$$\text{Velocity gradient } \frac{dv}{dz} = \frac{v}{h}$$

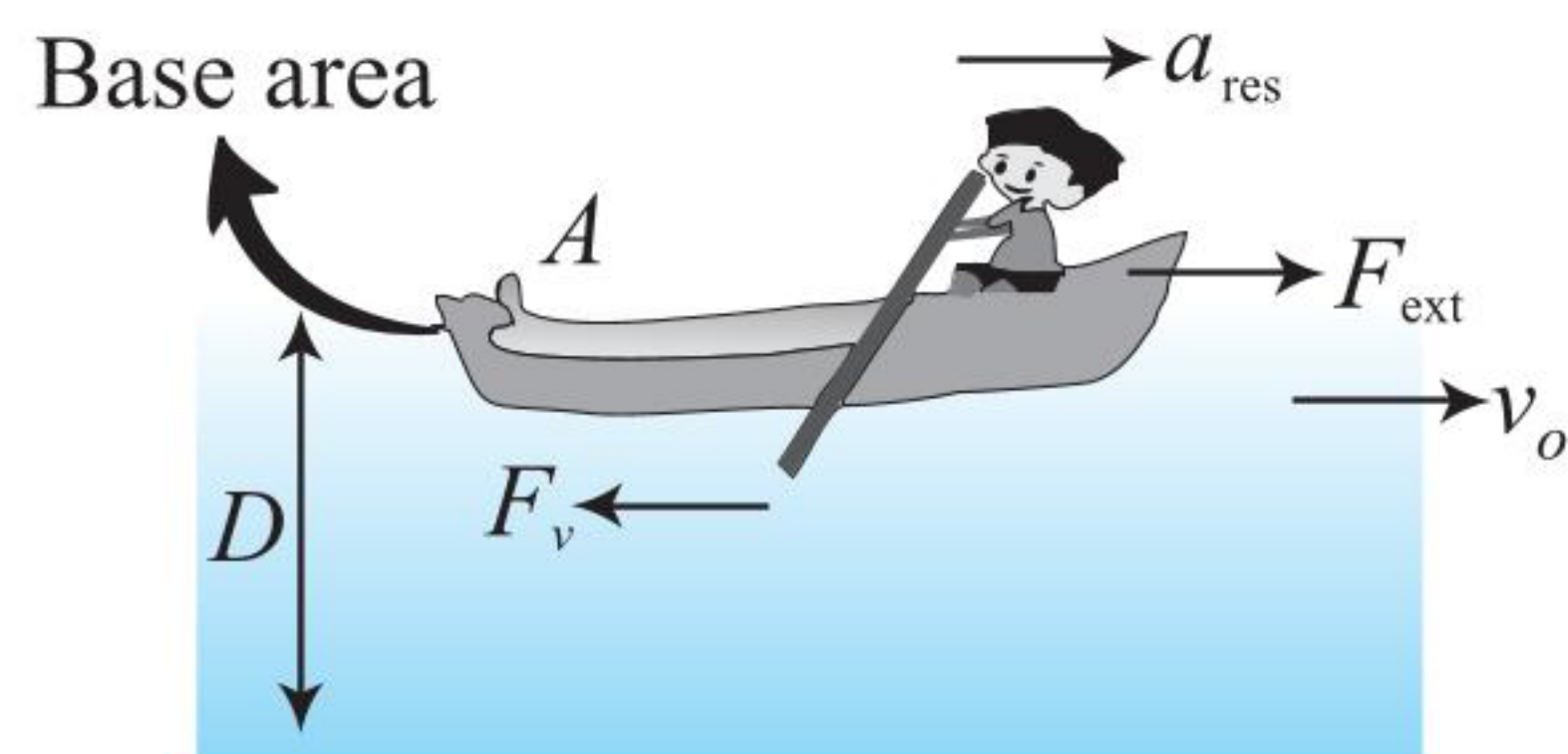


Hence $20 \times 10 \times \sin 30^\circ = \eta \times 4 \times \frac{10}{h}$

$$h = \frac{10^{-2} \times 4 \times 10}{20 \times 10 \times \frac{1}{2}} = \frac{40 \times 10^{-2}}{100} [\eta = 10^{-1} \text{ poise} = 10^{-2} \text{ N s m}^{-2}]$$

$$= 4 \times 10^{-3} \text{ m} = 4 \text{ mm}$$

54. (3) We can draw the free-body diagram of boat



$$F_{\text{ext}} - F_v = ma_{\text{res}}$$

As boat moves with constant velocity $a_{\text{res}} = 0$

$$F_{\text{ext}} = F_v$$

But $F_v = \eta A \frac{dv}{dz}$ but $\frac{dv}{dz} = \frac{v_0 - 0}{D} = \frac{v_0}{D}$

then $F_{\text{ext}} = F_v = \frac{\eta A v_0}{D}$

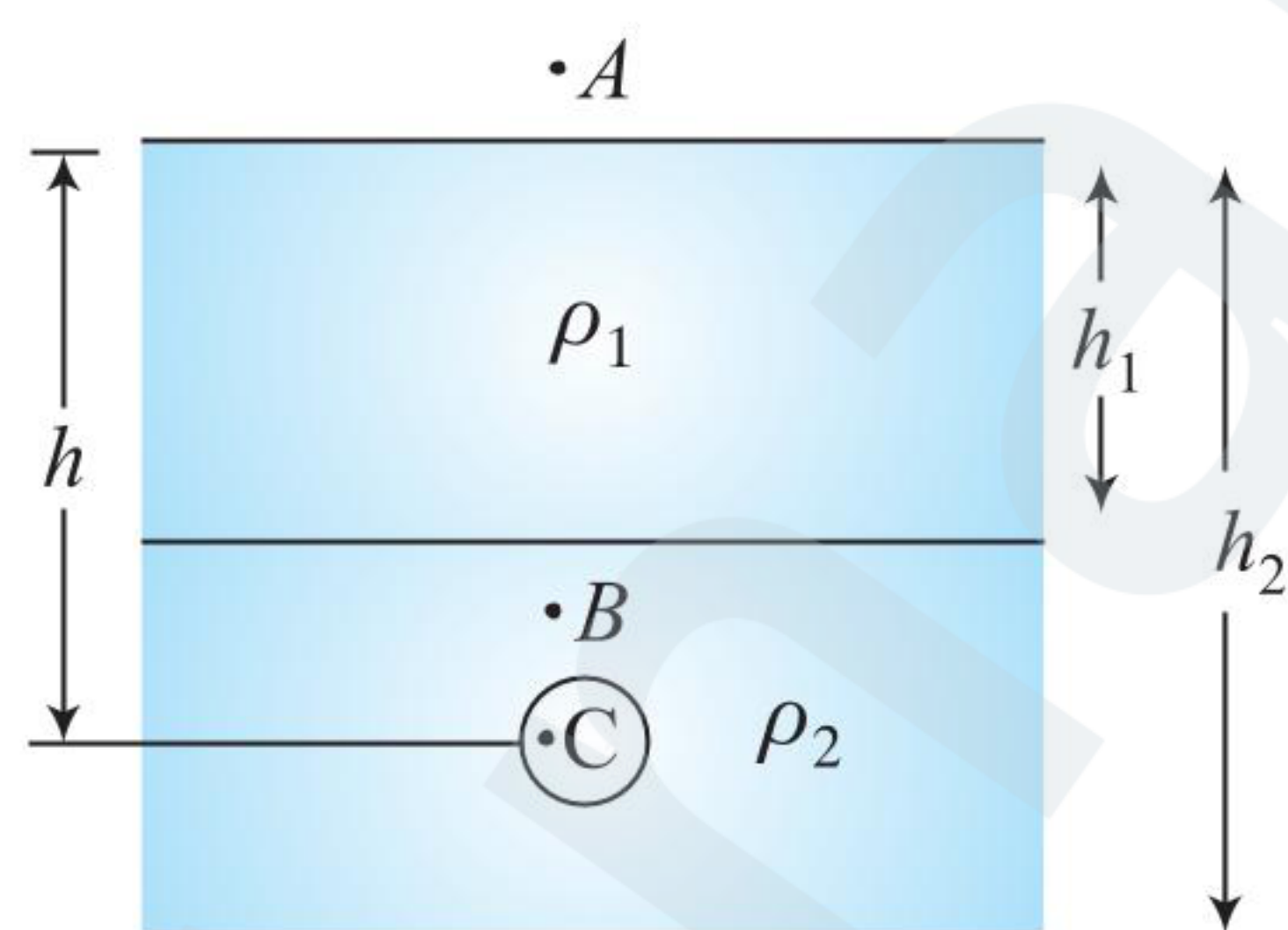
55. (3) $P_1 - P_0 = \frac{4S}{2R}$... (i)

$P_2 - P_1 = \frac{4S}{R}$... (ii)

From (i) and (ii) $P_2 - P_0 = \frac{6S}{R}$

56. (2) $P_C = P_B + \frac{2T}{r}$

$$P_B = P_A + \rho_1 g h_1 + \rho_2 g (h - h_1)$$



where $P_A = P_0$

$$P_C = P_0 + \rho_1 g h_1 + \rho_2 g (h - h_1) + \frac{2T_2}{r}$$

Then evaluate by putting the numerical values.

57. (3) $P_0 + \rho_1 g h - \rho_2 g h + \frac{2T}{r} = P_0$

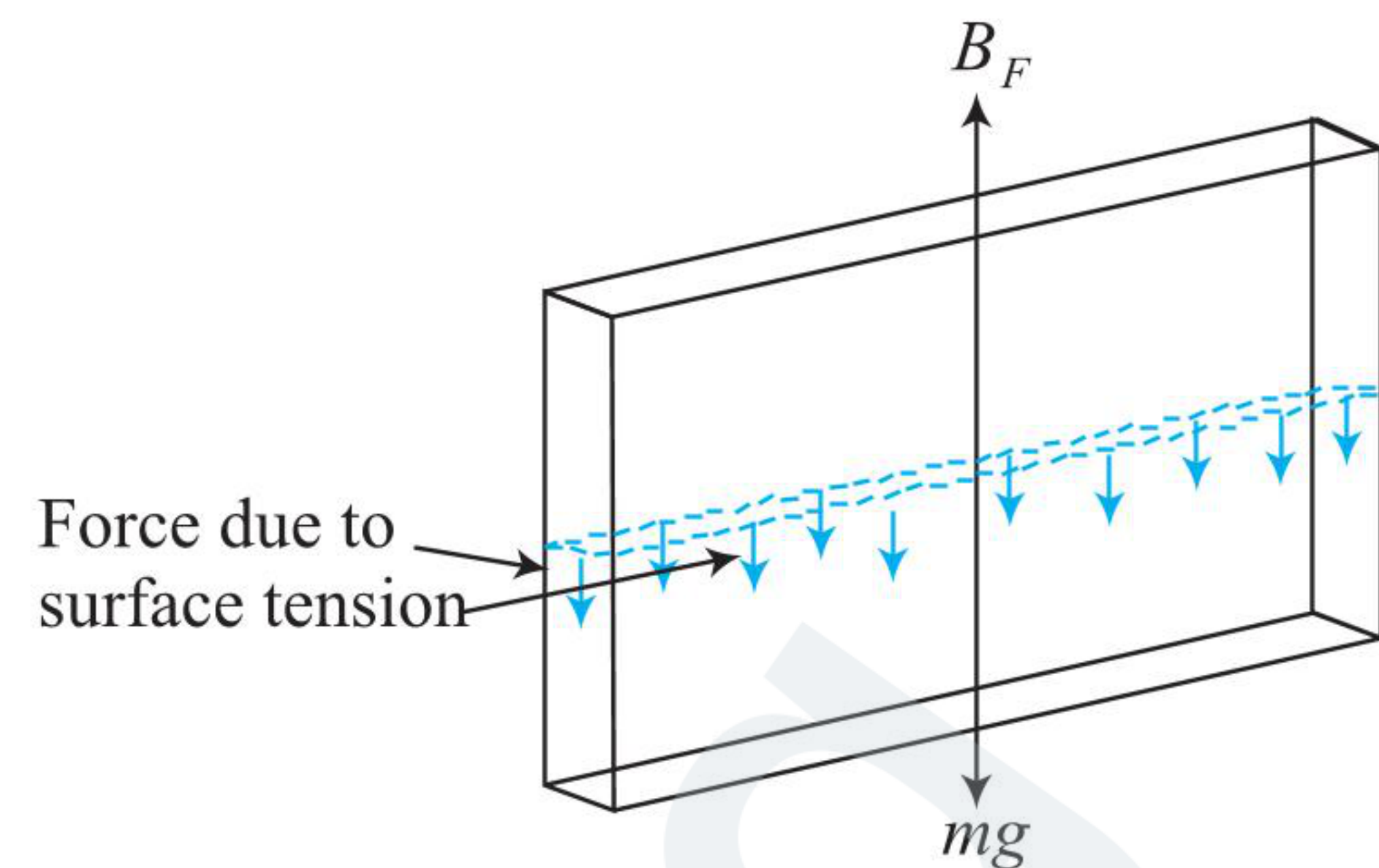
$$\Rightarrow T = \frac{r}{2} (\rho_2 - \rho_1) g h$$

58. (2) The forces acting on the plate are

(i) mg ($\downarrow$)

(ii) force due to surface tension (F_T) ($\downarrow$)

(iii) Buoyant force (B_F) ($\uparrow$)



$$w = mg = (8.2 \times 10^{-3}) \times 9.8 = 80.36 \times 10^{-3} \text{ N}$$

The water is in contact along the length,

$$L = 2(10 + 0.2) = 20.4 \text{ cm}$$

$$\therefore F_T = (20.4 \times 10^{-2})(7.3 \times 10^{-2}) = 148.92 \times 10^{-4} \text{ N}$$

$$B_F = \left[\frac{10 \times 0.2 \times 1.54}{2} \times 10^{-6} \right] \times 1000 \times 9.8 = 15.092 \times 10^{-3} \text{ N}$$

$\therefore$ Apparent weight

$$w + F_T - B_F = (80.36 + 14.892 - 15.092) \times 10^{-3} = 80.16 \times 10^{-3} \text{ N}$$

59. (3) Viscous force $F \propto$ (area)

So let $F = kA$

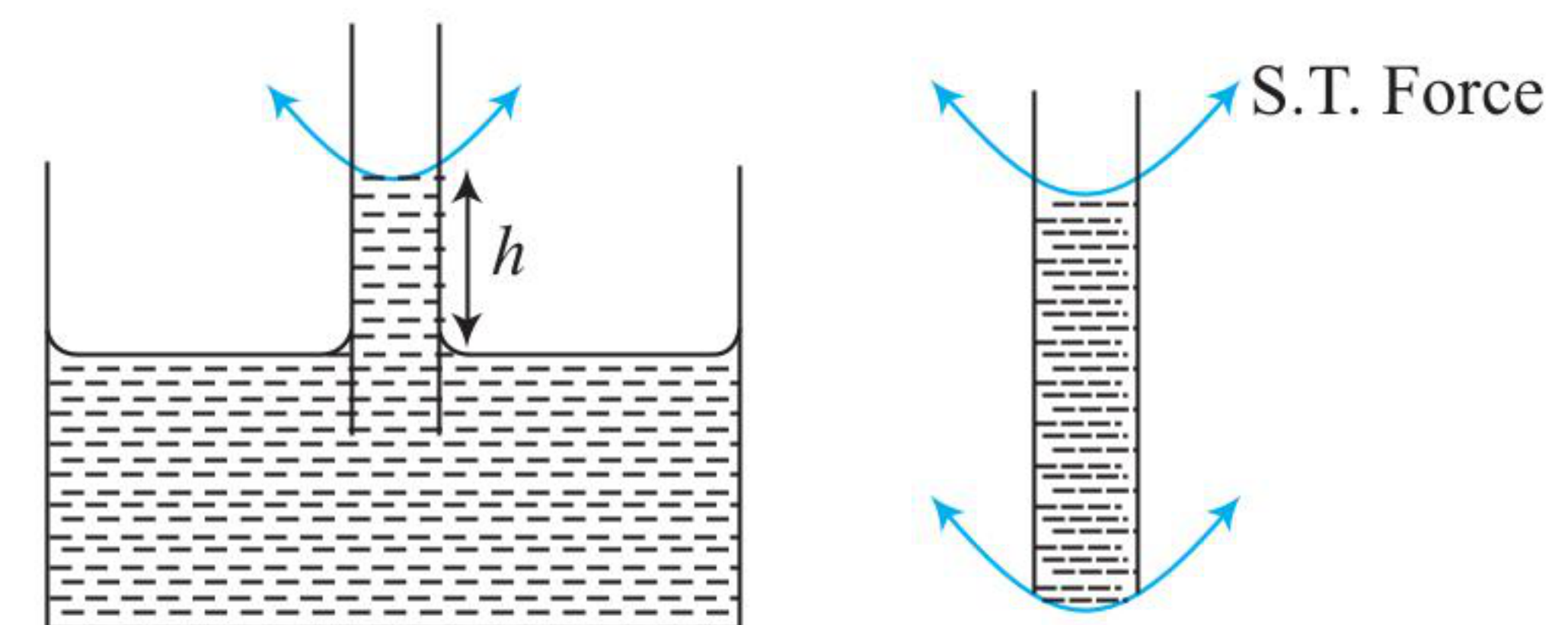
$$F_0 = k(A_1 + A_2) \quad \dots (i)$$

and $T = kA_1$... (ii)

Dividing Eq. (ii) by Eq. (i) we get,

$$\frac{T}{F_0} = \frac{A_1}{A_1 + A_2} \therefore T = \frac{F_0 A_1}{A_1 + A_2}$$

60. (1) When the capillary is inside the liquid, the surface tension force supports the weight of liquid of height 'h'.



When the capillary is taken out from the liquid similar type of surface tension force acts at the bottom also, as shown in second figure. Hence, now it can support weight of a liquid of height $2h$.

61. (2) Let L be the width of plates (perpendicular to paper inwards).

Surface tension force in upward direction = weight of liquid of height h

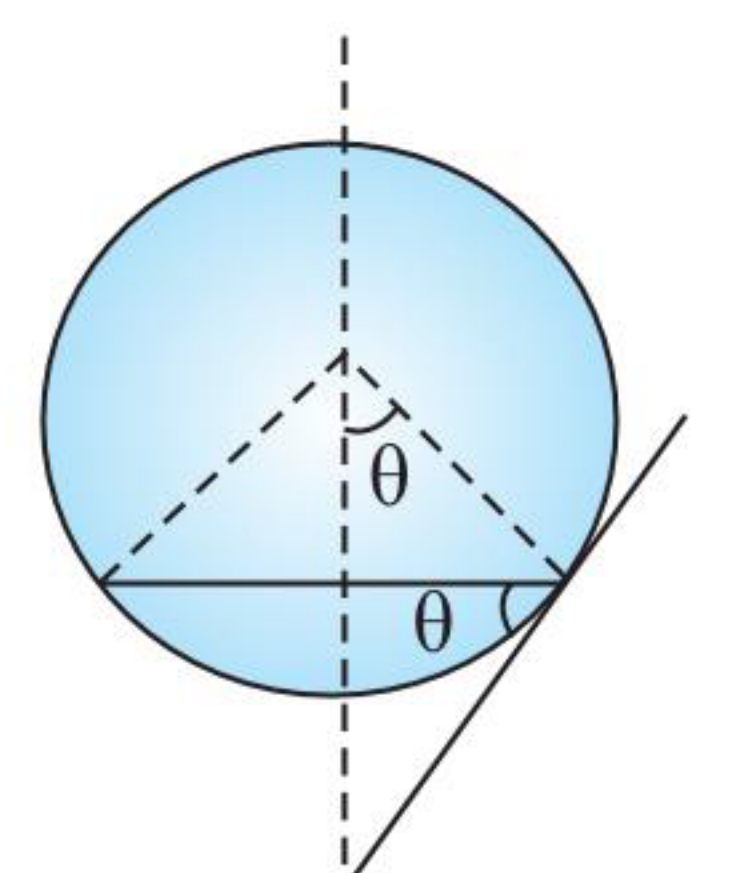
$$\therefore Tl \cos \theta = Vdg \quad (V = \text{volume})$$

$$\text{or } T(2L) \cos \theta = (Lxh)dg$$

$$h = \frac{2T \cos \theta}{xdg}$$

62. (2) $\cos \theta = \frac{R-h}{R}$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{R-h}{R} \right)$$



Multiple Correct Answers Type

1. (1), (2), (3), (4)

- Excess pressure inside a spherical liquid drop = $2T/R$, because there is only one free surface here
- In case of spherical meniscus of radius of curvature R , also excess pressure = $2T/R$ because again there is only one free surface.
- Excess pressure inside a cylindrical drop of radius $R = T/R$. Hence, for a cylindrical bubble in air, excess pressure is $2T/R$, because there exist two free surfaces in this case.
- For a spherical bubble in water, excess pressure = T/R , as there is only one free surface.

Hence, all the four options are correct.

2. (1), (2)

$$h = \frac{T}{Rdg}$$

Given that h , T , d and g are fixed, hence R must be same.

As weight of the liquid is balanced by force due to surface tension ($T \cos \theta$ and R are fixed for a given liquid, given material of capillary at a constant temperature) hence, weights of liquids in both the capillaries must be equal

So, options (1) and (2) are correct.

3. (1), (3)

Viscous force is somewhat like a frictional force but not exactly the same because it differs from it due to the two main following statements:

- Viscous force is velocity dependent while frictional force is not.
- Viscous force is temperature dependent while frictional force is not.

So, options (1) and (3) are correct.

4. (3), (4)

The angle of contact at the free liquid surface inside the capillary tube will change such that the vertical component of the surface tension forces just balances the weight of the liquid column.

5. (1), (2), (3)

The two free liquid surfaces must provide a net upward force due to surface tension to balance the weight of liquid column.

6. (1), (3)

Let S = surface tension = surface energy per unit area; r = radius of each small drop; and R = radius of a single drop

$$n \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3 \quad \text{or} \quad R = rn^{1/3}$$

Initial surface energy, $E_i = n \times 4\pi r^2 \times S = nE$

Final surface energy, $E_f = 4\pi R^2 S = 4\pi r^2 n^{2/3} S = n^{2/3} E$

Therefore, energy released = $E_i - E_f = E(n - n^{2/3})$

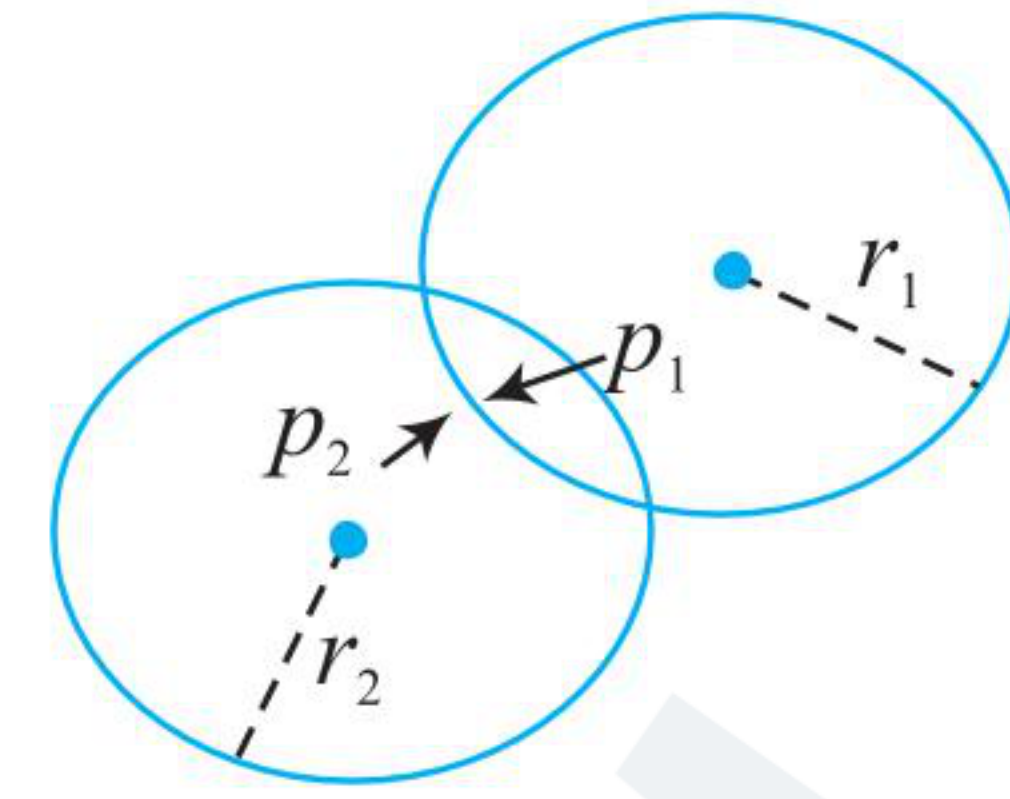
7. (1), (4)

Volume becomes eight times, so radius r becomes double. We know that terminal velocity: $v_t \propto r^2$, hence terminal velocity becomes four times. And drag force: $F \propto rv_t$, hence drag becomes eight times

8. (1), (2), (3)

Acceleration will be less than $g \left(\frac{\rho - \sigma}{\rho} \right)$. Time will depend on density ρ . Viscous force may be maximum or minimum both are possible.

9. (1), (3)



Excess pressure is always present on concave side.

So, interface is concave towards small bubble.

$$p_1 = p_0 + \frac{4T}{r_1}$$

$$p_2 = p_0 + \frac{4T}{r_2}$$

$$\text{Excess pressure} = p_1 - p_2 = 4T \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

This excess pressure for the interface is also equal to $\frac{4T}{R}$.

$$\therefore 4T \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = \frac{4T}{R}$$

$$R = \frac{r_1 r_2}{r_2 - r_1} = \frac{(0.002)(0.004)}{0.004 - 0.002} = 0.004 \text{ m}$$

10. (1), (2), (3)

Magnitude of viscous force, $F = \eta A \frac{dv}{dr}$

$$\Rightarrow \text{Viscous force per unit area } \sigma = \frac{F}{A} = \eta \frac{dv}{dr}$$

$$v = v_0 \left(1 - \frac{r^2}{R^2} \right)$$

$$\frac{dv}{dr} = -\frac{2v_0 r}{R^2} \Rightarrow \sigma = \eta \cdot \frac{2v_0 r}{R^2} \quad \dots(i)$$

Consider an annular element at r from axis, width dr .

$$dA = 2\pi r dr; \quad dQ = v \cdot dA = v_0 \left(1 - \frac{r^2}{R^2} \right) 2\pi r dr$$

$$Q = \int dQ = 2\pi v_0 \left[\frac{r^2}{2} - \frac{r^4}{4R^2} \right]_0^R = \frac{\pi}{2} R^2 v_0$$

$$\Rightarrow v_0 = \frac{2Q}{\pi R^2}$$

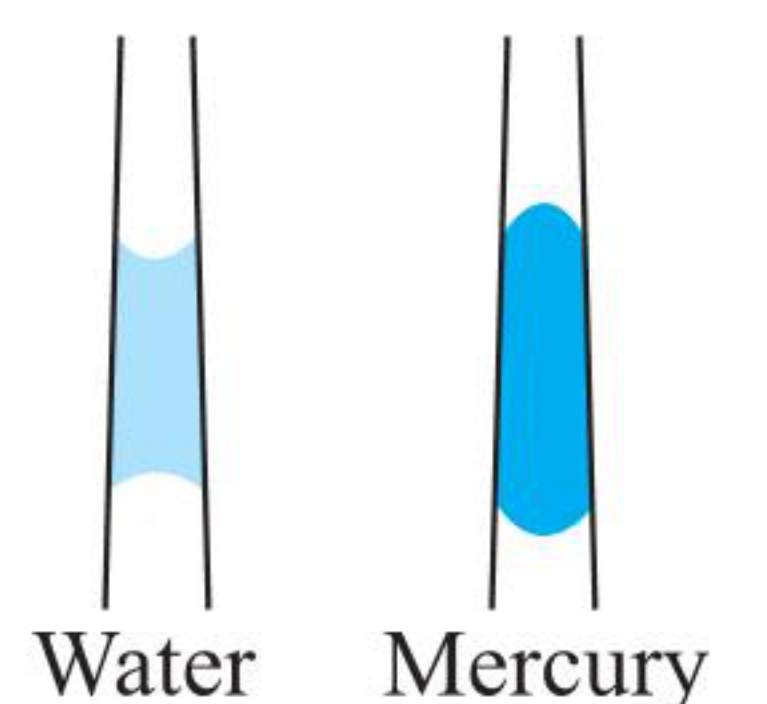
$$\therefore \sigma = \eta \frac{4Q}{\pi R^4} r, \quad R = 0.1 \text{ m} \quad (\text{from Eq. (i)})$$

$$\text{At } r = 0.04 \text{ m}, \quad \sigma = (0.75) \times 4 \times \frac{\pi}{2} \times 10^{-2} \times \frac{0.04}{\pi \times 10^{-4}} = 6 \text{ Nm}^{-2}$$

11. (1), (3)

$\Delta P = \frac{2\sigma}{r}$, where σ is surface tension.

Pressure difference is reciprocal to radius of free surface. Check the movement of liquid by pressure difference.



Linked Comprehension Type

For Problems 1–2

1. (4) If ρ_w is the density of water, then the density of the lead sphere and the density of the liquid are given by

$$\rho_0 = 11.20 \rho_w \quad \text{and} \quad \rho = 1.26 \rho_w$$

Thus, the equation $T = 2\pi \sqrt{\frac{l}{2g}}$ may be written as

$$\eta = \frac{2}{9} \frac{r^2}{v} (11.20 - 1.26) \rho_w g$$

Here, $r = 0.5 \times 10^{-3} \text{ m}$; $v = 0.7 \times 10^{-2} \text{ m s}^{-1}$; $g = 9.8 \text{ m s}^{-2}$; and $\rho_w = 10^3 \text{ kg/m}^3$.

$$\eta = \frac{2}{9} \frac{(0.5 \times 10^{-3})^2}{0.7 \times 10^{-2}} (11.20 - 1.26) (10^3) (9.8) = 0.77 \text{ N s/m}^2$$

2. (1) Reynolds number, $R_e = \frac{\rho v d}{\eta}$

$$R_e = \frac{(1.26 \times 10^3)(0.7 \times 10^{-2})(1 \times 10^{-3})}{0.77}$$

$$R_e = 0.01 < 0.20 \text{ (maximum limit of Stokes' law).}$$

For Problems 3–4

3. (3) Here $s = 7.0 \times 10^{-2} \text{ N/m}$; $r = 0.2 \times 10^{-3} \text{ m}$; $\rho = 10^3 \text{ kg/m}^3$

We know that the height of liquid rise in the tube is

$$h = 2\sigma \cos \theta / \rho R g$$

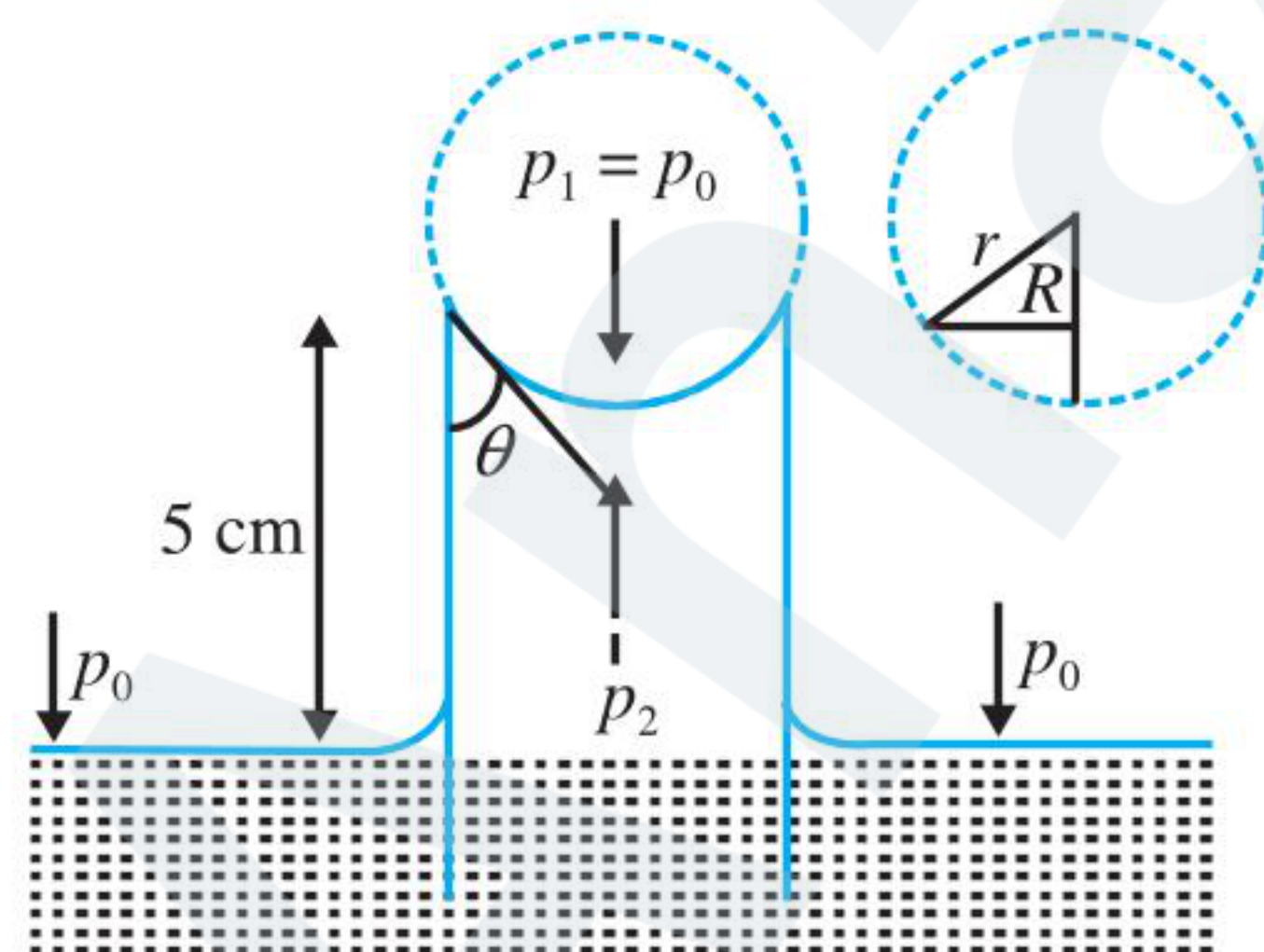
where R is the radius of the meniscus, and θ is the angle of contact.

Since $\theta = 0$ (given), radius of the meniscus is equal to the radius of the capillary tube, i.e., $R = r$.

$$\therefore h = \frac{2(7.0 \times 10^{-2})}{(10^3)(0.2 \times 10^{-3})(10)} = 0.07 \text{ m}$$

$$\text{or } h = 7.0 \text{ cm}$$

4. (2) When the length of the capillary tube above the free surface of the liquid is less than the height of liquid that rise in the tube, radius R of the free surface is not equal to the radius of the tube. It is greater than r as the surface tends to be flatter.



According to the equation, $p_1 - p_2 = (4\sigma/R)$, the pressure difference across the surface is given by

$$\Delta p = \frac{2\sigma \cos \theta}{R}$$

If p_1 and p_2 are the pressure just above and below the meniscus, respectively, then $p_1 - p_2 = \rho g h_0$

$$\therefore \rho g h_0 = \frac{2\sigma}{R} \quad \dots(i)$$

In part (a) we have seen that when $h_0 = h$; $\theta = 0$, $R = r$.

$$\text{and } \rho g h = \frac{2\sigma}{r} \quad \dots(ii)$$

Therefore, dividing Eq. (i) by Eq. (ii), we have $\frac{r}{R} = \frac{h_0}{h}$

From the figure, it is clear that $\cos \theta = \frac{r}{R}$

Therefore, the angle of contact is

$$\theta = \cos^{-1} \left(\frac{h_0}{h} \right) = \cos^{-1} \left(\frac{5}{7} \right) \approx 44^\circ$$

For Problems 5–7

5. (3) Using Poiseuille's equation, we get

$$Q = \frac{(p_1 - p_2) \pi D^4}{128 \eta L}$$

Here, $p_1 - p_2 = 38.4 \times 10^3 \text{ N/m}^2$

$D = 2.5 \text{ cm} = 2.5 \times 10^{-2} \text{ m}$

$L = 30 \text{ m}$; $h = 0.12 \text{ N/m}^2$

$$\therefore Q = \frac{(38.4 \times 10^3)(3.14)(2.5 \times 10^{-2})^4}{128(0.12)(30)} = 1.0 \times 10^{-4} \text{ m}^3/\text{s}$$

6. (1) We know that $\tau = -\frac{dp}{dx} \frac{r}{2}$

At the wall of the pipe, i.e., $r = R$

$$\tau_{\max} = -\frac{dp}{dx} \frac{R}{2} = \left(\frac{p_1 - p_2}{L} \right) \frac{D}{4}$$

$$= \left(\frac{38.4 \times 10^3}{30} \right) \frac{2.5 \times 10^{-2}}{4} = 8 \times 10^{-6} \text{ N/m}^2$$

7. (2) We know that $P = \frac{128 \eta Q^2 L}{\pi D^4} = (p_1 - p_2) Q$

$$\therefore P = (38.4 \times 10^3)(1 \times 10^{-4}) \quad \text{or} \quad P = 3.84 \text{ W}$$

For Problems 8–10

8. (1) Excess pressure is always on the concave side of the surface. For angle of contact of $2\pi/3$, the liquid should have a convex surface. So, the excess pressure should be in the upward direction.
9. (1) In soap bubble, there will be two free surfaces. So, excess pressure will be double than that in an air bubble. So, air bubble is $p/2$.
10. (3) Difference in pressure between the inner and outer surface is $(p_i - p_0)$.

$$\therefore \text{Force} = (p_i - p_0) \pi r^2$$

Since excess pressure is given, $p = (p_i - p_0)$

$$\therefore F = p \pi r^2$$

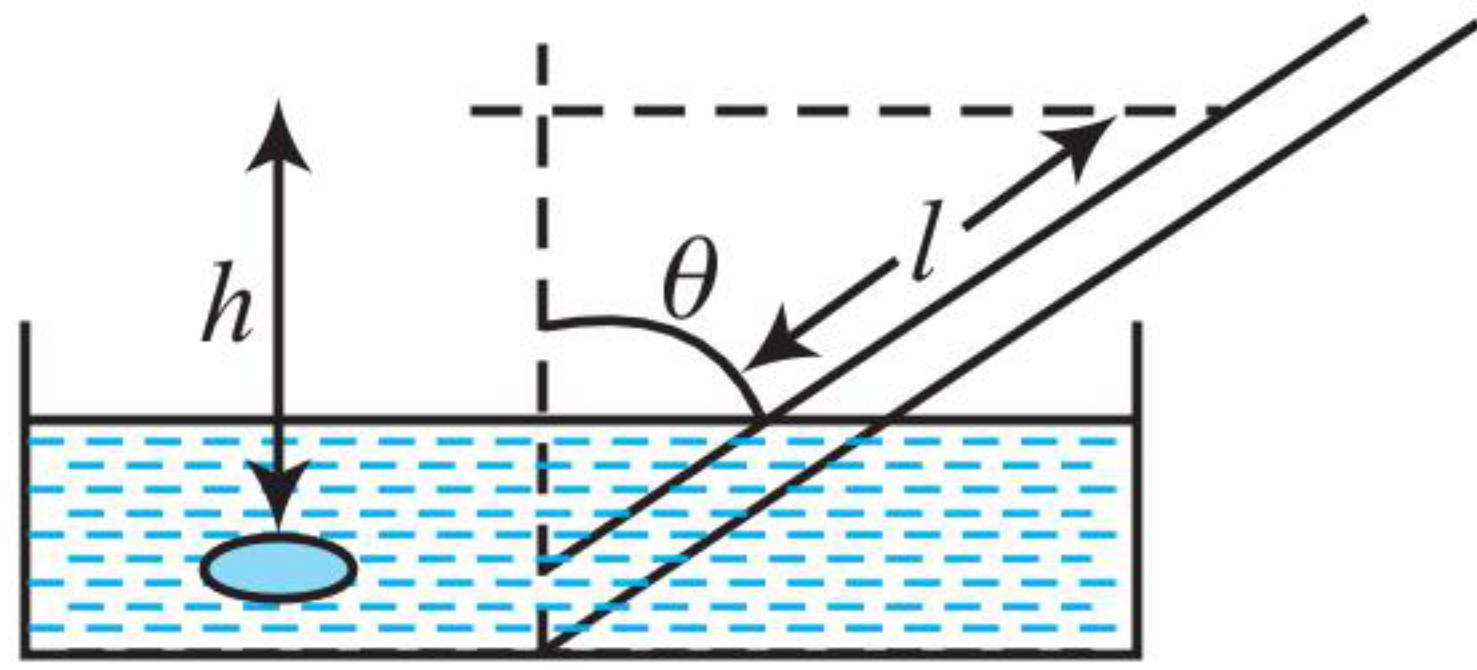
So option (3) is correct and the rest are incorrect.

For Problems 11–13

11. (3) $P_A = P_0 - \frac{2s}{r}$

The pressure inside a concave meniscus is less than the pressure outside (atmospheric). Assuming the meniscus to be spherical (as for thin capillaries), excess pressure is $2s/r$ where r is the radius of the hemispherical surface.

12. (2) When the capillary tube is tilted by an angle θ with the vertical, the capillary rise to distance l in the tube will be such that the meniscus will remain at the same vertical height above the level of water in the container.



Hence, $h = l \cos \theta$

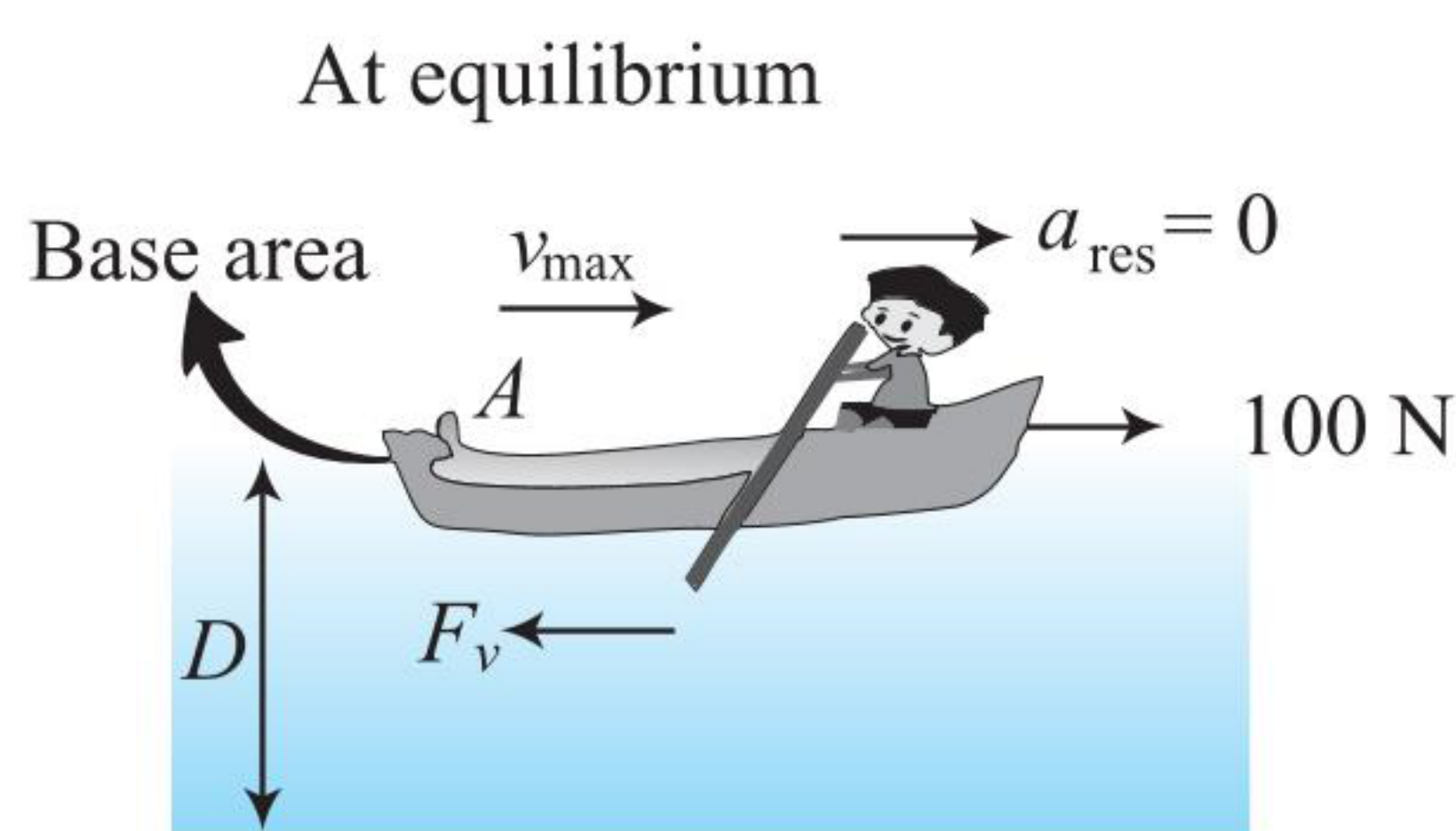
$$l = \frac{h}{\cos \theta} = \frac{10}{\cos 45^\circ} = 10\sqrt{2} \text{ cm}$$

13. (3) The capillary rise h is given by $h = \frac{2S \cos \theta}{\rho g r}$

where $hr = \text{constant}$ for a given glass and liquid. This is shown correctly in (3).

For Problems 14-15

14. (4) At equilibrium

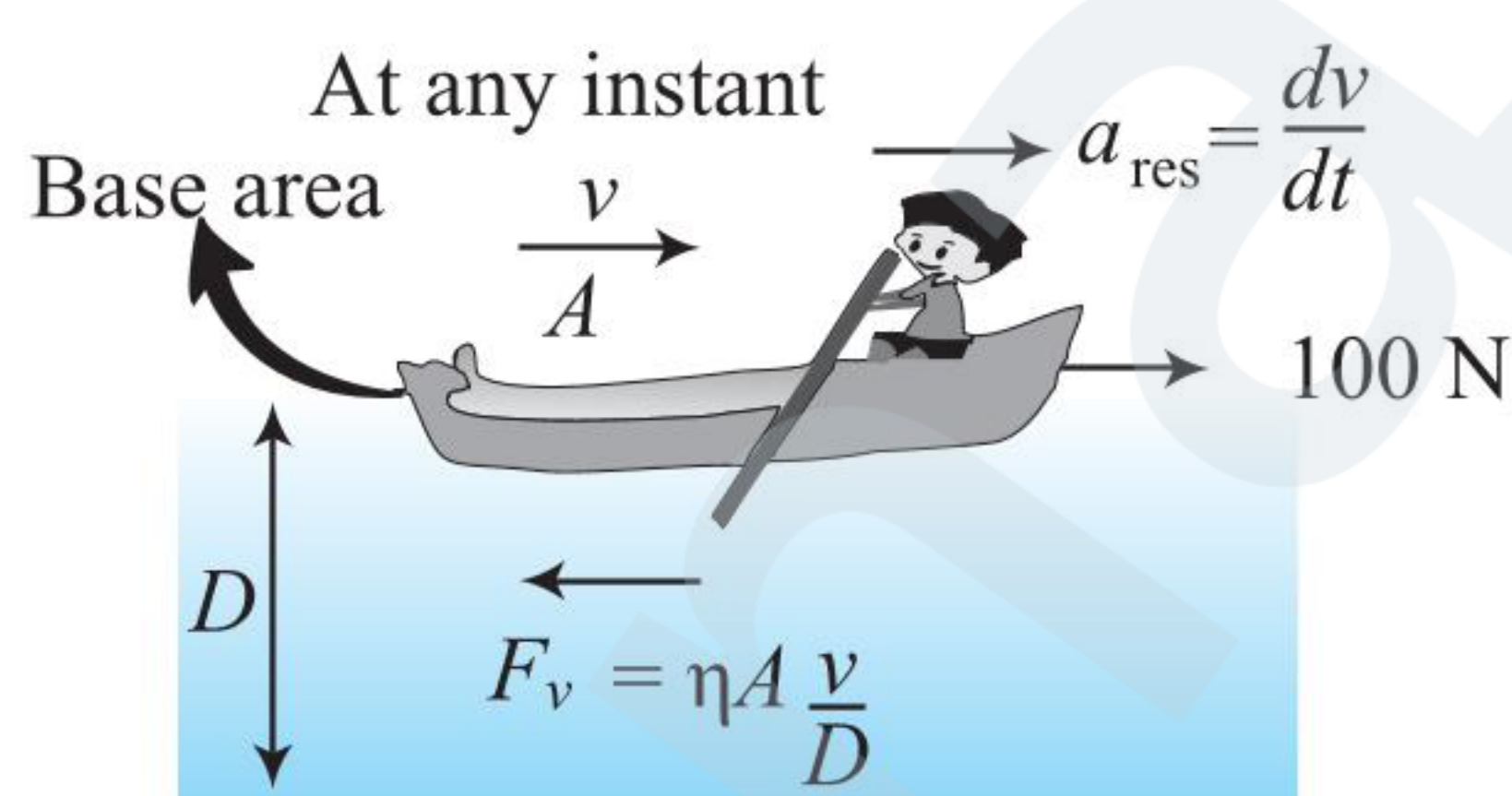


$$F_{\text{ext}} = F_v$$

$$F_{\text{ext}} = 100 \text{ N and } F_v = \eta A \frac{v_{\text{max}}}{D} = 1.5 \times 10 \times \frac{v_{\text{max}}}{D}$$

$$100 = 1.5 v_{\text{max}} \Rightarrow v_{\text{max}} = \frac{200}{3} \text{ m/s}$$

15. (3) At any instant



$$F_{\text{ext}} - F_v = m \frac{dv}{dt} \Rightarrow 100 - \eta A \frac{v}{D} = m \frac{dv}{dt}$$

$$100 - 1.5 v = 150 \frac{dv}{dt}$$

$$\int_0^t dt = \int_0^{v_{\text{max}}} 150 \frac{dv}{100 - 1.5v}$$

$$t = 150 \frac{\ln(100 - 1.5v)}{-1.5} \Big|_0^{100/2}$$

$$t = 100 \ln 2 \text{ seconds}$$

Matrix Match Type

1. i. $\rightarrow$ b., c.; ii. $\rightarrow$ a., d.; iii. $\rightarrow$ b., c.

For A and C: In this case, the final surface area is greater than the initial surface area, and hence surface energy increases the expanse of internal energy; hence it causes cooling.

For B, reverse of the above reasoning.

2. i. $\rightarrow$ b.; ii. $\rightarrow$ c.; iii. $\rightarrow$ d.

The forces acting on the plate are weight of the plate vertically downwards

$$W = mg = 20 \text{ gf}$$

- i. Buoyant force due to liquid,

$$F_b = V \rho_l g = \left(\frac{lb t}{2} \right) \rho_l g = \left(\frac{10 \times 4 \times 0.4}{2} \right) \times 1 \text{ gf} = 8 \text{ gf}$$

- ii. Force due to surface tension, vertically downward

$$F = T \times \text{perimeter of plate in contact with water}$$

$$= \frac{70}{980} \times 2(10 + 0.4) \text{ gf} = 1.5 \text{ gf}$$

- iii. Apparent weight of the plate $= (W + F) - F$

$$= (20 + 1.5) - 8 = 13.5 \text{ gf}$$

3. (1) (i) Stoke's law, $F = 6\pi \eta r v$

$$(ii) \text{ Terminal velocity, } v_T = \frac{2}{9} \cdot \frac{r^2 g (\rho_s - \rho_L)}{\eta}$$

$$(iii) \text{ Excess pressure inside a mercury drop is } \Delta p = \frac{2T}{r}$$

$$(iv) \text{ Viscous force is } F_v = -\eta A \frac{dv}{dy}$$

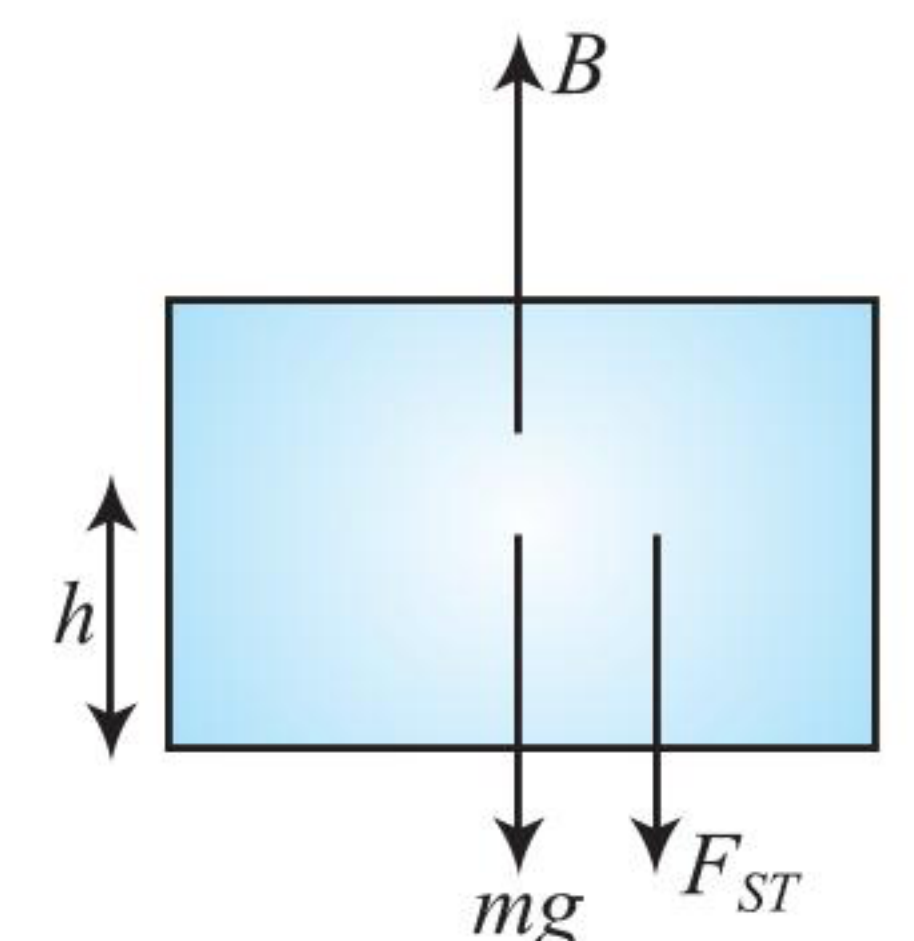
Numerical Value Type

1. (2) $Mg + F_{ST} = B$

$$mg + 4aT = a^2 h \rho_w g$$

$$10 + 4 \times \frac{10}{4} = 10h$$

$$h = 2 \text{ m}$$



2. (5) Case I: $y_1 = y_2 = h/2$

$$\frac{\partial u}{\partial y} = \frac{v}{h/2} = \frac{2v}{h}$$

$$\text{Viscous force} = \frac{\eta_0 2v}{h} A$$

The plate has two sides.

$$\text{Total force } F = \frac{4\eta_0 v A}{h}$$

$$\text{Case II: } F = \frac{\eta v A}{y} + \frac{\eta v A}{h-y} = \frac{\eta v A h}{y(h-y)}$$

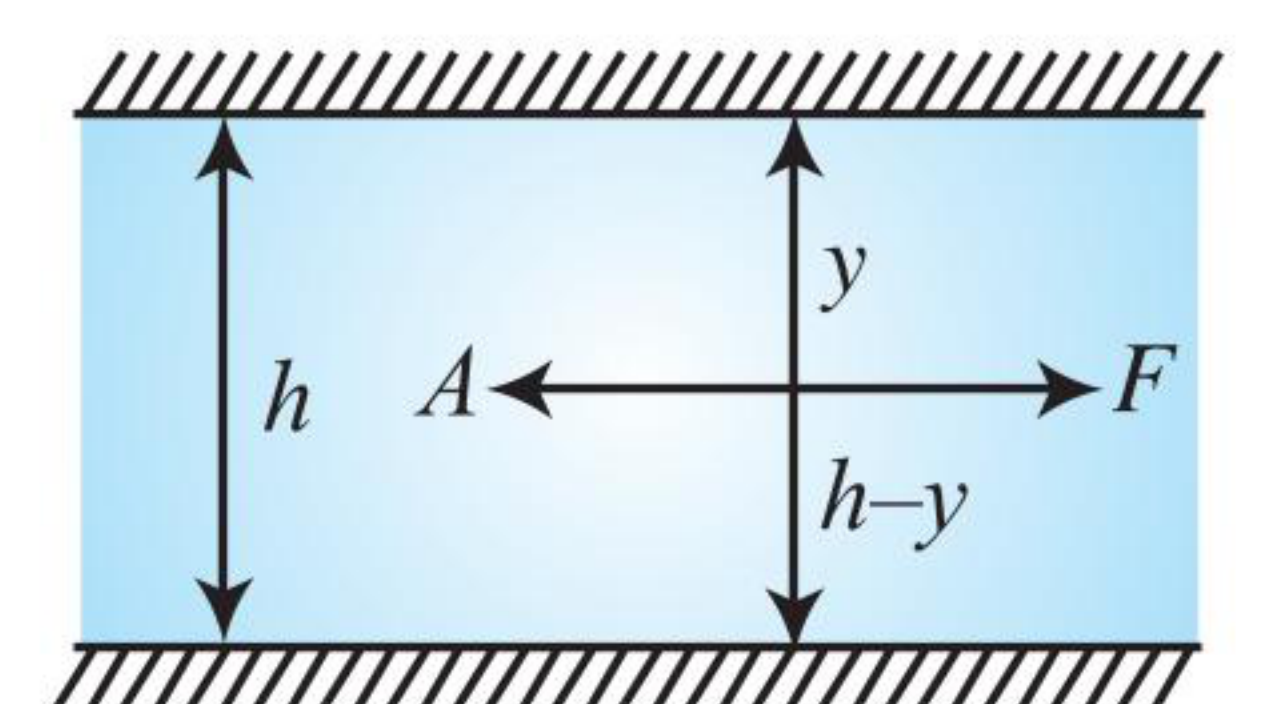
$$\therefore \frac{\eta v A h}{y(h-y)} = \frac{4\eta_0 v A}{h}$$

$$\frac{3}{16} h^2 = y(h-y)$$

$$\left(\because \eta = \frac{3}{4} \eta_0 \right)$$

$$\text{Solving, we get } y = \frac{h}{4}, \frac{3h}{4}$$

Hence, for minimum distance from top $y = 20/4 = 5 \text{ cm}$



3. (5) We have $pV = mRT$

$$\left(p_0 + \rho_w gh - \frac{4T}{d}\right) \frac{\pi}{6} d^3 = \left(p_0 + \frac{4T}{d} d\right) \frac{\pi}{6} n^3 d^3$$

(the bubble has only one surface)

$$\therefore h = \frac{d(n^3 - 1)p_0 + 4T(n^2 - 1)}{\rho_w g d}$$

$$\text{or } h = \frac{4 \times 10^{-6} (1 \times 1^3 - 1) \times 10^5 + 4 \times 0.07 (1 \times 1^2 - 1)}{1000 \times 9.8 \times 4 \times 10^{-6}} = 5 \text{ m}$$

$$4. (8) \frac{4}{3} \pi R^3 = n \times \frac{4}{3} \pi r^3$$

$$\text{or } R^3 = nr^3 \quad \text{or } R = n^{1/3} r$$

$$\text{or } R = 2n^{1/3} \text{ mm}$$

$$v_0 \propto r^2, v_0' \propto R^2$$

$$\text{Now, } \frac{v_0'}{v_0} = \frac{R^2}{r^2} = \frac{4n^{2/3}}{4}$$

$$\text{or } \frac{32}{8} = n^{2/3} \quad \text{or } n^{2/3} = 4$$

$$\text{or } n = 4^{3/2} = \sqrt{64}$$

$$\text{or } n = 8$$

5. (5) $Mg = PA$; A is sectional area

$$A \rho g = 10^5 A$$

$$l = \frac{10^5}{2 \times 10^3 \times 10} = 5 \text{ m}$$

$$6. (8.00) \frac{\pi r^4 P_1}{8 \eta l} = \frac{\pi (2r)^4 P_2}{8 \eta (2l)}$$

$$\Rightarrow \frac{P_1}{P_2} = 8$$

7. (4.00) $D = 2r = 2 \times 2 \times 10^{-3} \text{ m} = 4 \times 10^{-3} \text{ m}$

For laminar flow with maximum velocity. Reynold's number is

$$N_R = 2000$$

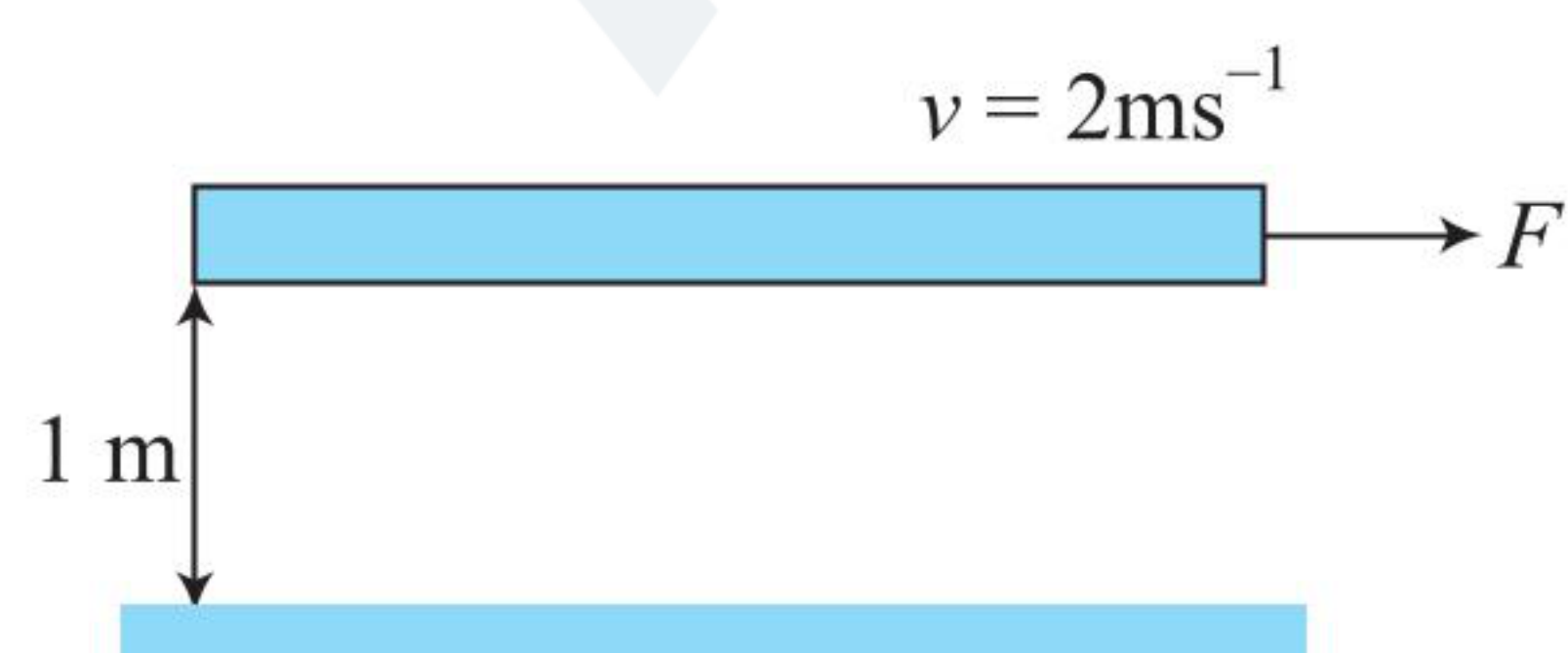
$$N_R = \frac{\rho v D}{\eta}$$

$$\Rightarrow v = \frac{N_R \eta}{\rho D} = \frac{(2000)(2.12 \times 10^{-3})}{(1.06 \times 10^3)(4 \times 10^{-3})} = 1 \text{ ms}^{-1}$$

Corresponding flow rate is $Q = Av = (\pi r^2)v$

$$Q = \pi (2 \times 10^{-3})^2 \times 1 = 4\pi \times 10^{-6} \text{ m}^3 \text{ s}^{-1}$$

8. (0.004) Velocity gradient $= \frac{\Delta v}{\Delta y} = \frac{2-0}{1-0} = 2 \text{ ms}^{-1} \text{ m}^{-1}$



Viscous force,

$$|F| = \eta A \frac{\Delta v}{\Delta y} = (0.01 \times 10^{-1})(2)(2) = 0.004 \text{ N}$$

So, to keep the plate moving with constant velocity, a force of $4 \times 10^{-3} \text{ N}$ has to be applied.

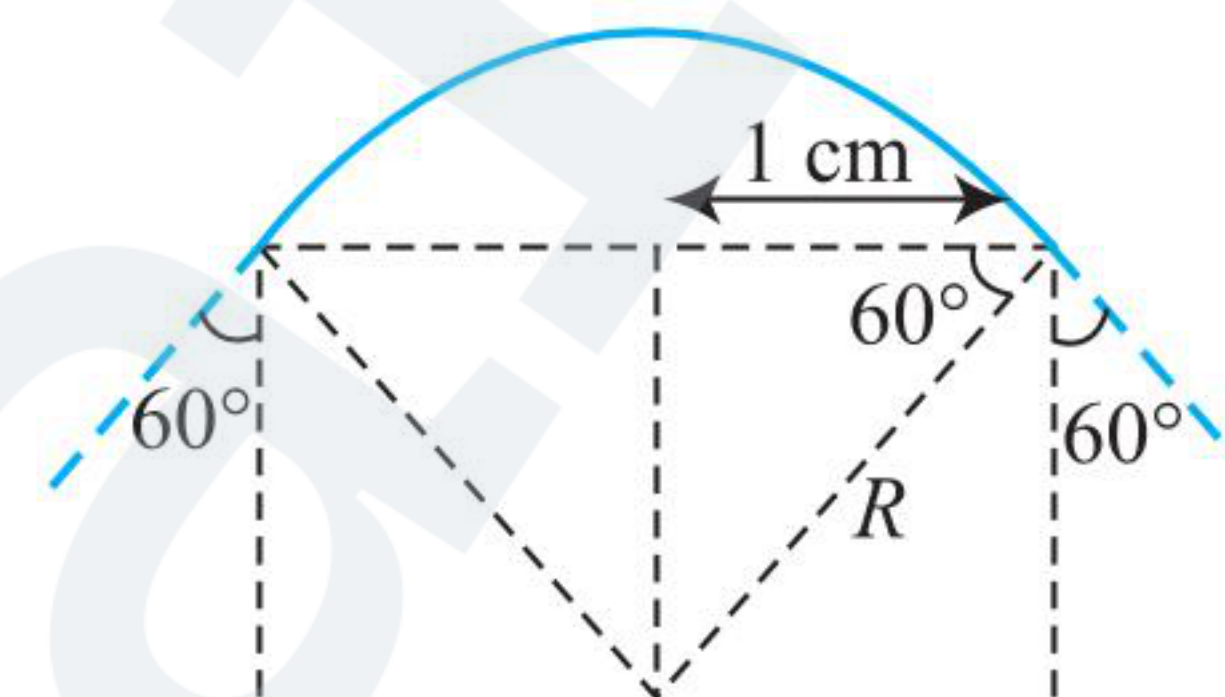
9. (25) Velocity gradient $= \frac{\Delta v}{\Delta y} = \frac{0.5-0}{1.25 \times 10^{-2}} = 40 \text{ ms}^{-1} \text{ m}^{-1}$

$$F = 2 \left[\eta A \frac{\Delta v}{\Delta y} \right] \Rightarrow 1 = 2 \times \eta \times 0.5 \times 40$$

$$\Rightarrow \eta = \frac{1}{40} \text{ kg-sm}^{-2}$$

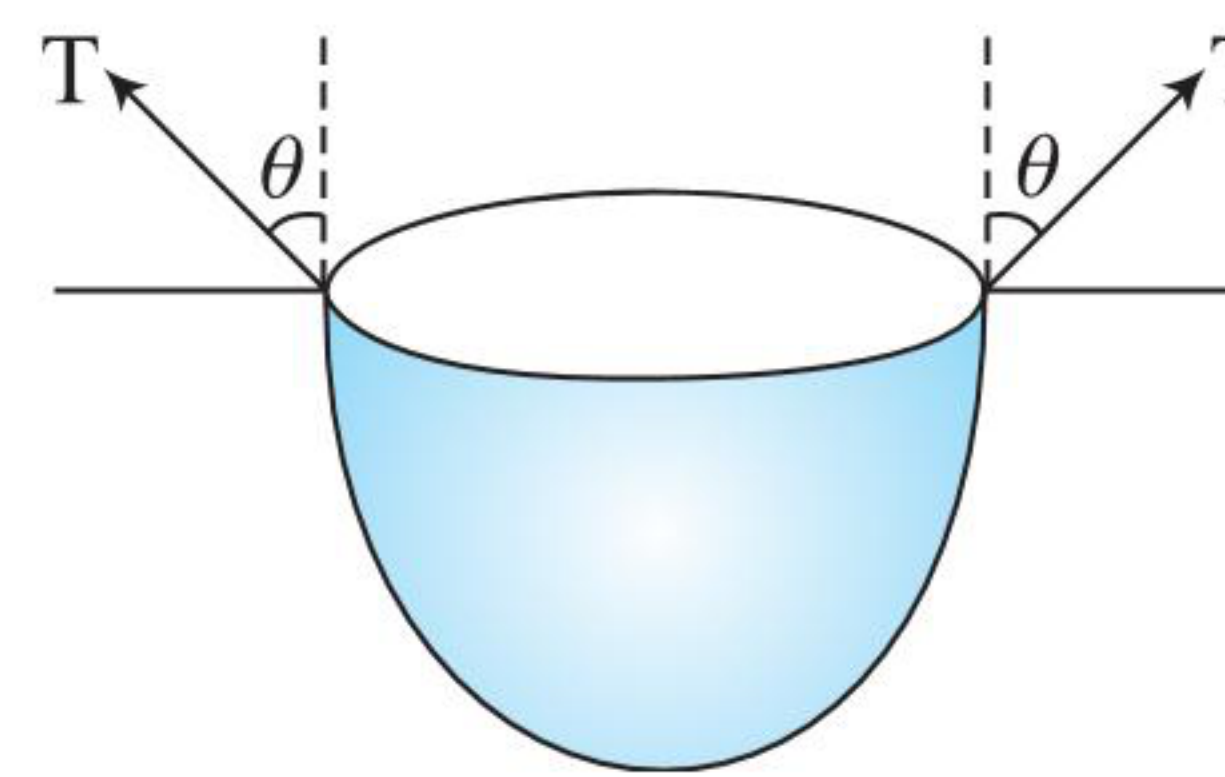
$$\text{or } \eta = 0.25 \times 10^{-1} = 25 \times 10^{-3} \text{ kg-sm}^{-2}$$

10. (10) $\cos 60^\circ = \frac{1}{R} \Rightarrow R = 2 \text{ cm}$



$$\Delta p = \frac{4T}{R} = \frac{4(0.05)}{0.02} = 10 \text{ Pa}$$

11. (3.00) The vertical force due to surface at the hole



$$= T \cos \theta \times L = T \cos \theta \times 2\pi r$$

This will balance weight $mg (= \pi r^2 h \rho g)$

$$\therefore T \cos \theta \times 2\pi r = \pi r^2 h \rho g$$

$$\Rightarrow h = \frac{2T \cos \theta}{r \rho g}$$

h is maximum, when $\cos \theta = 1$

$$\therefore h_{\max} = \frac{2T}{r \rho g} = \frac{2 \times 0.075}{0.0005 \times 1000 \times 10} = 0.03 \text{ m} = 3 \text{ cm}$$

12. (4.00) Mass of air entering in 1s is $m = \pi b^2 v \rho$

$$\text{Change in momentum in 1s} = mv - 0 = \pi b^2 \rho v^2$$

$$\text{Thus, } F = \pi b^2 \rho v^2$$

$$\text{Pressure due to this force is } \Delta p = \frac{F}{A} = \frac{\pi b^2 \rho v^2}{\pi b^2} = \rho v^2$$

When this excess pressure equals to $\frac{4T}{R}$, the bubble gets separated.

$$\therefore \rho v^2 = \frac{4T}{R} \Rightarrow R = \frac{4T}{\rho v^2}$$

So, $\beta = 4$

13. (20) The velocity attained by the sphere in falling freely from a height h is

$$v = \sqrt{2gh} \quad \dots(i)$$

This is the terminal velocity of the sphere in water. Hence by Stokes's law, we have

$$v = \frac{2}{9} \frac{r^2 (\rho - \sigma) g}{\eta}$$

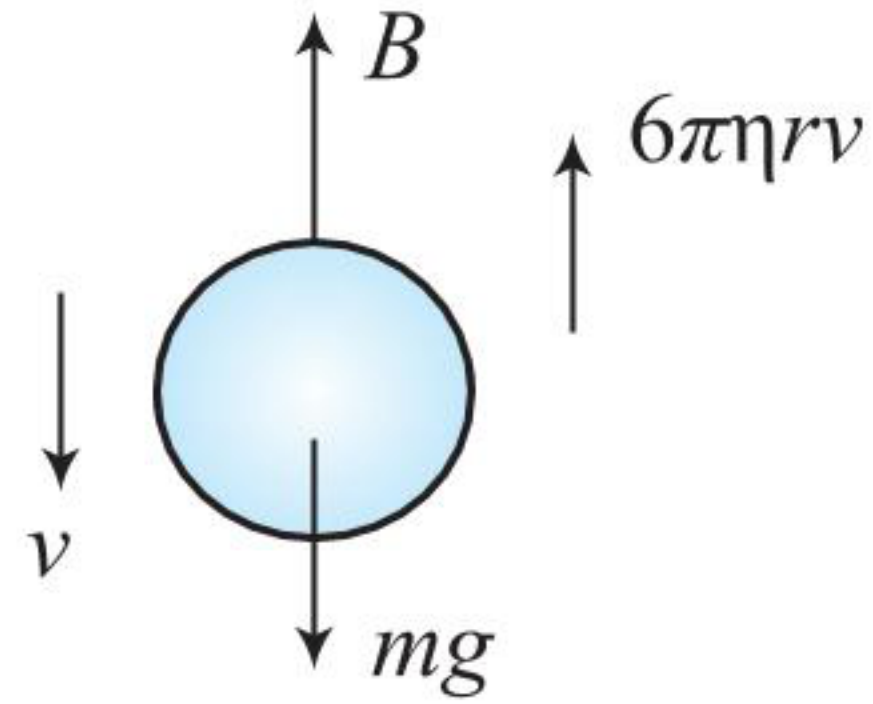
where r is the radius of the sphere, ρ is the density of the material of the sphere

$\sigma (=1.0 \times 10^3 \text{ kg/m}^3)$ is the density of water and η is coefficient of viscosity of water.

$$v = \frac{2 \times (1.0 \times 10^{-3})^2 (1.0 \times 10^4 - 1.0 \times 10^3) \times 10}{9 \times 1.0 \times 10^{-3}} = 20 \text{ m/s}$$

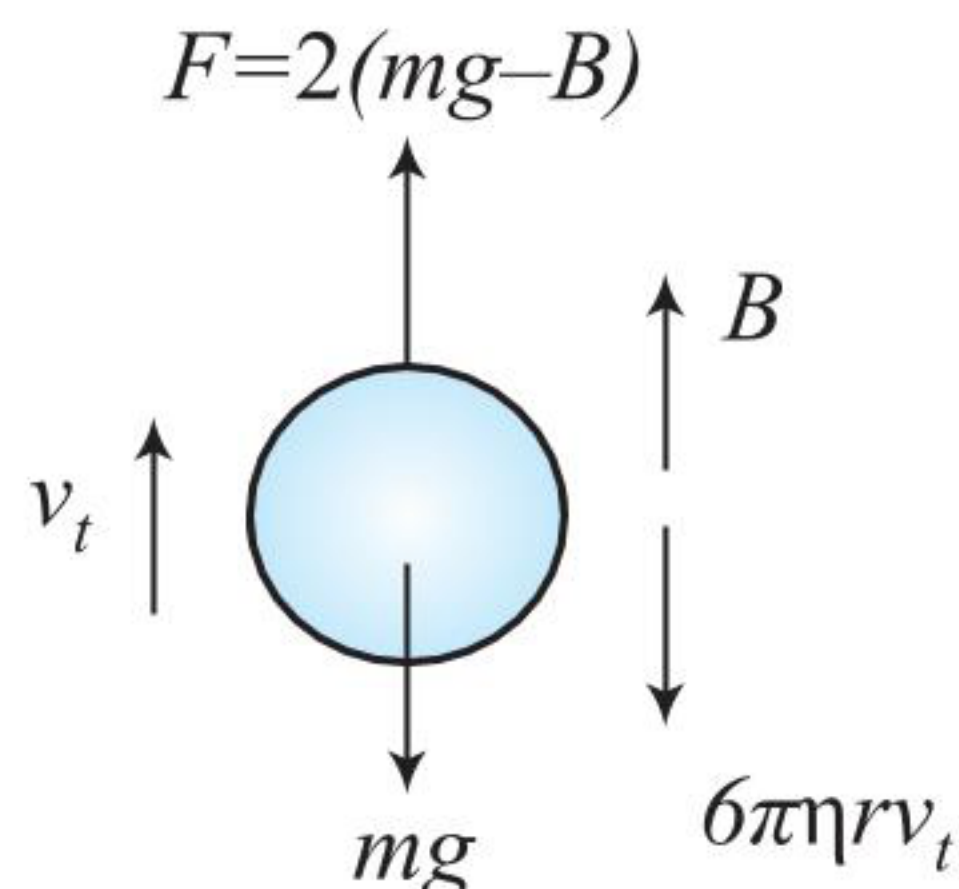
From Eq. (i), we have $h = \frac{v^2}{2g} = \frac{20 \times 20}{2 \times 10} = 20 \text{ m}$

14. (10) First case:



$$B + 6\pi\eta rv = mg \quad \dots(i)$$

For second case:



$$F + B = mg + 6\pi\eta rv_t \Rightarrow 2mg - 2B + B = mg + 6\pi\eta rv_t$$

$$\Rightarrow mg - B = 6\pi\eta rv_t \quad \dots(ii)$$

From Eqs. (i) and (ii), $v_t = v = 10 \text{ cm/s}$

15. (0.08) When the ring is about to leave the water surface, surface tension force on it is

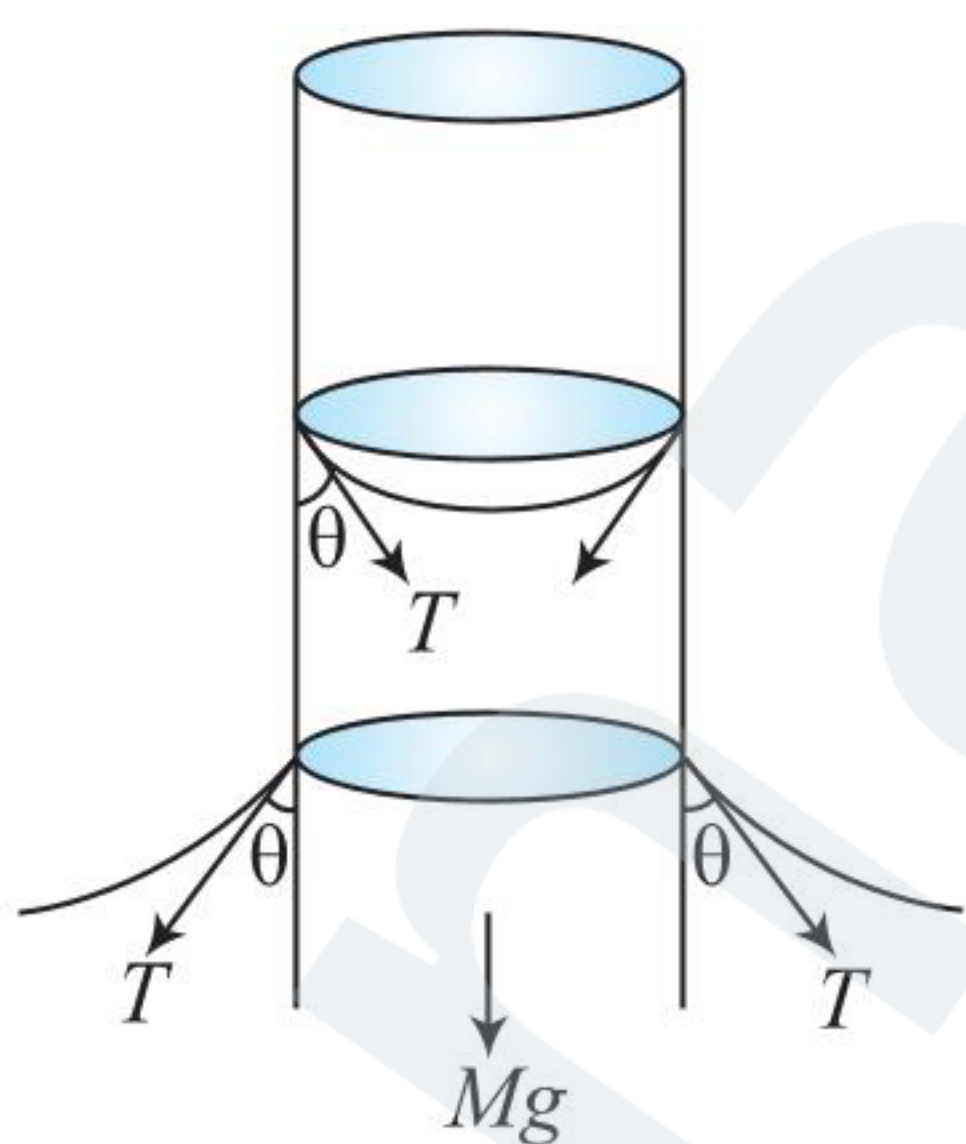
$$F_{ST} = 2\pi RT + 2\pi rT = 2\pi(R+r)T$$

Spring force $F_s = kx$

$$\therefore kx = 2\pi(R+r)T + mg$$

$$\therefore T = \frac{kx - mg}{2\pi(R+r)} = \frac{0.9 \times 4 \times 10^{-2} - 8 \times 10^{-4} \times 10}{2 \times 3.14 \times (50 + 20) \times 10^{-3}} = 0.080 \text{ Nm}^{-1}$$

16. (0.133) Force = Mg + force due to surface tension on the inner wall + force due to surface tension on the outer wall

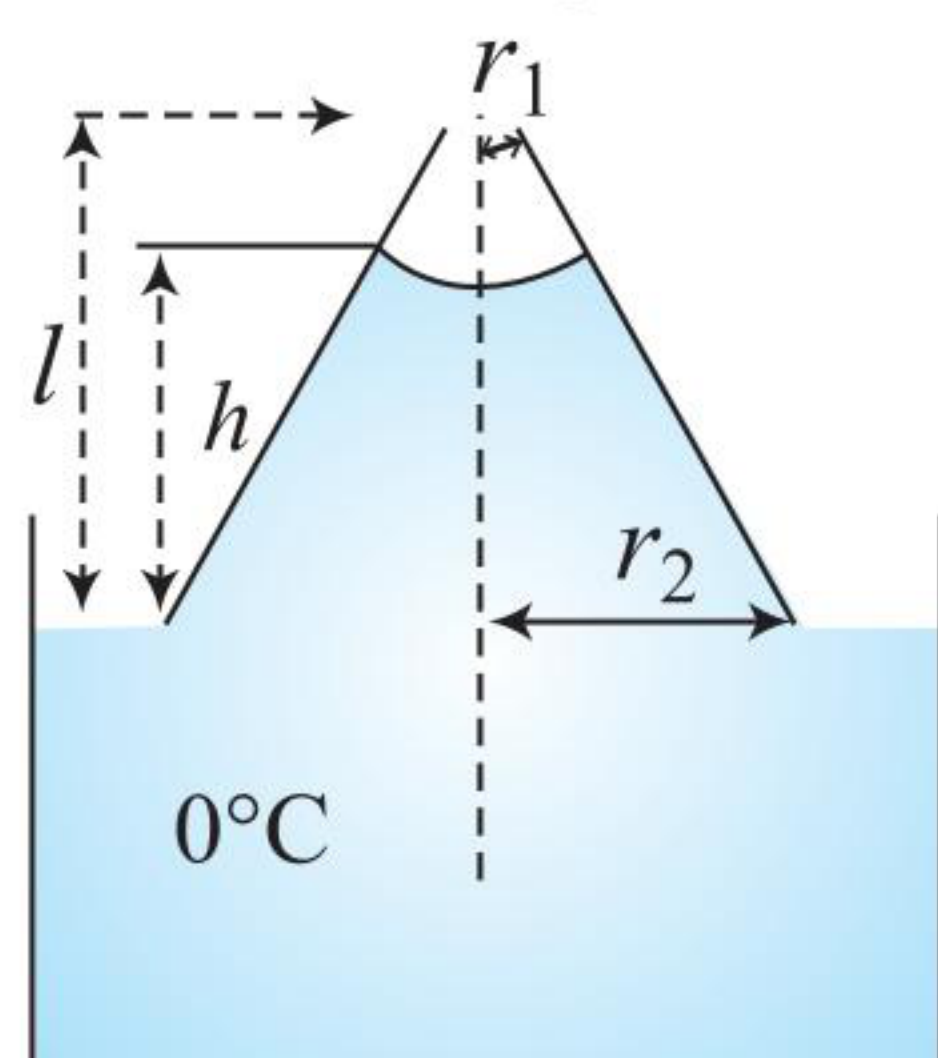


$$= Mg + 2\pi rT \cos \theta + 2\pi rT \cos \theta = Mg + 4\pi rT \cos \theta$$

$$= 10 \times 10^{-3} \times 10 + 4 \times \frac{22}{7} \times 7 \times 10^{-3} \times 0.75 \times \frac{1}{2}$$

$$= 0.133 \text{ N}$$

17. (1.4) The situation is shown in figure.



Let r_1 and r_2 be radii of upper and lower ends of the conical capillary tube. The radius r at the meniscus is given by

$$r = r_1 + (r_2 - r_1) \left(\frac{l-h}{l} \right)$$

$$= (2.5 \times 10^{-4}) + (2.5 \times 10^{-4}) \left(\frac{0.1 - 0.08}{0.1} \right) = 3.0 \times 10^{-4} \text{ m}$$

The surface tension at 0°C is given by

$$T_0 = \frac{r h \rho g}{2} = \frac{(3.0 \times 10^{-4})(8 \times 10^{-2})(1/4 \times 10^4)(9.8)}{2} = 0.084$$

For tube B: $\frac{T_0}{T_{50}} = \frac{h_0}{h_{50}} = \frac{6 \times 10^{-2}}{5.5 \times 10^{-2}} = \frac{12}{11}$

or $T_{50} = \frac{11}{12} \times T_0 = \frac{11}{12} \times 0.084 = 0.077 \text{ N/m}$

Considering that change in surface tension as linear, the change in surface tension with temperature is given by

$$\alpha = \frac{T_{50} - T_0}{50} = \frac{0.077 - 0.084}{50} = -1.4 \times 10^{-4} \frac{\text{N}}{\text{m}^\circ\text{C}}$$

18. (1130) Pressure inside the bubble = $p_0 + \rho gh + \frac{2T}{r}$

$\therefore$ Amount of pressure inside the bubble greater than the atmospheric pressure = $\rho gh + \frac{2T}{r}$

Substituting the values, we get

$$\Delta p = 10^3 \times 9.8 \times 0.1 + \frac{2 \times 0.075}{10^{-3}}$$

$$= 980 + 150 = 1130 \text{ N/m}^2 = 1130 \text{ Pa}$$

19. (20.00) When the tube is taken out, a convex meniscus is formed at the bottom end. The total upward force due to surface tension is

$$F = 2\pi rT + 2\pi rT = 4\pi rT$$

This balances the weight of water column of length H

$$\Rightarrow 4\pi rT = (\pi r^2 H) \rho g \Rightarrow H = \frac{4T}{r \rho g}$$

But $h = \frac{2T}{r \rho g}$ therefore $H = 2h = 20 \text{ cm}$

The length of the liquid column remaining = 20 cm

Archives

JEE Advanced

Single Correct Answer Type

1. (4) If R be the meniscus radius

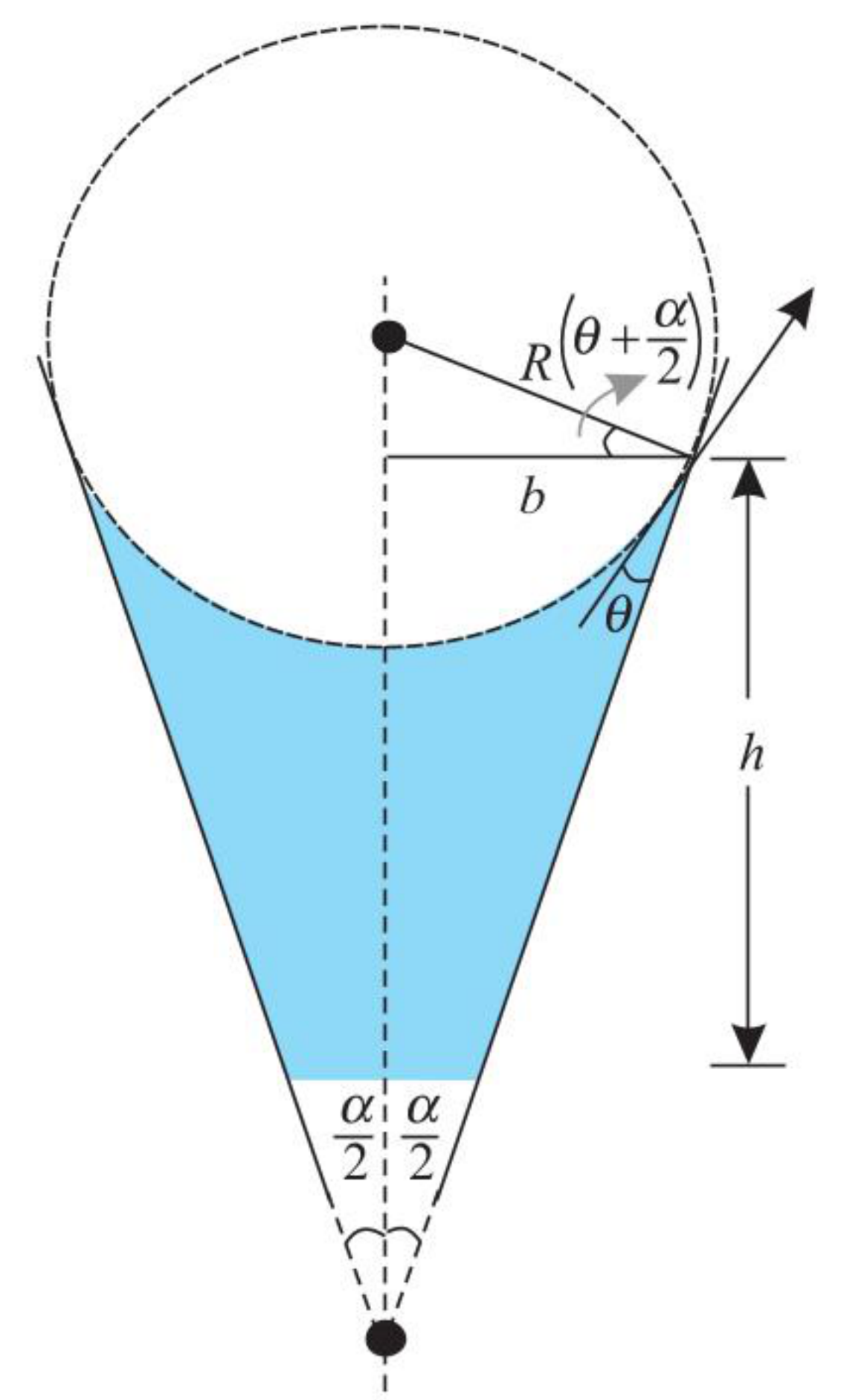
$$R \cos(\theta + \alpha/2) = b$$

Excess pressure on concave

side of meniscus = $\frac{2S}{R}$

$$h \rho g = \frac{2S}{R} = \frac{2S}{b} \cos\left(\theta + \frac{\alpha}{2}\right)$$

$$\Rightarrow h = \frac{2S}{b \rho g} \cos\left(\theta + \frac{\alpha}{2}\right)$$



2. (3) Density of sphere is $\rho = \frac{m}{V} = \frac{3m}{4\pi R^3}$

$$\Rightarrow \frac{1}{\rho} \frac{d\rho}{dt} = -\frac{3}{R} \frac{dR}{dt}$$

Since $\frac{1}{\rho} \frac{d\rho}{dt}$ is constant

$$\therefore \frac{dR}{dt} \propto R$$

Velocity of any point on the circumference v is equal to $\frac{dR}{dt}$ (rate of change of radius of outer layer).

Hence, $h \propto \frac{1}{r}$

Hence option (1) is correct. h depends on σ , hence option (2) is wrong.

If list is going up with constant acceleration.

$$g_{\text{eff}} = (g + a)$$

In this case the height of liquid in capillary

$$h = \frac{2\sigma \cos \theta}{r\rho(g+a)}, \text{ it means } h \text{ decreases}$$

It means option (3) is correct. h is proportional to $\cos \theta$ not θ , hence option (4) is wrong.

3. (1),(3),(4)

Viscous force is given by $F = -\eta A \frac{dv}{dy}$



Since h is very small therefore, magnitude of viscous force is given by

$$F = \eta A \frac{\Delta v}{\Delta y}$$

$$\therefore F = \frac{\eta A u_0}{h} \Rightarrow F \propto \eta \text{ and } F \propto u_0; \quad F \propto \frac{1}{h}, F \propto A$$

Since plate is moving with constant velocity, same force must be acting on the floor.

4. (1), (3), (4)

Height of water column in capillary tube

$$h = \frac{2T \cos \theta}{\rho g R}$$

$$\text{For tube } T_1, h_1 = \frac{2 \times 0.075 \times \cos 0^\circ}{1000 \times 10 \times 0.2 \times 10^{-3}}$$

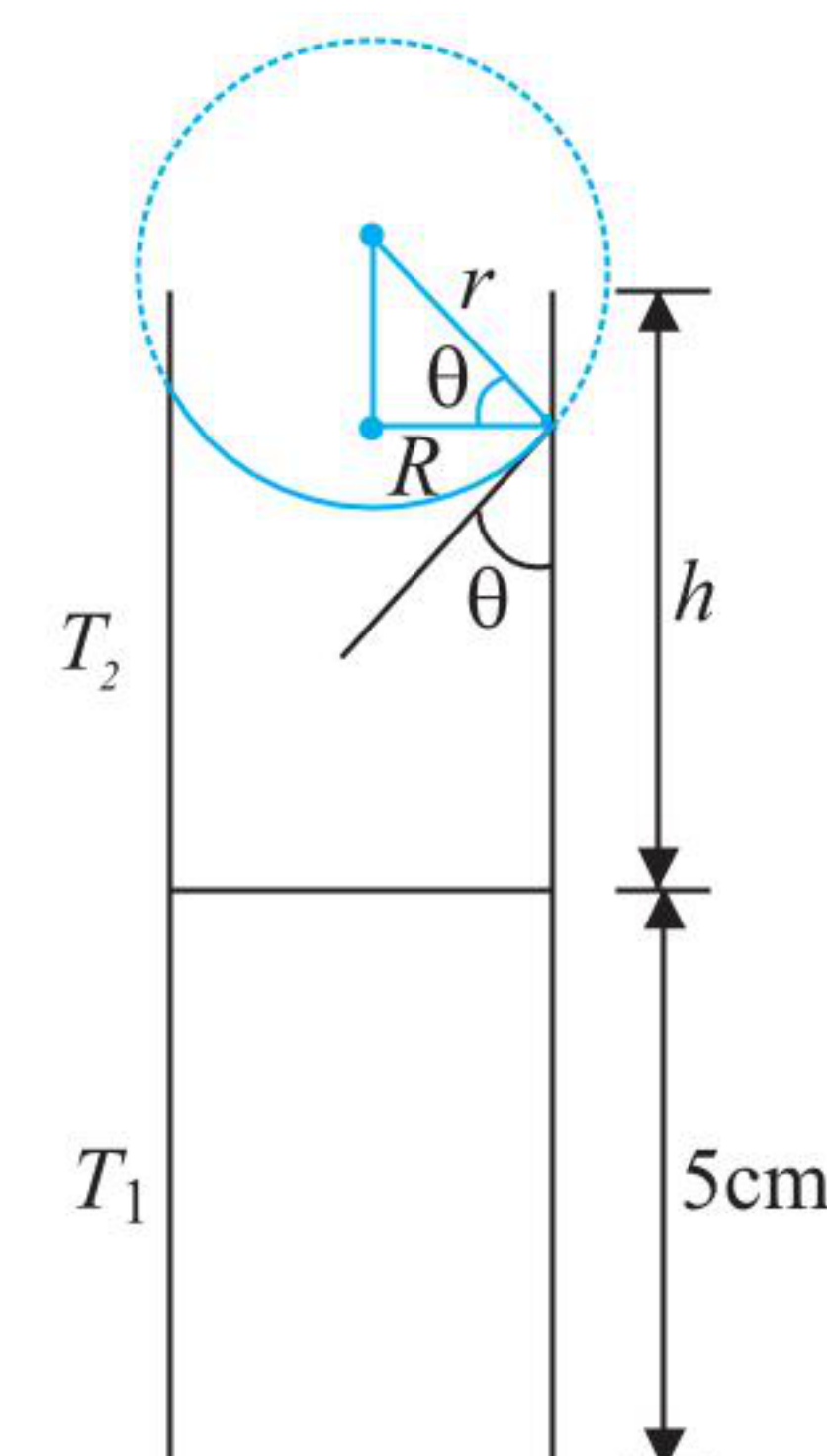
$$\Rightarrow h_1 = 75 \text{ mm (in } T_1) \text{ [If we assume entire tube of } T_1]$$

$$\text{For tube } T_2, h_2 = \frac{2 \times 0.075 \times \cos 60^\circ}{1000 \times 10 \times 0.2 \times 10^{-3}} = 37.5 \text{ mm}$$

(in T_2) [If we assume entire tube of T_2]

Option (1): Since contact angles are different so correction in the height of water column raised in the tube will be different in both the cases, so option (1) is correct.

Option (2): If joint is 5 cm above water surface, then let's say water crosses the joint by height h , then



$$\Rightarrow P_0 - \frac{2T}{r} + \rho g h + \rho g \times 5 \times 10^{-2} = P_0$$

Multiple Correct Answers Type

1. (1),(3)

Free body diagram of P and Q when they are connected by a string.

For P : $\sigma_1 V = \rho_1 V + T$

For Q : $T + \sigma_2 V = \rho_2 V$

Adding Eqs. (i) and (ii)

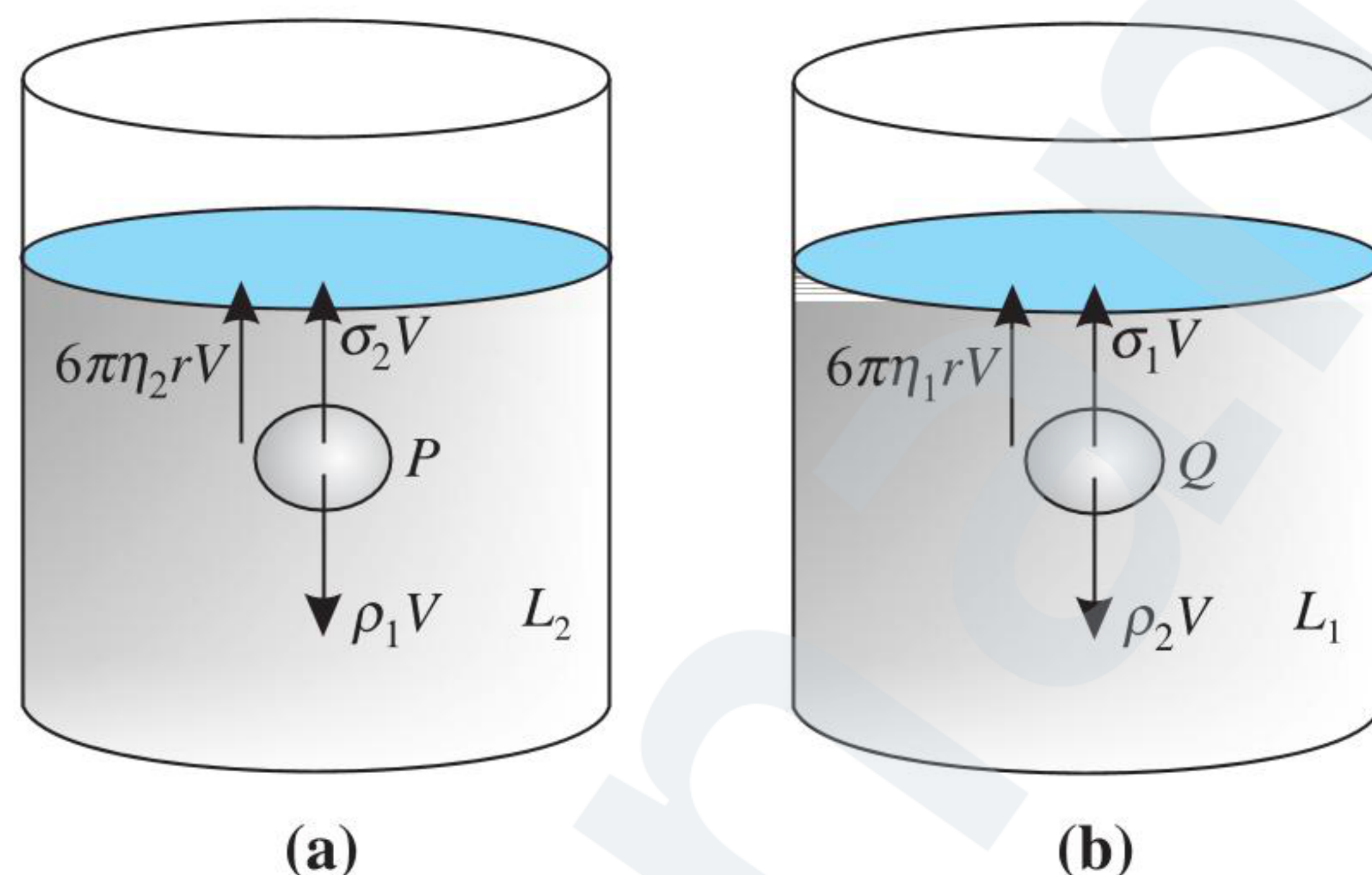
$$\sigma_1 - \rho_2 = \rho_1 - \sigma_2$$

If the sphere P along in L_2 (in Fig. (a))

Terminal velocity of P ; $V_P = \frac{2r^2}{9\eta_2}(\rho_1 - \sigma_2)$

If the sphere Q along in L_1 (in Fig. (b))

Terminal velocity of Q ; $V_Q = \frac{2r^2}{9\eta_1}(\rho_2 - \sigma_1)$



$$\left| \frac{V_P}{V_Q} \right| = \frac{\eta_1}{\eta_2} \text{ and } \vec{V}_P \cdot \vec{V}_Q < 0$$

as if $\rho_1 - \sigma_2 > 0$

$$\Rightarrow \rho_2 - \sigma_1 < 0$$

2. (1),(3)

The height of liquid in capillary is given by

$$h = \frac{2\sigma \cos \theta}{r\rho g}$$

Here $r \rightarrow$ radius of capillary; $\theta \rightarrow$ contact angle; $\sigma \rightarrow$ surface tension; and $\rho \rightarrow$ density of liquid.

For given material of capillary tube, $\theta \rightarrow$ constant

$$\Rightarrow \rho g (h \times 5 \times 10^{-2}) = \frac{2T}{r}$$

$$\Rightarrow \cos \theta = \frac{R}{r}, r = \frac{R}{\cos \theta}$$

$$\Rightarrow \rho g (h \times 5 \times 10^{-2}) = \frac{2T \cos \theta}{R}$$

$$\Rightarrow h = \frac{2 \times 0.075 \times \cos 60^\circ}{0.2 \times 10^{-3} \times 1000 \times 10} - 5 \times 10^{-2}$$

$\Rightarrow h = -ve$, not possible, so liquid will not cross the interface, but angle of contact at the interface will change, to balance the pressure,

So option (2) is wrong.

Option (3): If interface is 8 cm above water then water will not even reach the interface, and water will rise till 7.5 cm only in T_1 , so option (3) is right.

Option (4): If interface is 5 cm above the water in vessel, then water in capillary will not even reach the interface. Water will reach only till 3.75 cm, so option (4) is right.

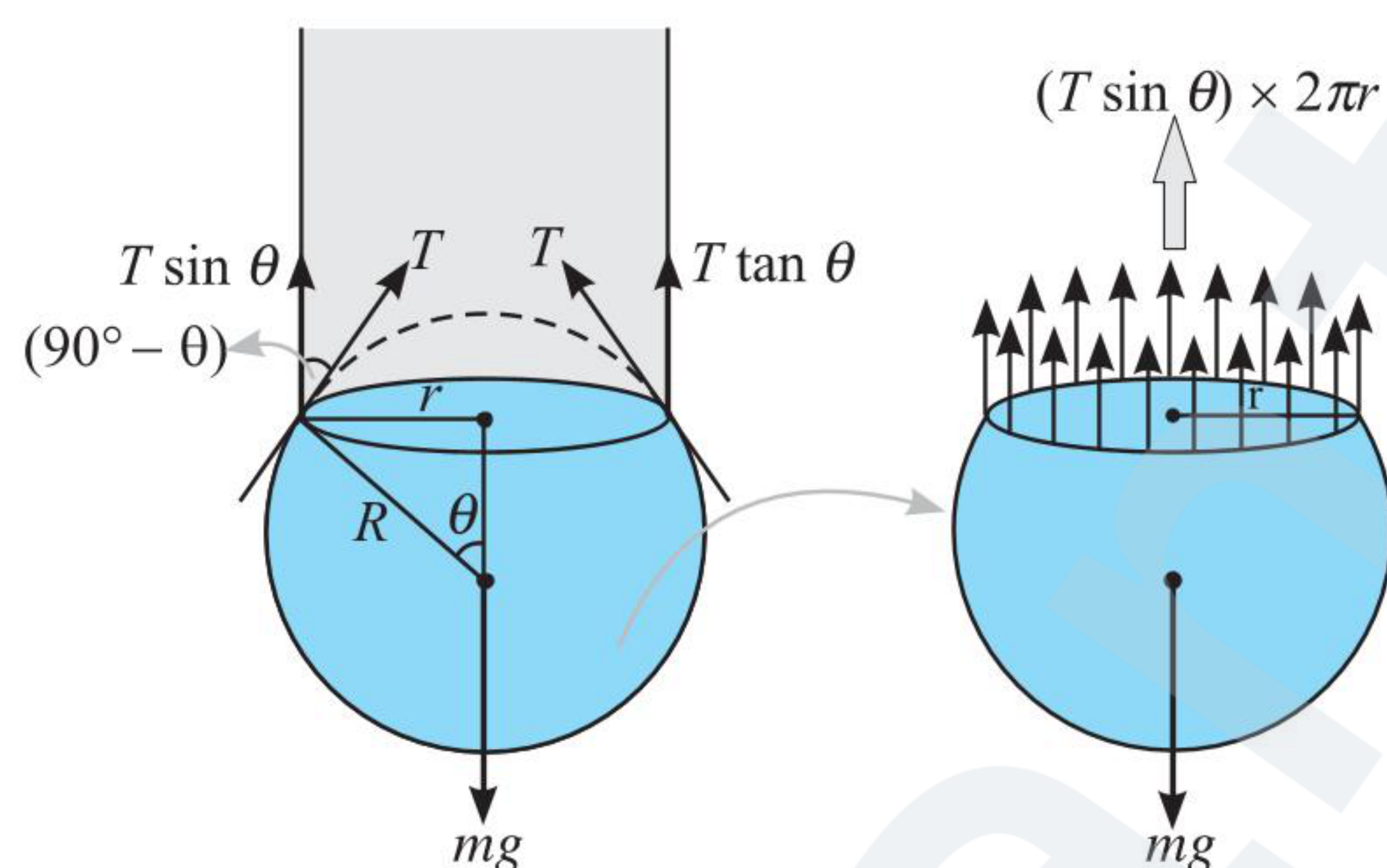
Linked Comprehension Type

1. (3) From force body diagram of liquid drop, the vertical component of surface tension on the drop is

$$(F_{\text{surface}})_{\text{vertical}} = (T \sin \theta) - 2\pi r$$

$$= T \cdot 2\pi r \sin \theta = T \cdot 2\pi r \times \frac{r}{R}$$

$$\Rightarrow (F_{\text{surface}})_{\text{vertical}} = \frac{2\pi r^2 T}{R}$$



2. (1) The liquid drop detaches from the dropper when the vertical component of the force due to surface tension equals the weight of the drop. At this time

$$\frac{2\pi r^2 T}{R} = mg = \frac{4}{3} \pi R^3 \rho g$$

$$\text{or } R^4 = \frac{3}{2} \frac{\pi^2 T}{\rho g}$$

$$\Rightarrow R = \left(\frac{3}{2} \frac{r^2 T}{\rho g} \right)^{1/4}$$

Substituting the values, we get

$$R = 1.4 \times 10^{-3} \text{ m}$$

3. (2) The surface energy of the drop

$$E_{\text{surface}} = 4\pi R^2 \times T$$

$$\text{or } E_{\text{surface}} = 4 \times \frac{22}{7} \times \pi \times (1.4 \times 10^{-3})^2 \times 0.11$$

$$\Rightarrow E_{\text{surface}} = 2.7 \times 10^{-6} \text{ J}$$

Numerical Value Type

1. (6) The excess of pressure above atmospheric pressure, due to surface tension in a bubble $\Delta p = \frac{4T}{r}$

The surrounding pressure for 1st bubble

$$P_A = P_0 + \frac{4T}{r_A} = 8 + \frac{4 \times 0.04}{0.02} = 16 \text{ N/m}^2$$

Similarly for 2nd bubble

$$P_B = P_0 + \frac{4T}{r_B} = 8 + \frac{4 \times 0.04}{0.04} = 12 \text{ N/m}^2$$

Using, $PV = nRT$

$$(16) \frac{4}{3} \pi (0.02)^3 = n_A RT \quad \dots(i)$$

$$(12) \left(\frac{4}{3} \pi (0.04)^3 \right) = n_B RT \quad \dots(ii)$$

Dividing Eq. (ii) with Eq. (i), we get $\frac{n_B}{n_A} = 6$

$$2. (3) V_T \propto \frac{r^2 [d_m - d_L]}{n}$$

$$\frac{V_{TP}}{V_{TQ}} = \left(\frac{r_P}{r_Q} \right)^2 \times \frac{n_{L_2}}{n_{L_1}} \times \left[\frac{d_m - d_{L_1}}{d_m - d_{L_2}} \right]$$

$$\frac{V_{TP}}{V_{TQ}} = \left(\frac{2}{1} \right)^2 \times \frac{2}{3} \times \left[\frac{8 - 0.8}{8 - 1.6} \right]$$

$$\frac{V_{TP}}{V_{TQ}} = 3$$

$$3. (6) \frac{4}{3} \pi R^3 = \frac{4}{3} \pi r^3 \times K \quad (\text{Conservation of volume})$$

R = radius of bigger drop

r = radius of smaller drop

$$R^3 = r^3 \times K$$

$$U_i = S (4\pi R^2)$$

$$U_f = KS (4\pi r^2)$$

$$KS (4\pi r^2) - S \times 4\pi R^2 = 10^{-3}$$

$$4\pi S [Kr^2 - R^2] = 10^{-3}$$

$$K^{1/3} R^2 - R^2 = 10^{-2}$$

$$10^{\alpha/3} - 1 = 100$$

$$10^{\alpha/3} = 101$$

$$\alpha \approx 6$$

$$4. (3.74)$$

Pressure at the bottom of disc = pressure due to surface tension

$$\rho gh = T \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$R_1 \gg R_2$$

$$\text{So } \frac{1}{R_1} \ll \frac{1}{R_2} \text{ and } R_2 = \frac{h}{2}$$

$$\therefore \rho gh = T \left[\frac{1}{R_1} + \frac{1}{R_2} \right] = T \left[0 + \frac{1}{h/2} \right]$$

$$h^2 = \frac{2T}{\rho g}$$

$$h = \sqrt{\frac{2T}{\rho g}} = \sqrt{\frac{2 \times 0.07}{10^3 \times 10}} = \sqrt{\frac{14 \times 100}{10^4 \times 100}}$$

$$h = 3.741$$

